ECERTA PROJECT

Towards the assessment of aerodynamic modelling uncertainty in aeroelastic predictions

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Overview

- Introduction
- Basic concept of framework
- Results
- Outlook

Introduction

Introduction

- Eigenvalue based stability prediction method
  and LCO response prediction method
- Development of methods to propagate structural uncertainty
  (including structural damping)
- Exploitation of eigenvalue based method to search flight envelope
  for risk of aeroelastic instability
- Exploitation of eigenvalue based method to investigate sensitivity
  of the stability to trim state and variation in the atmospheric conditions
- Assessing the uncertainty from aerodynamic models
  and updating the models with more reliable data once available

Basic concept of framework

Basic concept of framework

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➢ Four main levels of aerodynamic modelling considered

- Level 1: inviscid, irrotational and linear flow
  $\Longrightarrow$ linear potential methods (Laplace or Prandtl–Glauert equation)
- Level 2: plus nonlinear effects
  $\Longrightarrow$ nonlinear potential methods (TSD or FP equation)
- Level 3: plus rotational effects
  $\Longrightarrow$ Euler (Euler equations)
- Level 4: plus viscous and heat–conducting effects
  $\Longrightarrow$ Navier–Stokes (RANS equations plus turbulence/transition model)

Basic concept of framework

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  - Level 4: plus viscous and heat–conducting effects
    $\Longrightarrow$ Navier–Stokes (RANS equations plus turbulence/transition model)
- Basic framework conceived as follows
  - nonlinear potential model as simplest model being able to predict shock waves
  - Clebsch variable model to correct for shock generated entropy and vorticity
  - integral boundary layer model to correct for viscosity
    (estimate of the boundary layer displacement effect and representation of shallow separations)
- Issue of costs
  - compare 5 (7) unknowns of Euler (RANS) model with 2 unknowns of FP
  - additional models (Clebsch and BL) only add little to costs

Basic concept of framework

- Interaction of fluid and structure

$$D\frac{d\boldsymbol{w}}{dt} = \boldsymbol{R}, \qquad \boldsymbol{w} = \left[\boldsymbol{w}_f, \boldsymbol{w}_s\right]^T, \ \boldsymbol{R} = \left[\boldsymbol{R}_f, \boldsymbol{R}_s\right]^T$$

- To solve use Newton's method
  - steady state solution $\boldsymbol{w_0}$ given by vanishing residual $\boldsymbol{R}(\boldsymbol{w_0}) = 0$:

$$A\,\delta\boldsymbol{w}^{\nu} = -\boldsymbol{R}(\boldsymbol{w}^{\nu})$$

  - unsteady simulation by dual time–stepping with pseudo–residual $\boldsymbol{R}^* = \boldsymbol{R}(\boldsymbol{w}) - D\frac{d\boldsymbol{w}}{dt}$:

$$\left(A - D\frac{3}{2\Delta t}\right)\delta\boldsymbol{w}^{\nu} = -\boldsymbol{R}^*(\boldsymbol{w}^{\nu})$$

  - eigenvalue approach

$$A\boldsymbol{p} = \lambda D\,\boldsymbol{p}$$

Basic concept of framework

- Jacobian matrix $A = \dfrac{\partial \boldsymbol{R}}{\partial \boldsymbol{w}}$
- Conveniently partitioned in blocks expressing the dependencies

$$A = \left(\begin{array}{c||c} A_{ff} & A_{fs} \\ \hline\hline A_{sf} & A_{ss} \end{array} \right)$$

- Fluid feels the motion (location, speed) of the structure
  Structure feels the pressure distribution of the surrounding fluid
- To address uncertainties in aerodynamic modelling, look at fluid part

Basic concept of framework

- Starting point: unsteady full potential model plus circulation convection
  - continuity equation with velocity $\boldsymbol{q} = \nabla\phi$
  - density relation derived from unsteady Bernoulli equation
  - circulation convection to model unsteady shedding of vorticity

$$D_{pp}\frac{d\boldsymbol{w}_p}{dt} = \boldsymbol{R}_p, \qquad \boldsymbol{w}_p = [\rho, \phi, \Gamma]^T,$$

$$\boldsymbol{R}_p = \begin{pmatrix} -\nabla \cdot (\rho\nabla\phi) \\ \dfrac{1-q^2}{2} - \dfrac{\rho^{\gamma-1}-1}{(\gamma-1)\,M_r^2} \\ -u\,\Gamma_\xi \end{pmatrix}$$

- Limitation: no strong shock waves, no viscous effects

Basic concept of framework

- Viscous effects modelled by integral boundary layer model
  - two equation dissipation–type closure model plus stress transport equation
  - fixed transition (original model contains free transition model)
  - used for free wakes by setting skin friction to zero
  - fully–simultaneously coupled with inviscid solver

$$D_{vv}\frac{d\boldsymbol{w}_v}{dt} + D_{vp}\frac{d\boldsymbol{w}_p}{dt} + D_{ps}\frac{d\boldsymbol{w}_s}{dt} = \boldsymbol{R}_v$$

$$\boldsymbol{w}_v = \left[\delta^*, \theta, \tilde{C}_\tau\right]^T, \ \boldsymbol{w}_s = \left[h, \dot{h}, \alpha, \dot{\alpha}\right]^T, \ \boldsymbol{R}_v = \left[\mathcal{R}_{\delta*}, \mathcal{R}_\theta, \mathcal{R}_{\tilde{C}_\tau}\right]^T$$

$$\mathcal{R}_{\delta*} = -u_e\left(\frac{\partial \ln\theta}{\partial \ln\xi} + \left(H + 2 - Ma_e^2\right)\frac{\partial \ln u_e}{\partial \ln\xi} - \frac{\xi}{\theta}\frac{C_f}{2}\right)\frac{\theta}{\xi}$$

$$\mathcal{R}_\theta = -u_e\left(\frac{\partial \ln H^*}{\partial \ln\xi} + \frac{\partial \ln\theta}{\partial \ln\xi} + \left(2H^{**} + H^*\left(3 - Ma_e^2\right)\right)\frac{1}{H^*}\frac{\partial \ln u_e}{\partial \ln\xi} - \frac{\xi}{\theta}\frac{2C_D}{H^*}\right)\frac{\theta}{\xi}H^* - \mathcal{R}_{\delta*}$$

$$\mathcal{R}_{\tilde{C}_\tau} = -u_e\left(\frac{\partial \ln\tilde{C}_\tau}{\partial \ln\xi} - \xi\frac{K_c}{2\delta}\left(\tilde{C}_{\tau eq} - a_2\tilde{C}_\tau\right) - \xi\, Q_{eq} - \frac{\partial \ln u_e}{\partial \ln\xi}\right)\frac{u}{u_e}\frac{\tilde{C}_\tau}{\xi}.$$

Basic concept of framework

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- Modification to Jacobian matrix

$$A = \left(\begin{array}{c||c} A_{ff} & A_{fs} \\ \hline\hline A_{sf} & A_{ss} \end{array}\right) \Longrightarrow A = \left(\begin{array}{c|c||c} A_{pp} & A_{pv} & A_{ps} \\ \hline A_{vp} & A_{vv} & A_{vs} \\ \hline\hline A_{sp} & A_{sv} & A_{ss} \end{array}\right)$$

  - A_{pv}: displacement effect of BL modelled by blowing velocity, $v_n \approx u_e \dfrac{d\delta^*}{d\xi}$
  - A_{sv}: zero in current formulation
  - A_{vp}, A_{vs}: BL feels inviscid edge solution, ϕ_e and ρ_e

- fully–simultaneous inviscid/viscous coupling matrix in upper–left 2×2 block

Basic concept of framework

➢ Vorticity and entropy effects added by Clebsch variable formulation

- derived from continuity, unsteady Crocco equation and entropy equation
- velocity rewritten as $\boldsymbol{q} = \nabla\phi + S\nabla\psi$ with S as entropy and ψ as Clebsch variable
- two convection equations

$$D_{cc}\frac{d\boldsymbol{w}_c}{dt} = \boldsymbol{R}_c, \qquad \boldsymbol{w}_c = \left[S, \psi\right]^T$$

$$\boldsymbol{R}_c = \begin{pmatrix} -\boldsymbol{q}\cdot\nabla S \\ \rho^{\gamma-1} - \boldsymbol{q}\cdot\nabla\psi \end{pmatrix}$$

