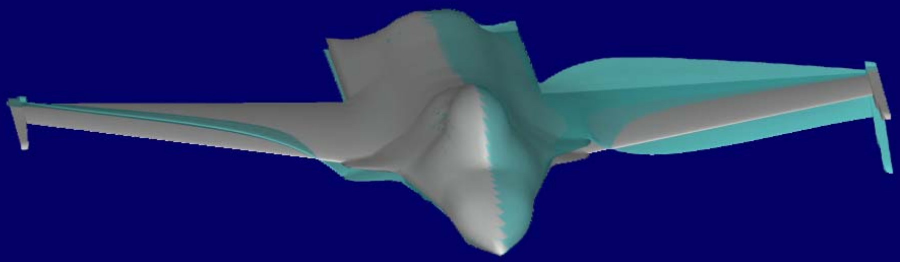
International Workshop on Fluid-Structure Interaction:
Theory, Numerics and Applications - Herrsching, September 29 - October 1, 2008



Transonic Flutter Predictions for a Generic Fighter Configuration

S. Marques, H. Khodaparast,
K. Badcock, J. Mottershead

E C E R T A – Enabling Certification by Analysis

UNIVERSITY OF LIVERPOOL

Marie Curie
Excellence Team

LABORATORY

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CONTENTS

- Motivation
- Schur Complement Method
- Test Cases Description
  - Goland Wing
  - Generic Fighter Configuration
    - Aerodynamics Updating
    - Structural Updating
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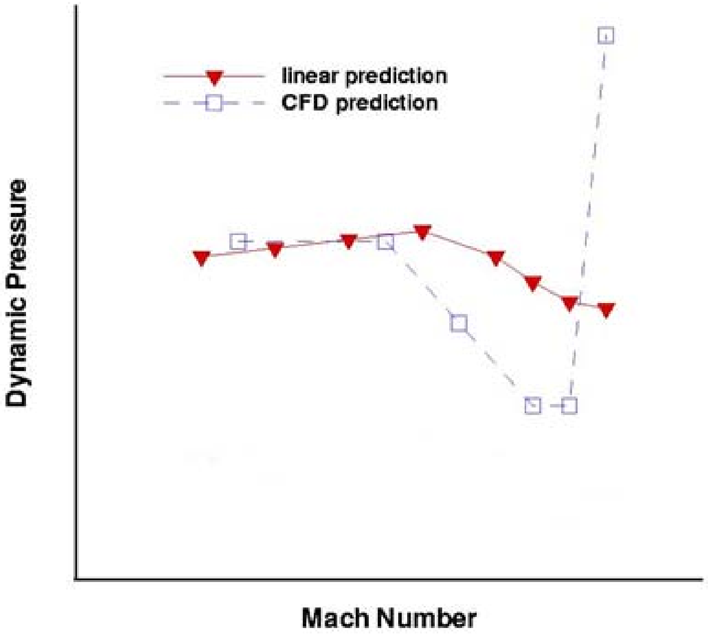
- Conclusions

MOTIVATION

F-16 LCO

MOTIVATION

F-16 LCO



Transonic Flutter Boundary

SCHUR METHOD

- The coupled CFD-CSD system can be described as:

$$\frac{d\mathbf{w}}{dt} = \mathbf{R}(\mathbf{w}, \mu)$$

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μ – Bifurcation Parameter

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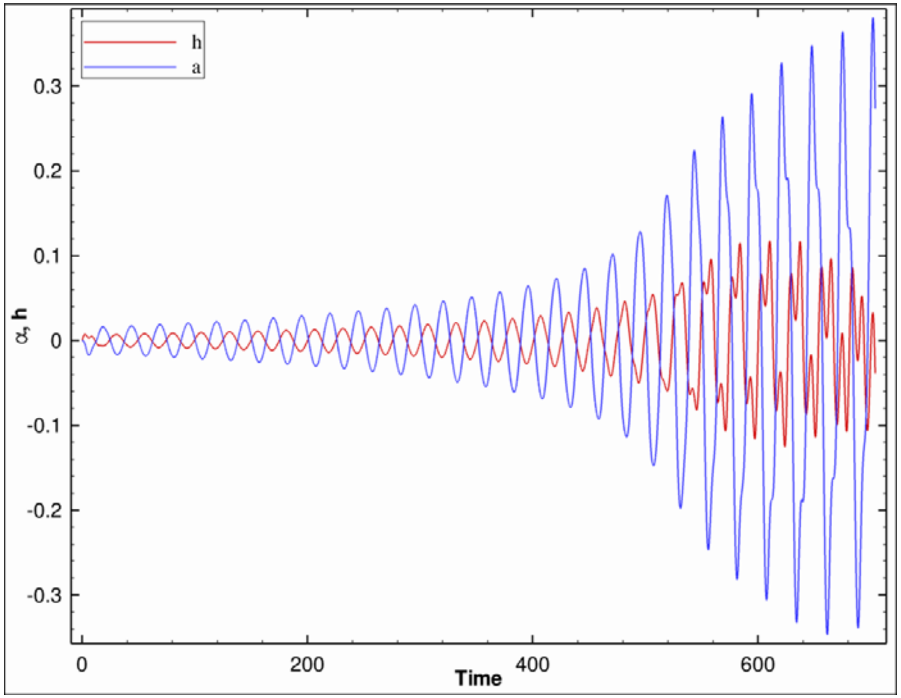

Pitch and Plunge Aerofoil Case

Computationally very intensive
Impractical for flight envelope search

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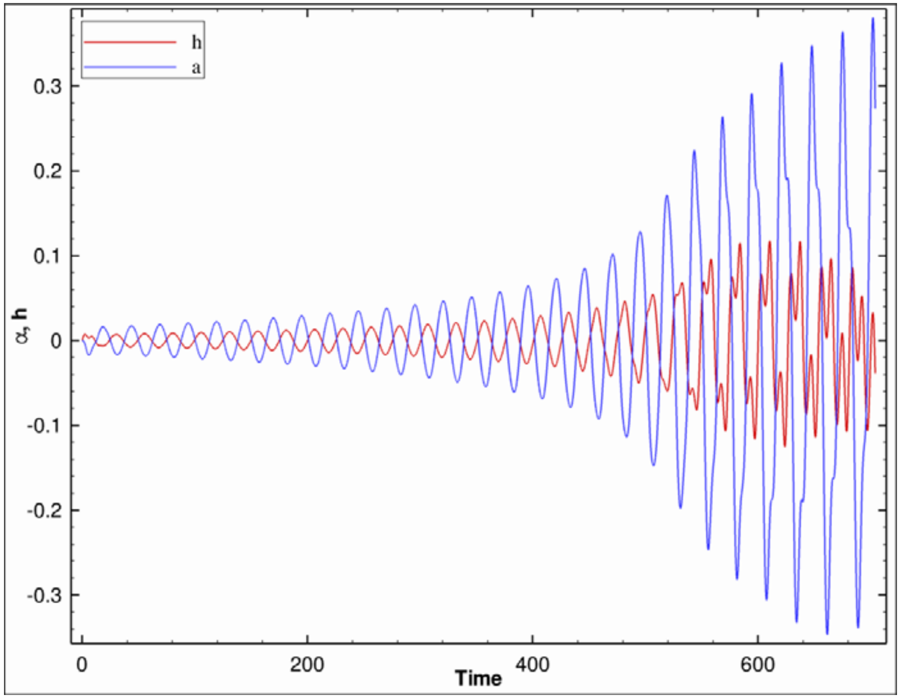

Pitch and Plunge Aerofoil Case

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However…

SCHUR METHOD

- The eigenvalue problem can be written as:

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \begin{bmatrix} p_f \\ p_s \end{bmatrix} = \lambda \begin{bmatrix} p_f \\ p_s \end{bmatrix}$$

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SCHUR METHOD

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- Shifted Inverse Power Method
  - –System becomes ill-conditioned
  - –Solving in Parallel Difficult

$$z_k = \begin{bmatrix} A_{ff} - \lambda_0 I & A_{fs} \\ A_{sf} & A_{ss} - \lambda_0 I \end{bmatrix}^{-1} x_{k-1}$$

Badcock et al, AIAA J, 45(6), 1370-1381,2007

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- Schur Complement formulation:

$$S(\lambda) p_s = \lambda p_s$$

$$S(\lambda) = A_{ss} - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}$$

λ is not an eigenvalue of A_{ff}

Bekas and Saad, SIAM Journal of Scientific Computing 27(2) 458, 2005

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New formulation for Non-linear Eigenvalue Problem

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$$\mathbf{u} = [\mathbf{p}_s \;\; \lambda]^T$$

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Full Evaluation, Expensive

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Full Evaluation, Expensive

But $$(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1} + \lambda^2 A_{ff}^{-1} A_{ff}^{-1} A_{ff}^{-1} + \dots$$