- upstream boundary condition: define location and speed of shock wave

Basic concept of framework

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  - upstream boundary condition: define location and speed of shock wave

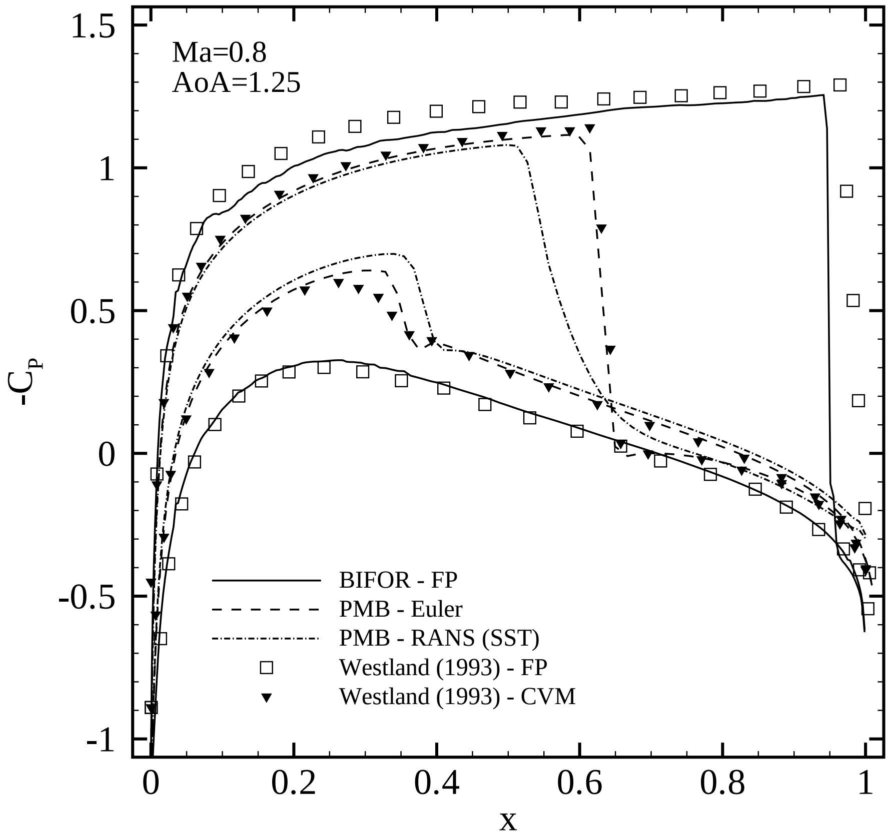
- Implementation
  - derivatives along streamlines could be approximated by derivatives in x–direction
  - streamlines could be defined from initial isentropic calculation, and auxiliary grid defined for solution of extra two variables
  - full convection equations could be solved on the same grid used for the full potential equations

- Modification to Jacobian matrix

$$\Longrightarrow \qquad A = \left(\begin{array}{c|c|c||c} A_{pp} & A_{pv} & A_{pc} & A_{ps} \\ \hline A_{vp} & A_{vv} & A_{vc} & A_{vs} \\ \hline A_{cp} & A_{cv} & A_{cc} & A_{cs} \\ \hline\hline A_{sp} & A_{sv} & A_{sc} & A_{ss} \end{array}\right)$$

Basic concept of framework

- NACA 0012 aerofoil at Mach 0.8 and incidence of 1.25 degree



Pressure distribution

Basic concept of framework

- Study of uncertainty (boundary layer model)
  - approximation of skin friction and velocity profile is needed for attached and separated boundary layers
  - found by empiricism from experimental data (high–fidelity CFD results)
  - used to close system of BL equations

  - adjust skin friction and velocity profile and investigate influence on aeroelastic stability
- Study of uncertainty (Clebsch model)
  - influence of approximation to implement the two convection equations

Results

- Forced motion
- Free motion
- Inviscid/viscous interaction
- Eigenvalue approach

Results

- Forced motion
- Oscillatory pitching motion about quarter chord

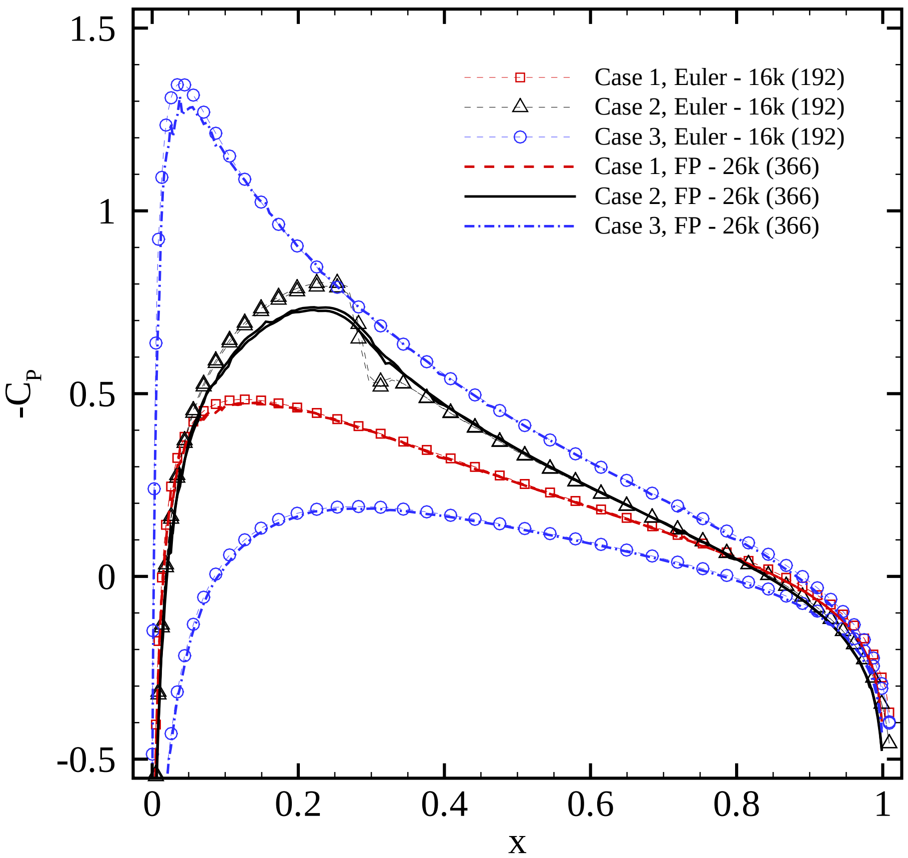
$$\alpha(t) = \alpha_m + \alpha_0 \sin(\omega t) \qquad (\omega = 2k)$$

- 3 cases

	k	Ma	$Re \times 10^6$	α_m	α_0
case 1	0.1000	0.500	–	0.000	2.00
case 2 (AGARD CT 5)	0.0814	0.755	5.5	0.016	2.51
case 3 (AGARD CT 1)	0.0808	0.600	4.8	2.890	2.41

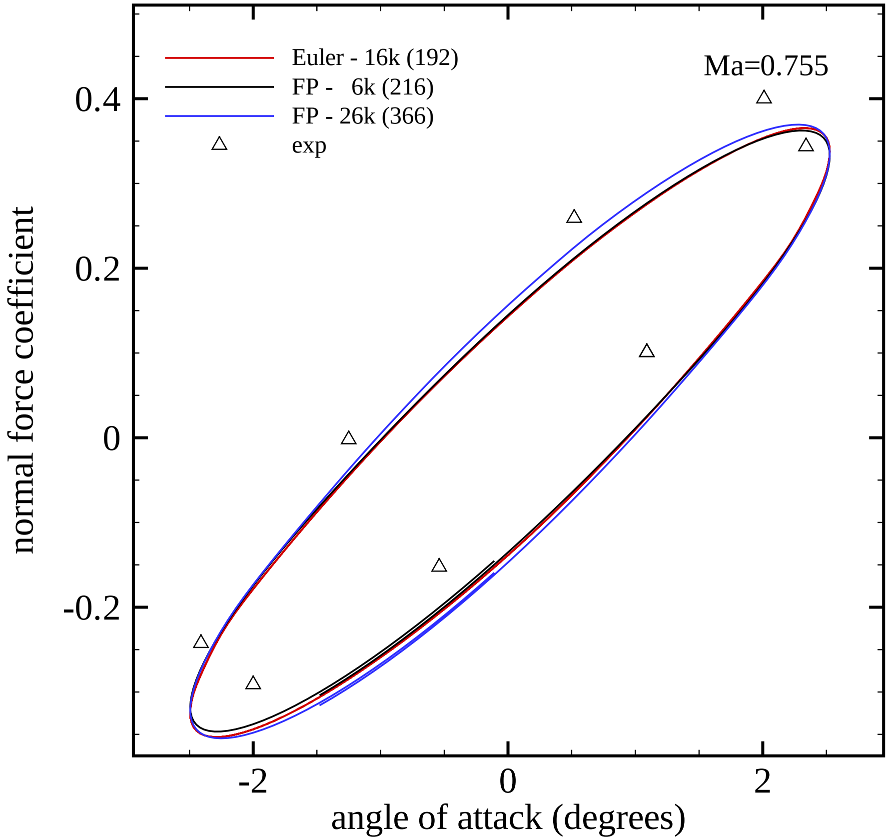
Results

- Forced motion – steady state results

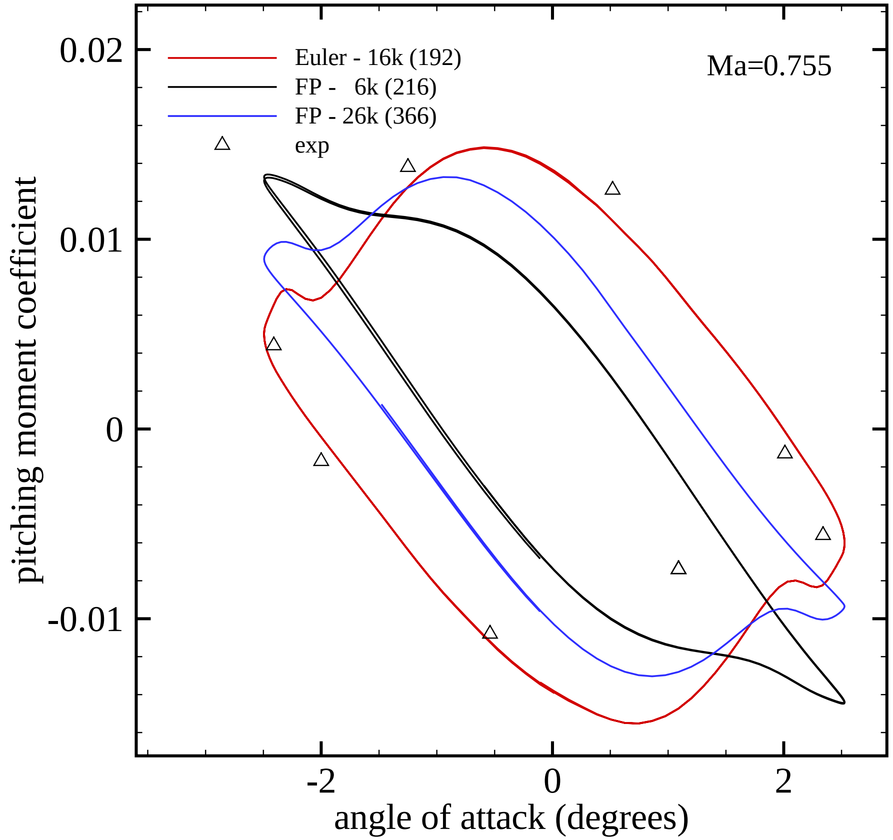


Steady state pressure coefficient

- Forced motion – case 2



Normal force coefficient

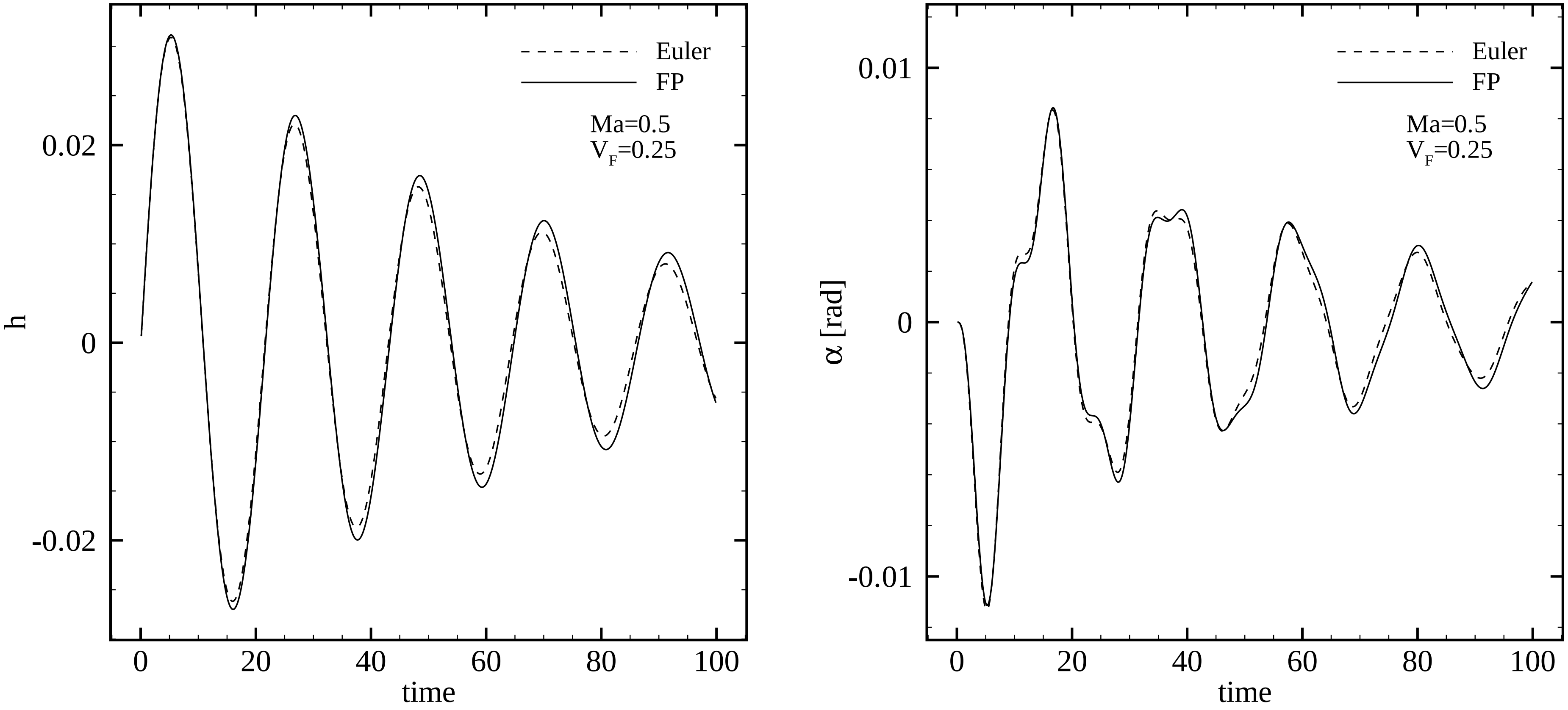


Pitching moment coefficient

Results

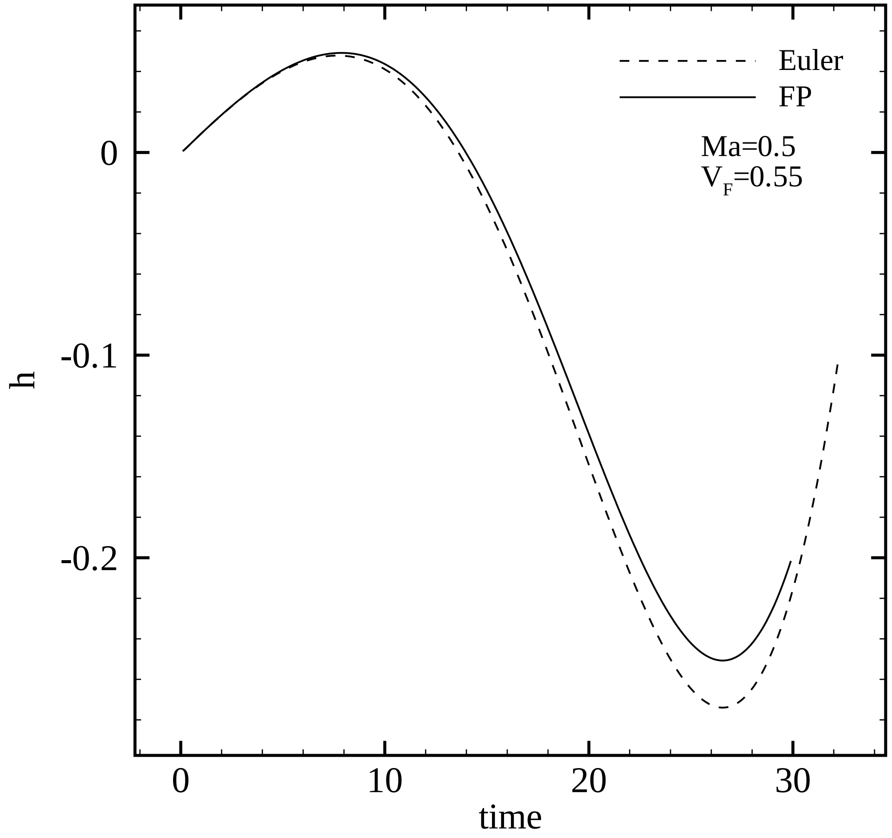
- Free motion
- NACA 0012 aerofoil configuration
- Parameters
  - elastic axis $x_{ea} = 0.4$
  - offset between center of gravity and elastic axis $x_\alpha = -0.2$
  - radius of gyration about the elastic axis $r_\alpha = 0.539$
  - aerofoil to fluid mass ratio $\mu_s = 100$
  - ratio of natural frequencies $\omega_r = 0.343$