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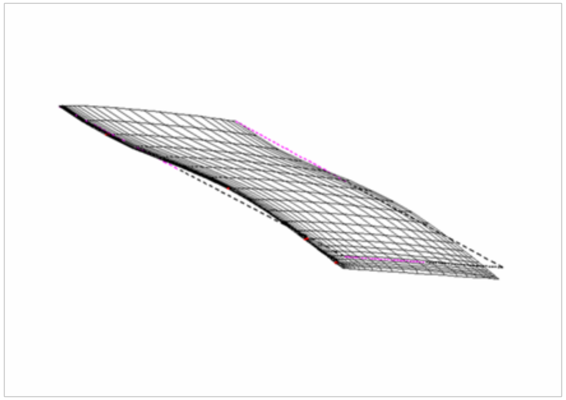
But $(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1} + \lambda^2 A_{ff}^{-1} A_{ff}^{-1} A_{ff}^{-1} + \dots$

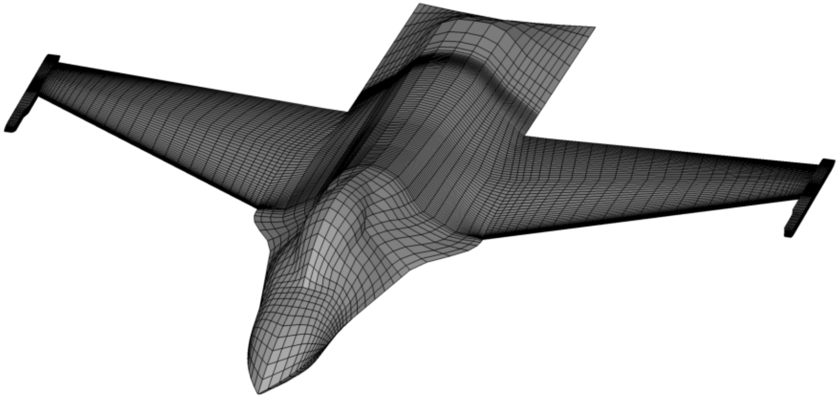
Pre-Compute $A_{sf} A_{ff}^{-1} A_{fs}$ and $A_{sf} A_{ff}^{-2} A_{fs}$

Badcock and Woodgate, AIAA paper 2008-1820, 2008

TEST CASE

- Two test cases are used to demostrate the method presented here:
  - Goland Wing
  - Generic Fighter Configuration