- Free motion – $\bar{U} = 2.5$

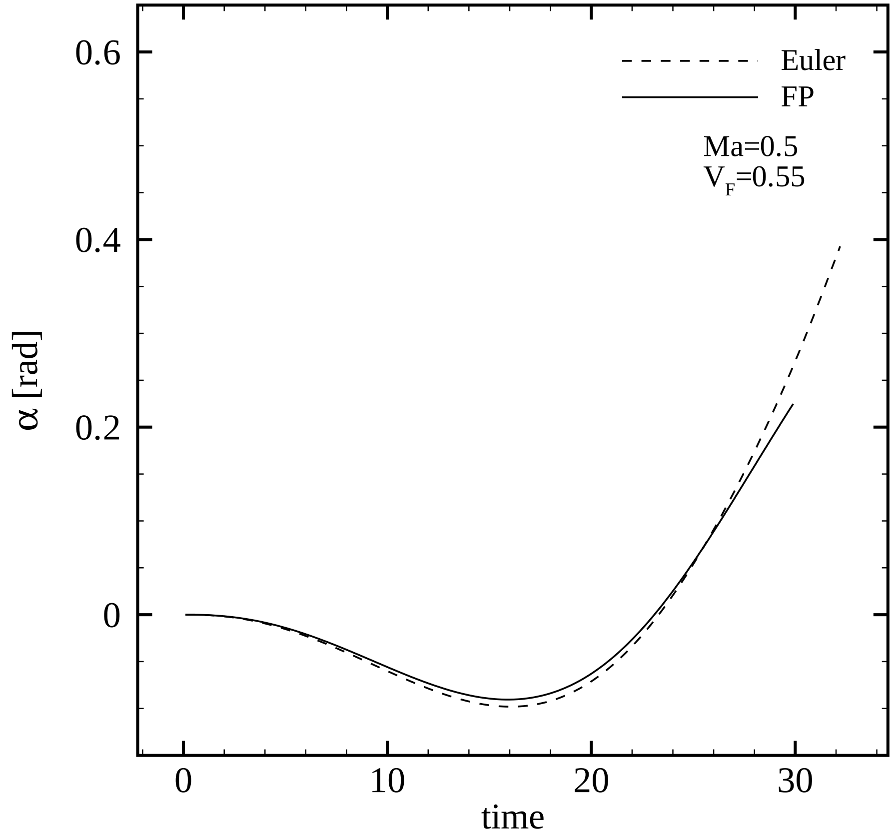


Plunge Pitch

- Free motion – $\bar{U} = 5.5$



Plunge

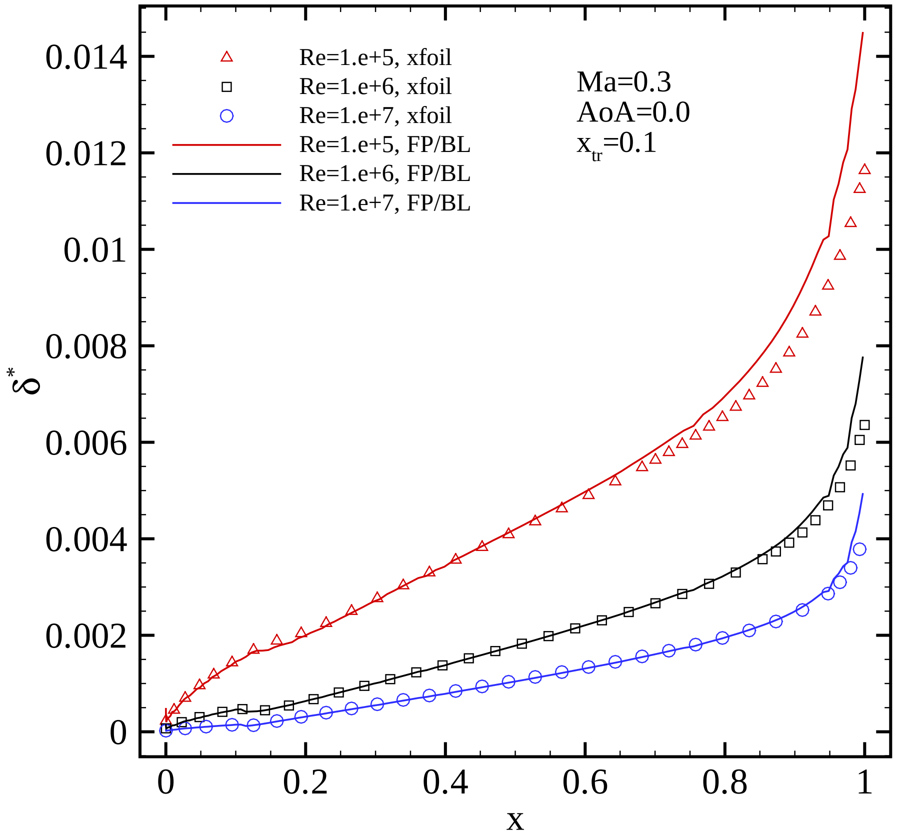


Pitch

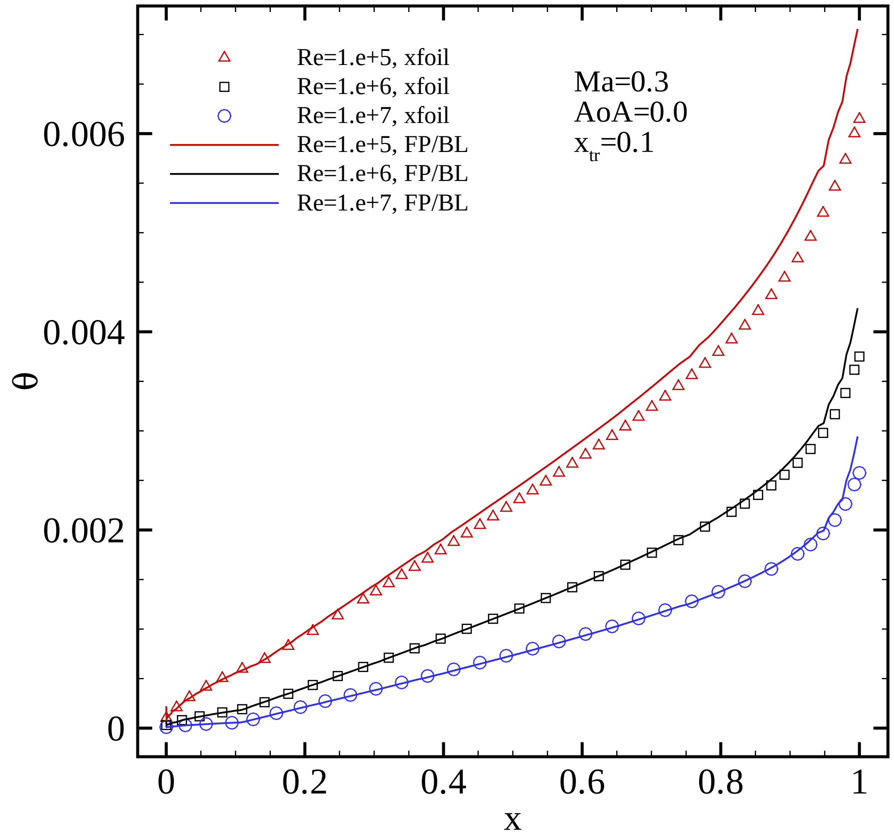
Results

- Inviscid/viscous interaction
  - Steady state results
  - Variation of
    - Mach numbers
    - Reynolds numbers
    - angle of attack
  - Results compared to Xfoil

- Inviscid/viscous interaction



Displacement thickness

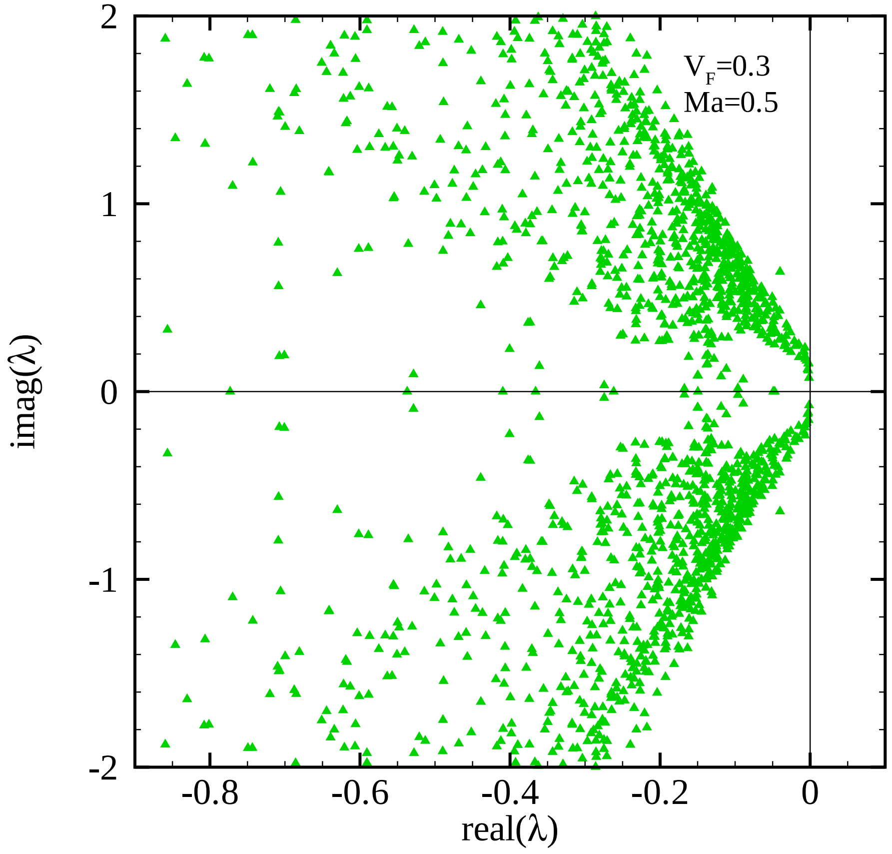


Momentum thickness

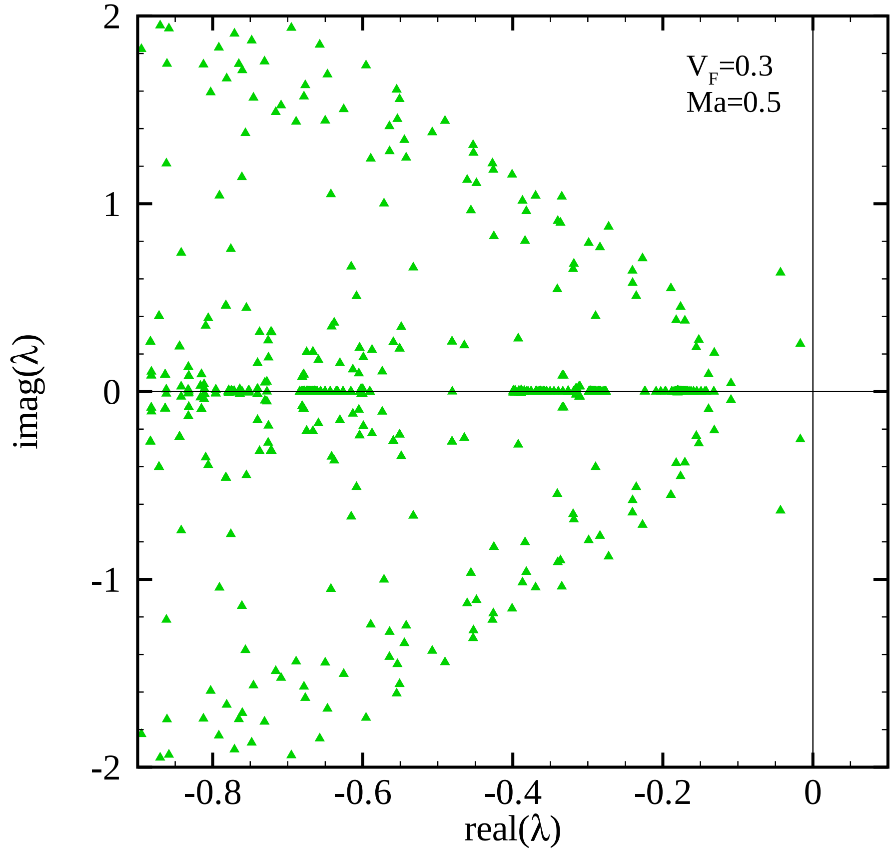
Results

- Eigenvalue approach
  - Complete eigenspectra calculated in Matlab and compared to Schur method
  - Instability boundaries
  - NACA 0012 aerofoil configuration, FP and Euler

- Eigenspectrum for NACA 0012 (medium view)