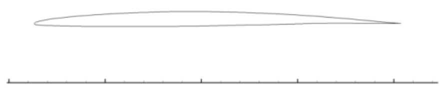


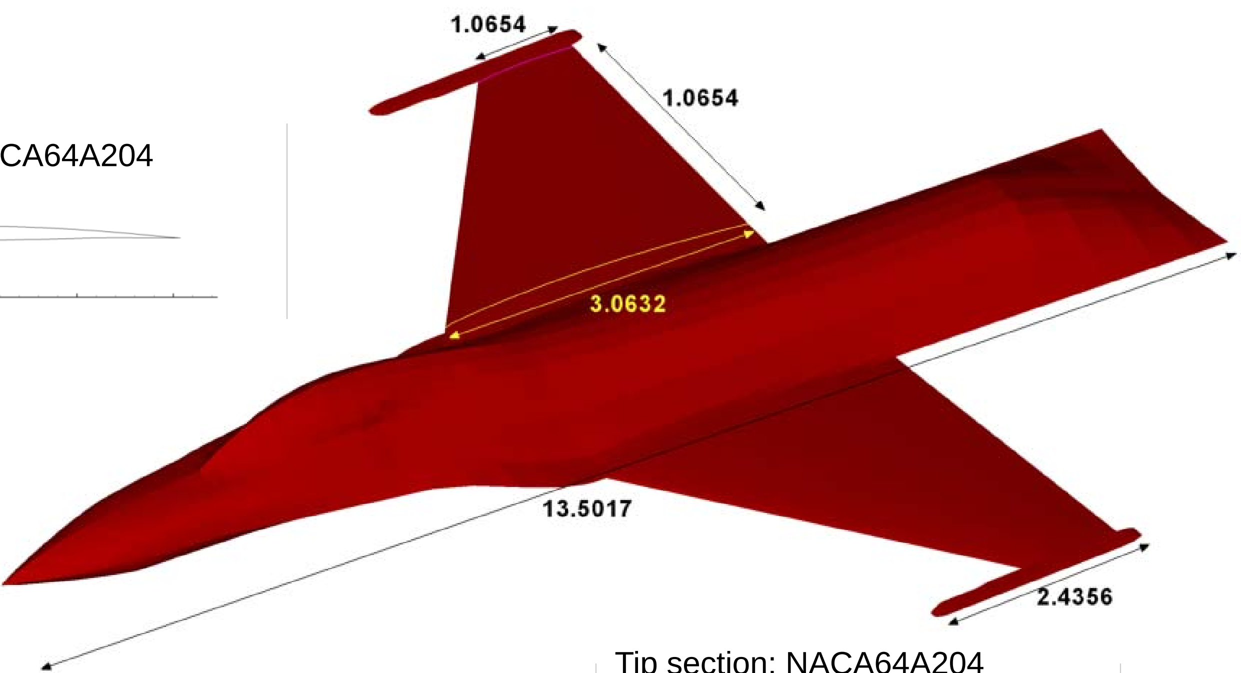


TEST CASE

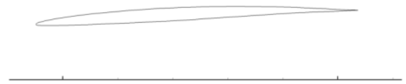
- Aerodynamic updating

Root section: NACA64A204
-1° twist



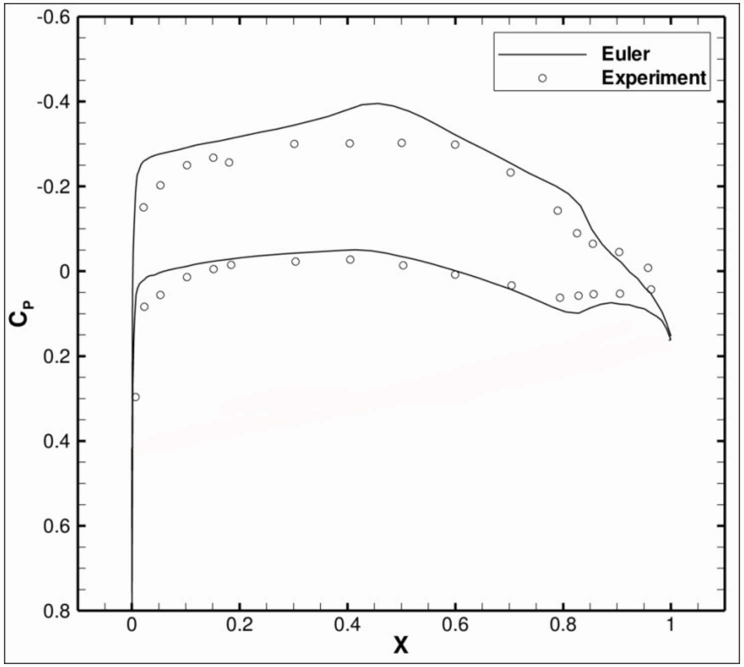


Tip section: NACA64A204
-2.4° twist

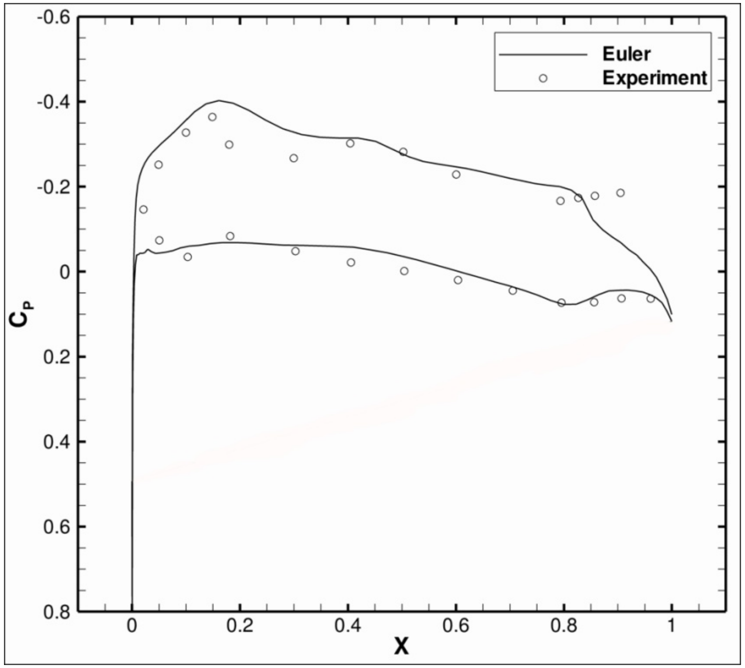


TEST CASE