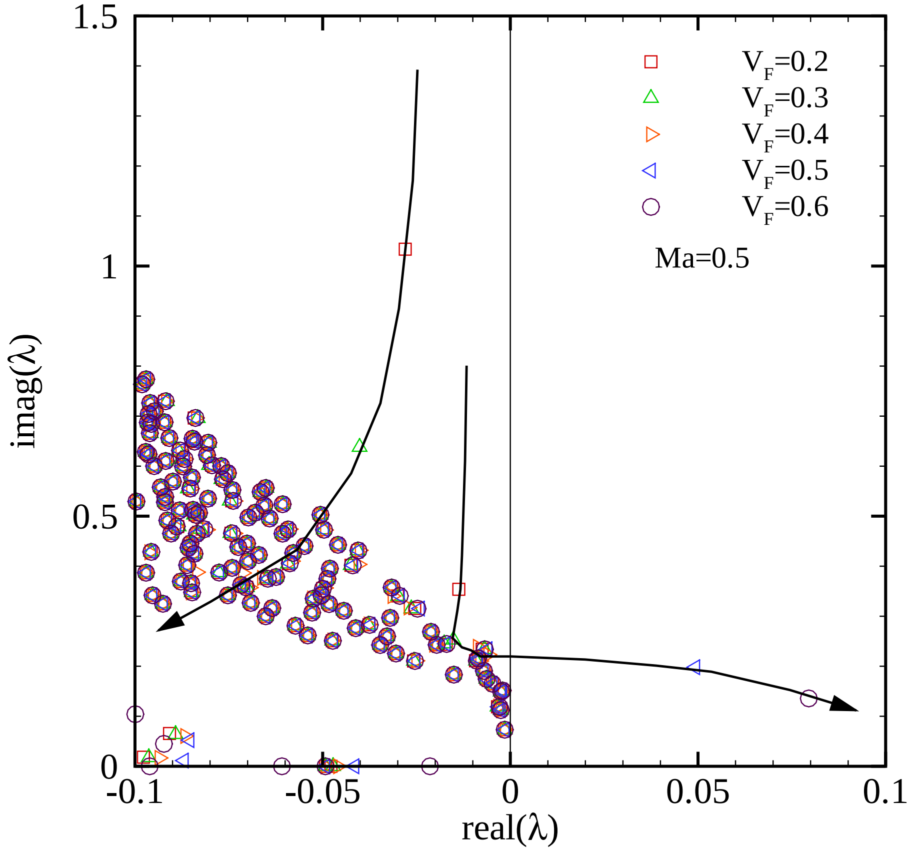
Full potential formulation



Euler formulation

Results

- Eigenspectrum for NACA 0012 (closeup view)



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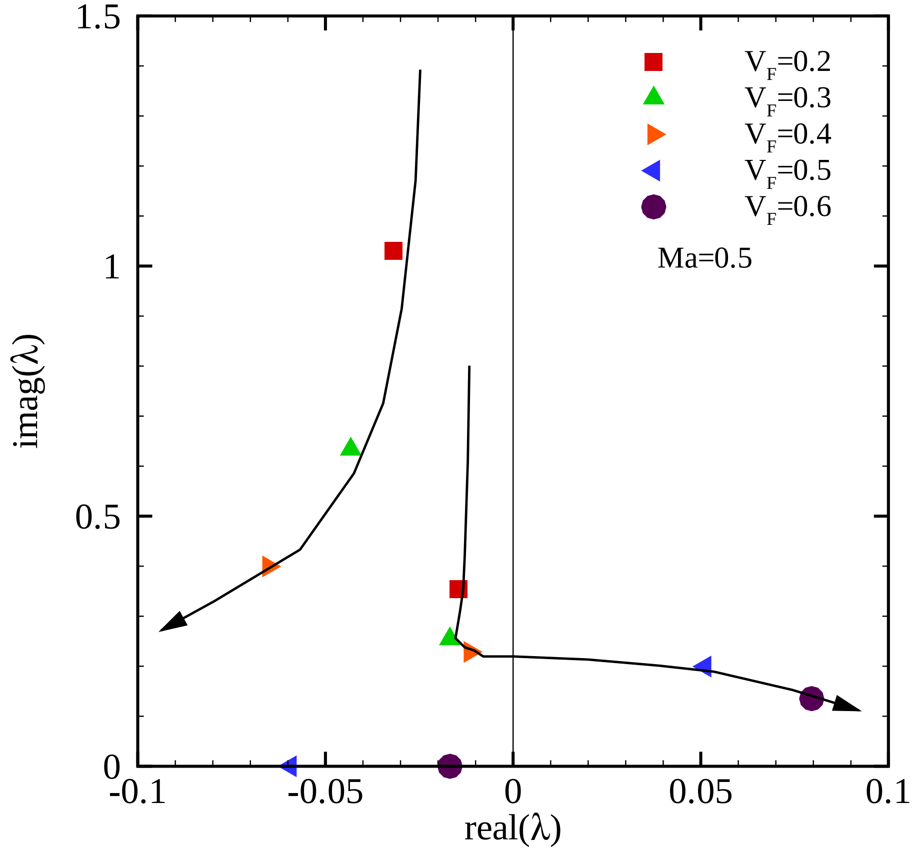

Euler formulation

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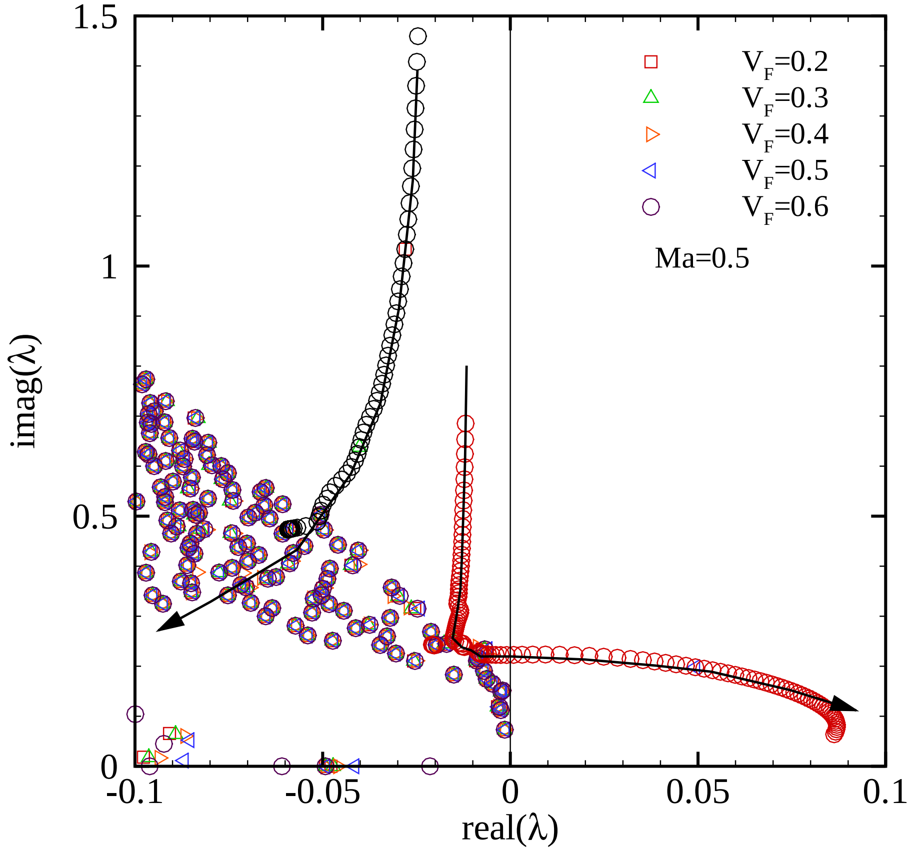
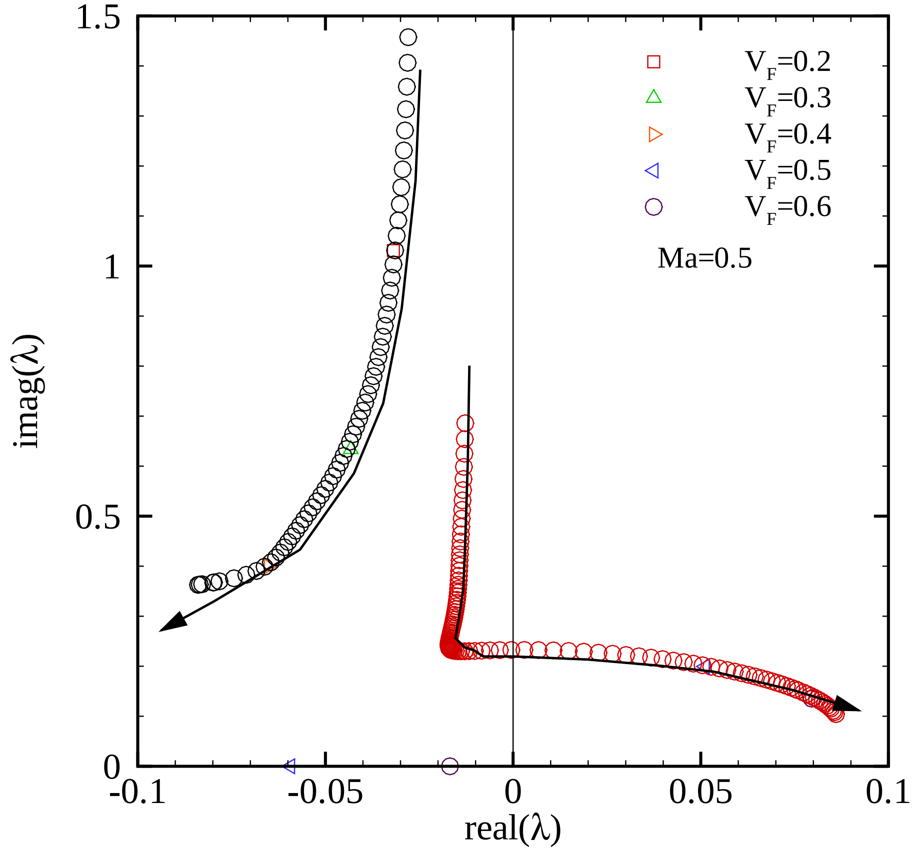
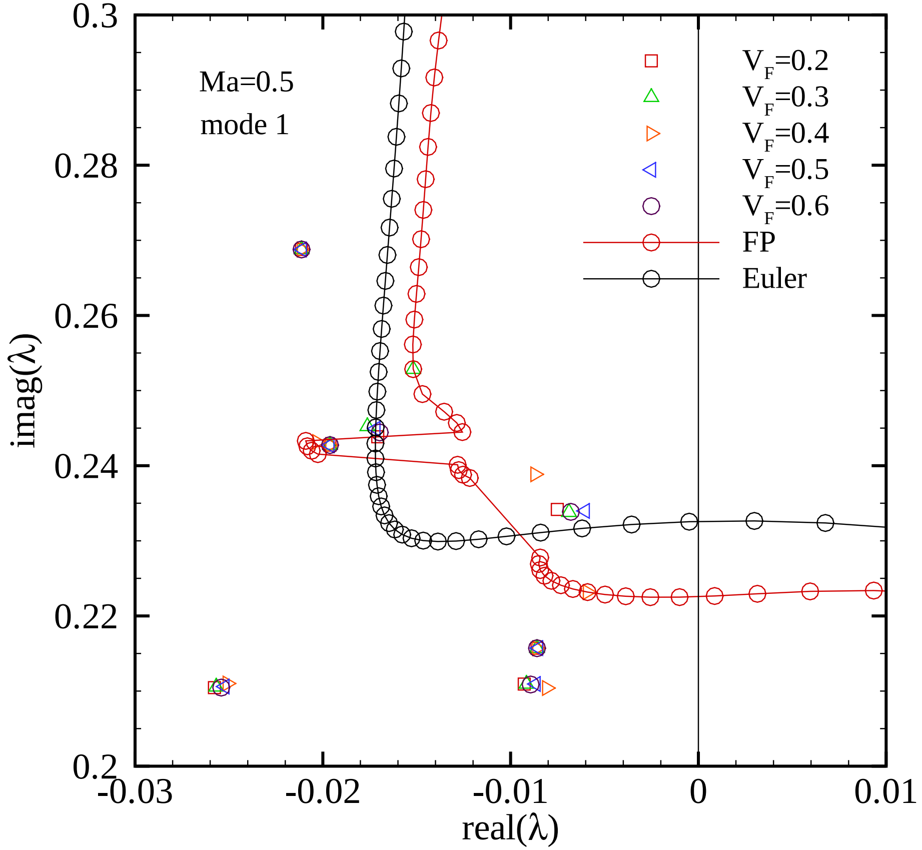


Full potential formulation

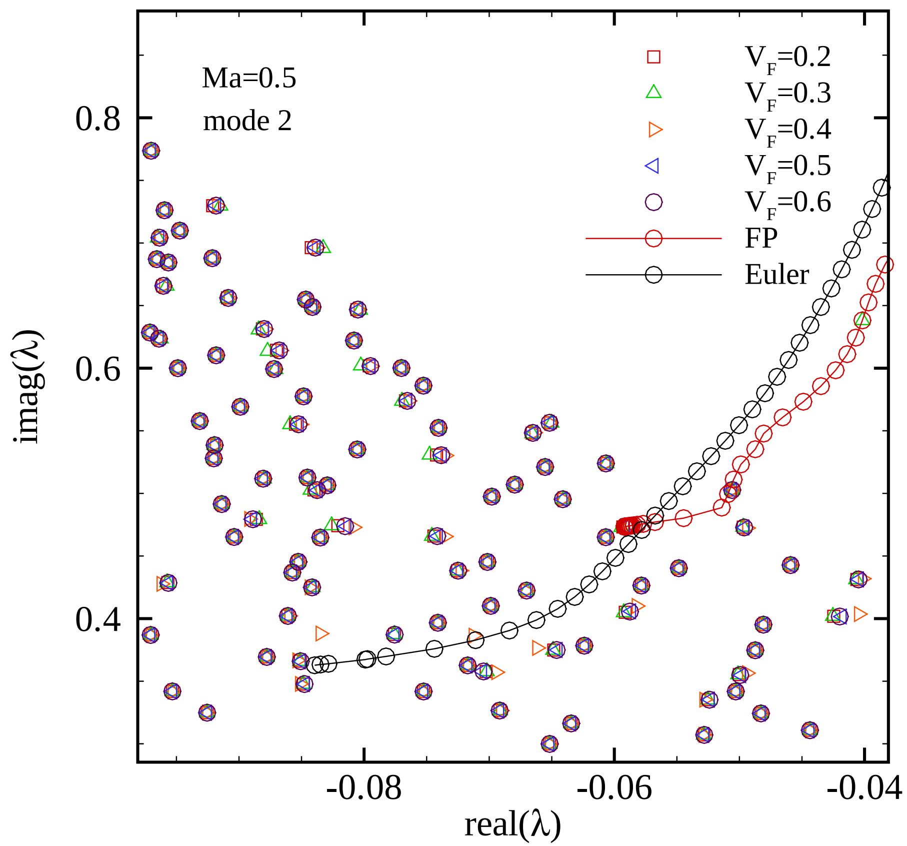


Euler formulation

- Eigenspectrum for NACA 0012 (details)