- Wing aerodynamic configuration was matched to publicly available data



AoA=2.12°; M=0.85; 59% Span

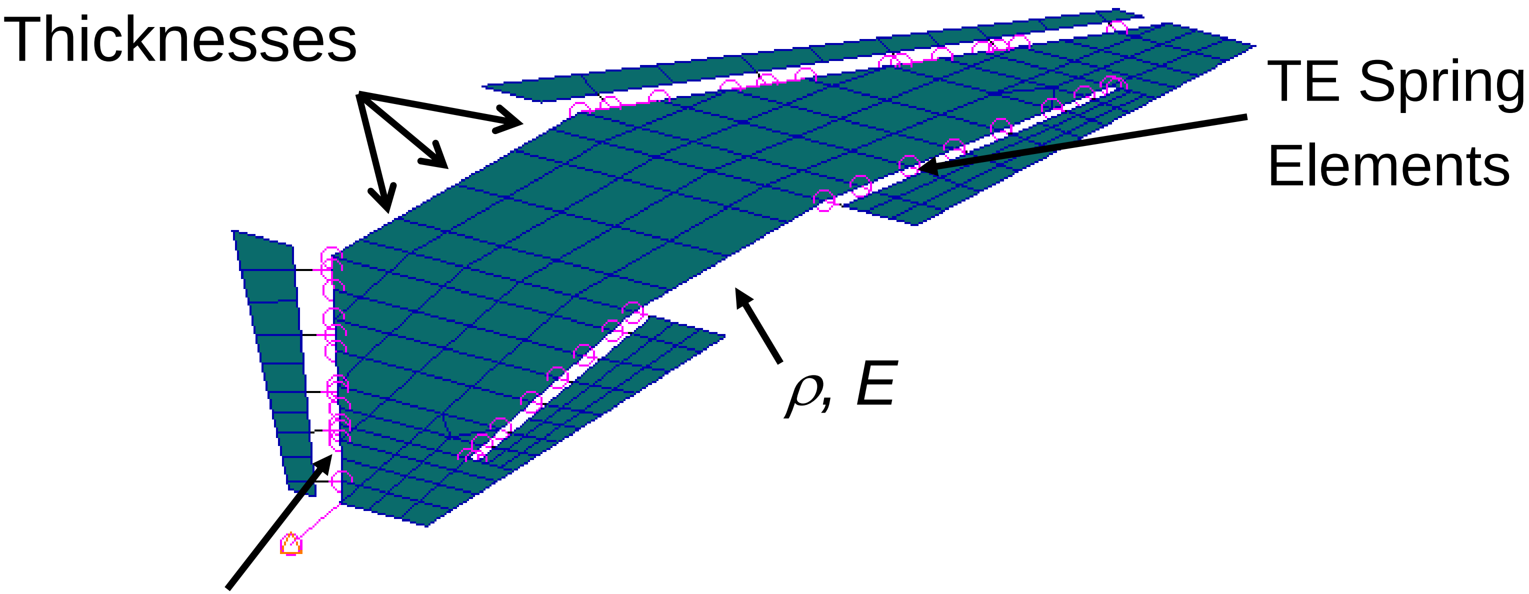


AoA=2.12°; M=0.85; 85% Span

C. Denegri and J. Dubben, IFASD, Munich, 2005

TEST CASE

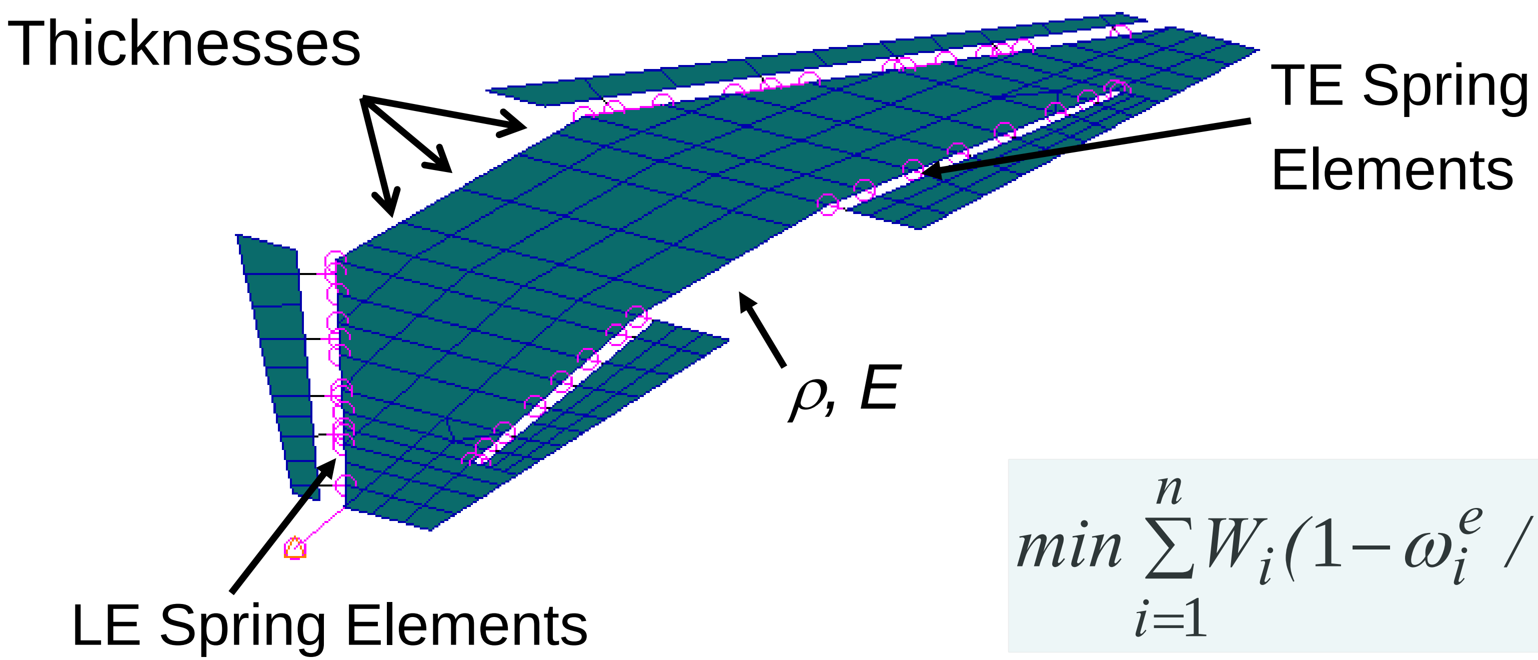
- FE Model Updating



21 Structural Parameters: Directional stiffness, material density, Young modulus, spanwise thickness