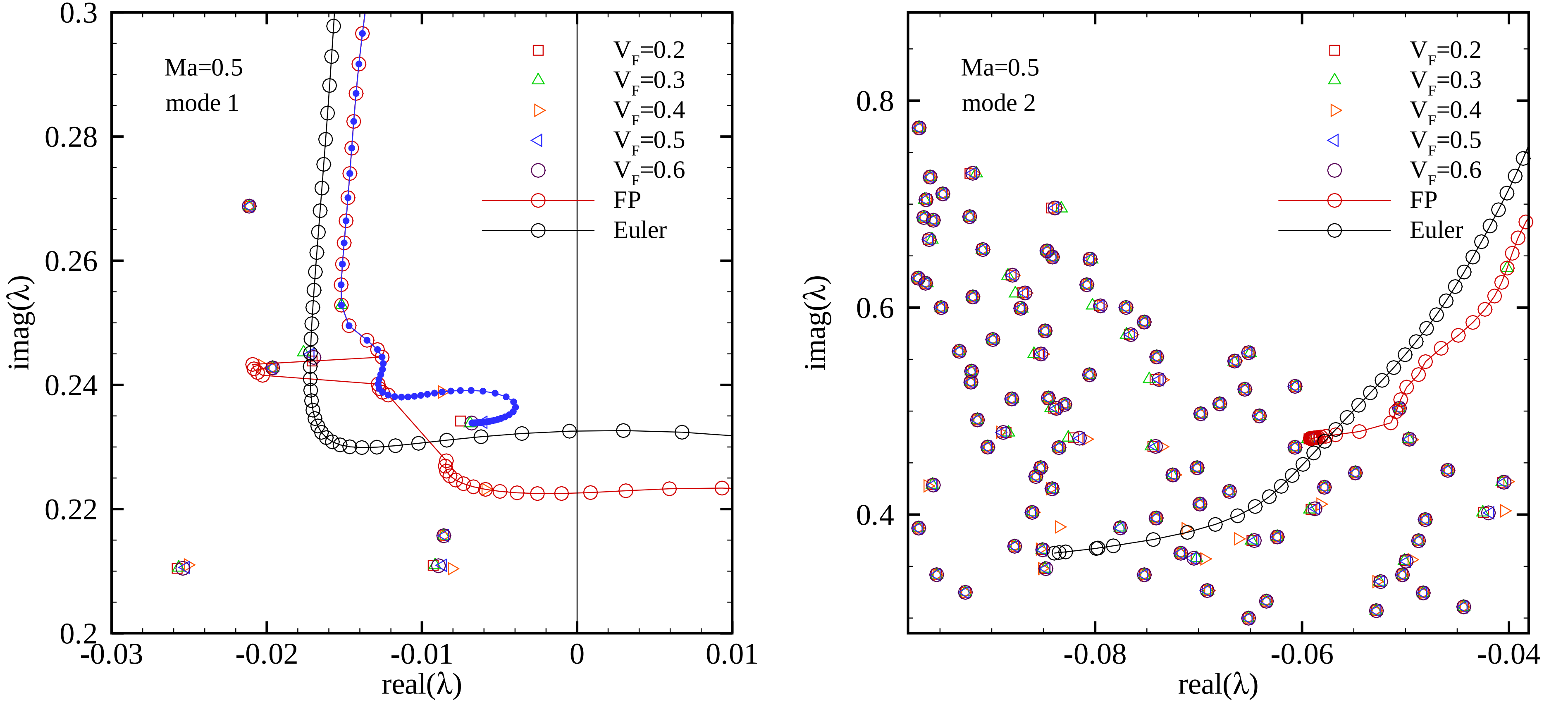


mode 1



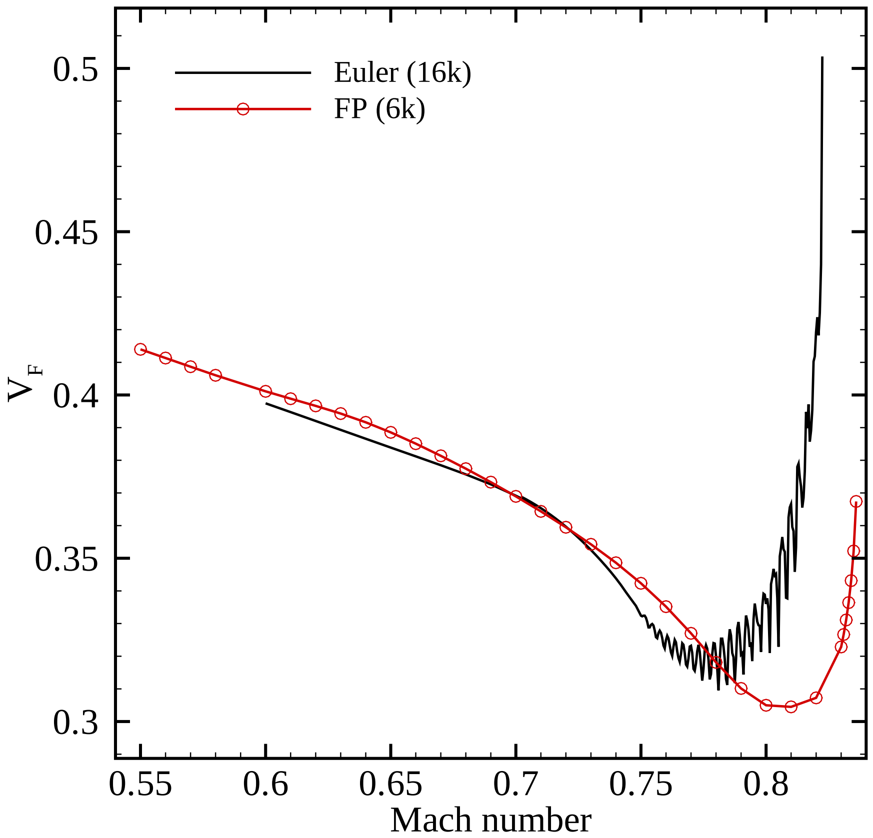
mode 2

- Eigenspectrum for NACA 0012 (details)

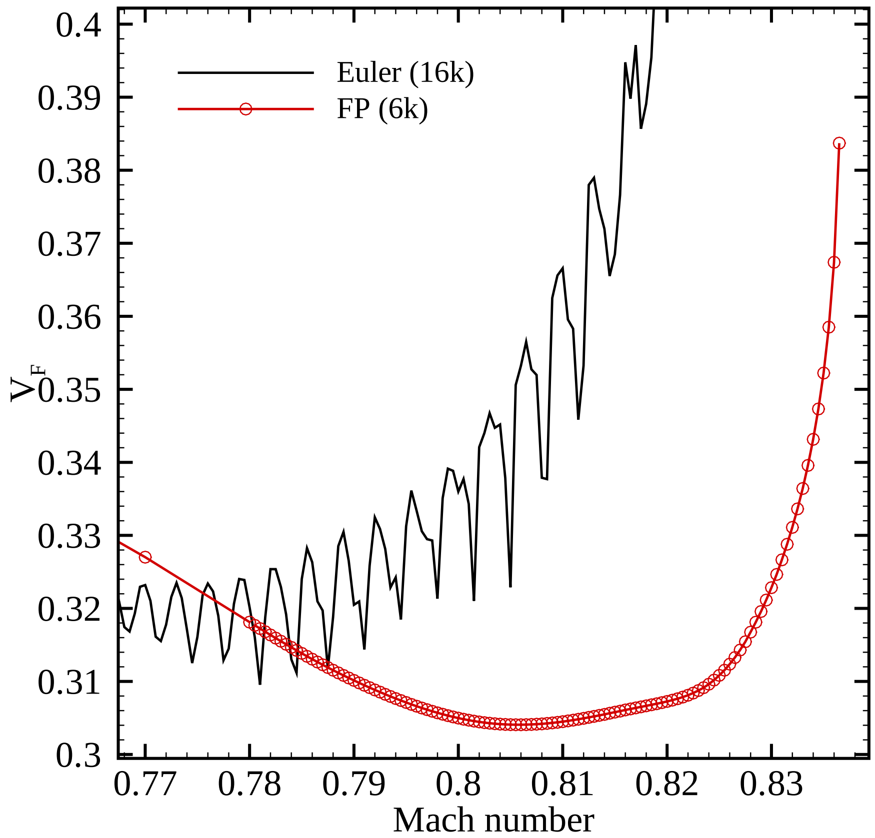


mode 1 mode 2

- Instability boundary for NACA 0012 configuration



Instability boundary



Instability boundary (detail)

Outlook

Outlook

- Improve FP spatial discretisation scheme
- Implement Clebsch variable formulation
- Implement required changes in BL formulation to simulate separated regions
- 'Open' BL closure relations to address uncertainty

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