TEST CASE

- FE Model Updating



$$min \sum_{i=1}^{n} W_i (1 - \omega_i^e / \omega_i^a)^2$$

21 Structural Parameters: Directional stiffness, material density, Young modulus, spanwise thickness

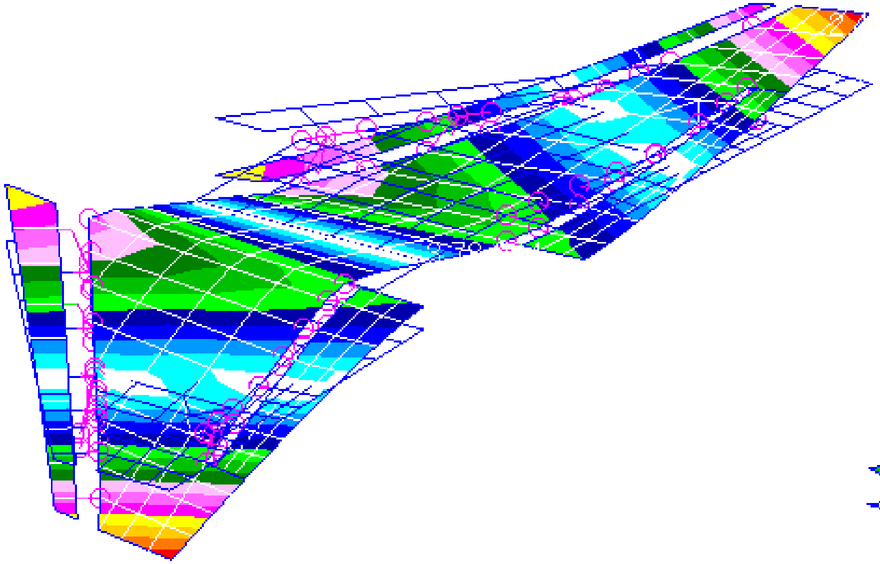
TEST CASE

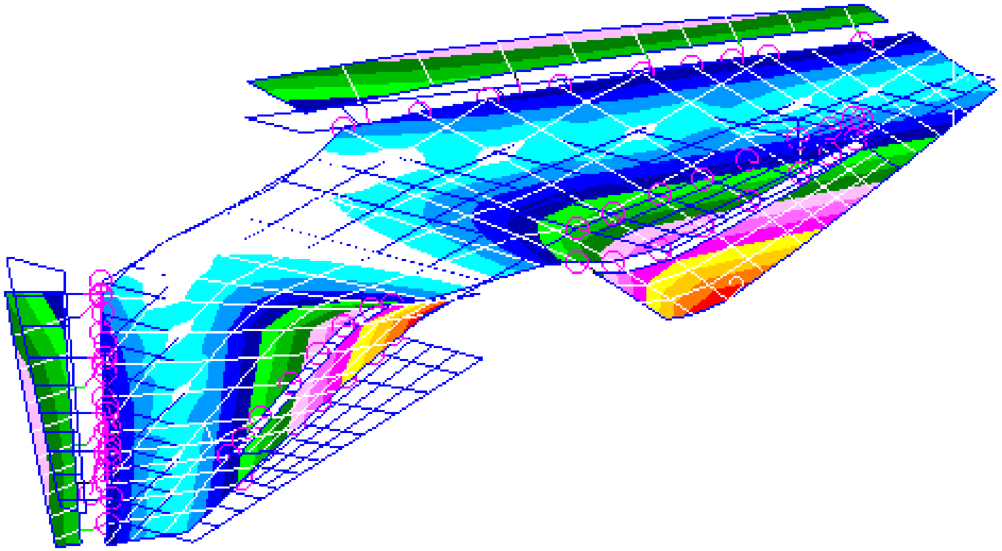
Mode	Initial FE model (Hz)	Denegri data (Hz)	Updated FE model (Hz)	Mode shape
1	7.329		3.920	symmetric
2	11.983	9.191	9.191	antisymmetric
3	17.165	9.964	9.964	antisymmetric
4	21.396		22.452	antisymmetric
5	31.019		22.608	symmetric
6	34.380		24.020	antisymmetric
7	41.109		26.772	symmetric
8	41.217		31.292	antisymmetric
9	44.905		40.040	symmetric
10	45.504		41.695	antisymmetric

C. Denegri, AIAA J. of Aircraft, 37(5), 2000

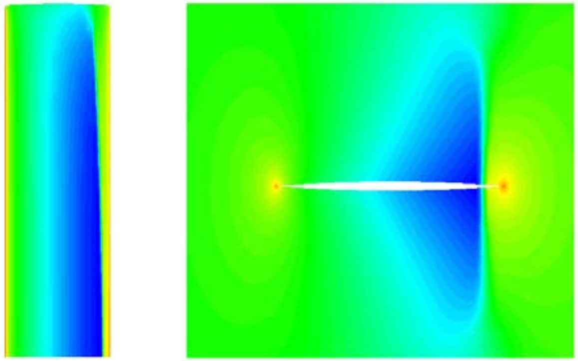
TEST CASE

- Updated model mode shapes





GOLAND WING



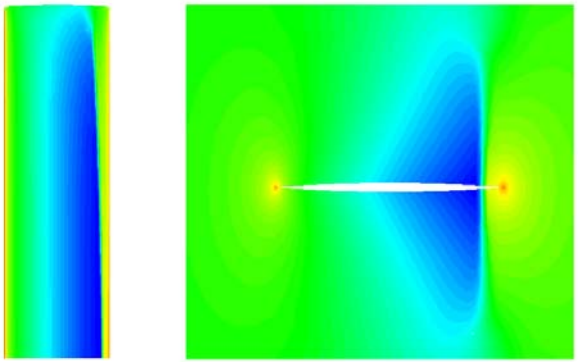
← Compute Steady State

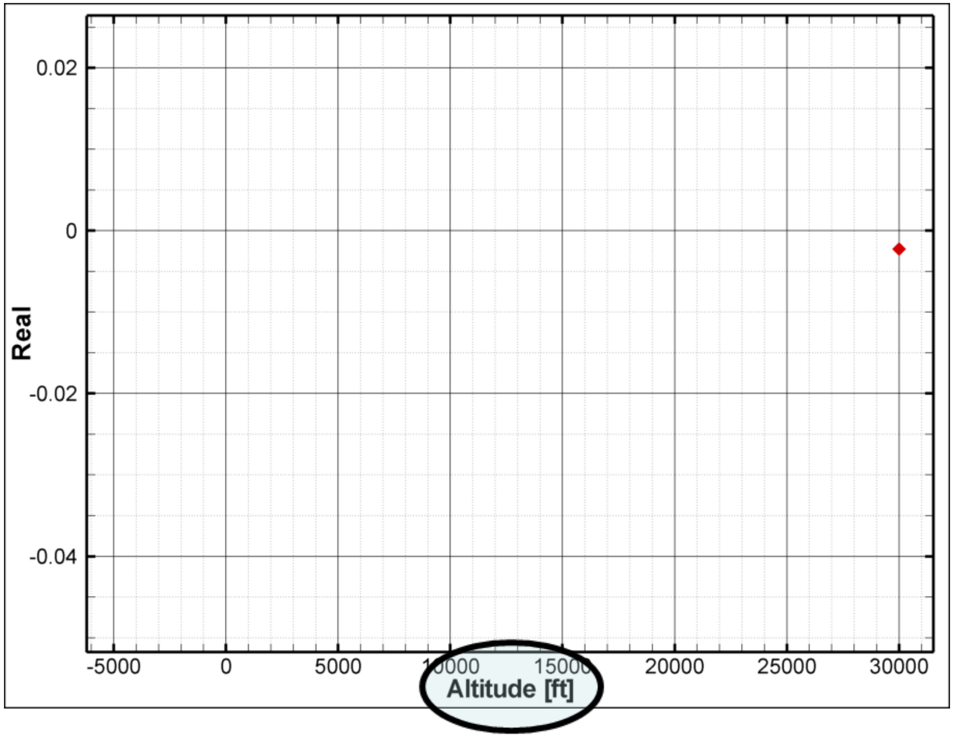
Choose Mode

Pre Compute

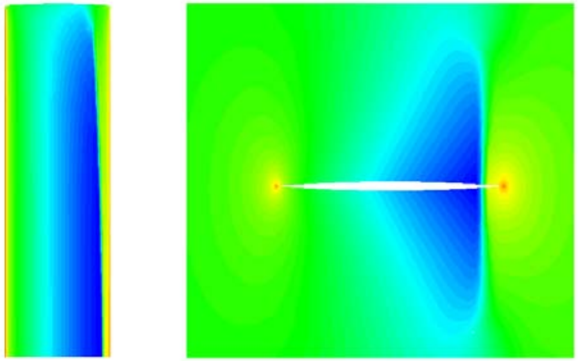
$A_{sf} A_{ff}^{-1} A_{fs}$ and $A_{sf} A_{ff}^{-2} A_{fs}$ →

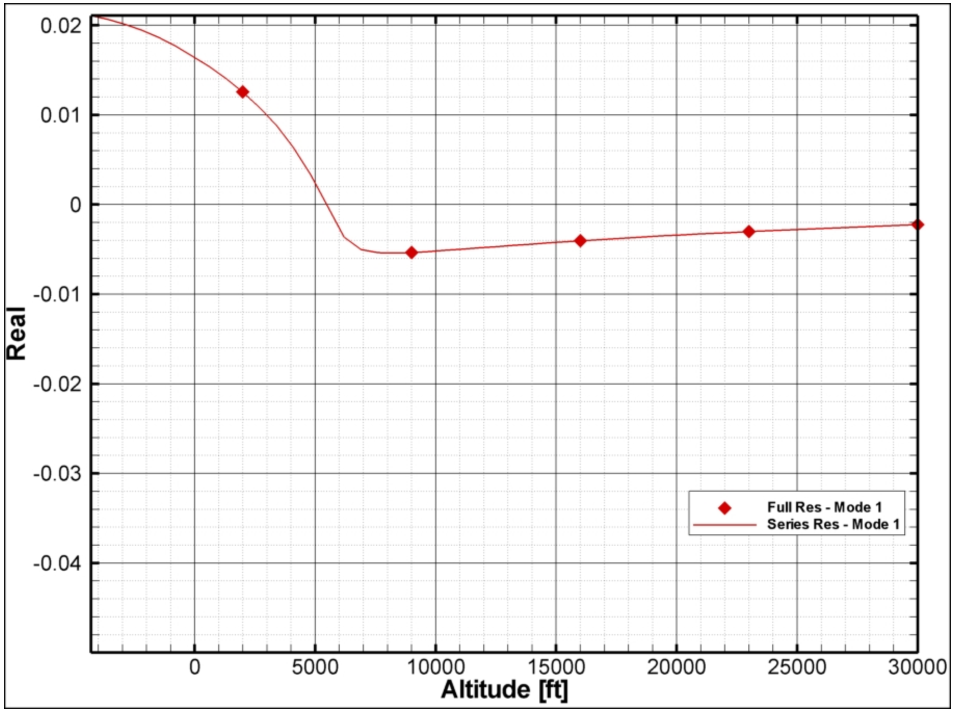
SCHUR METHOD





SCHUR METHOD





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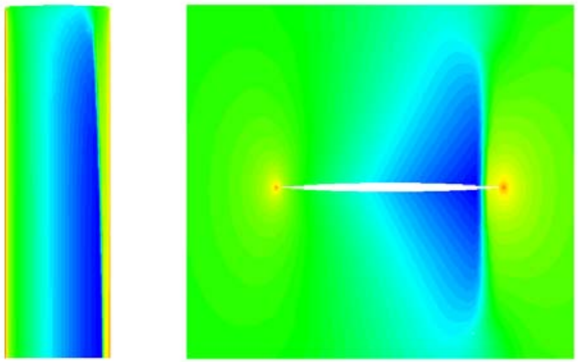

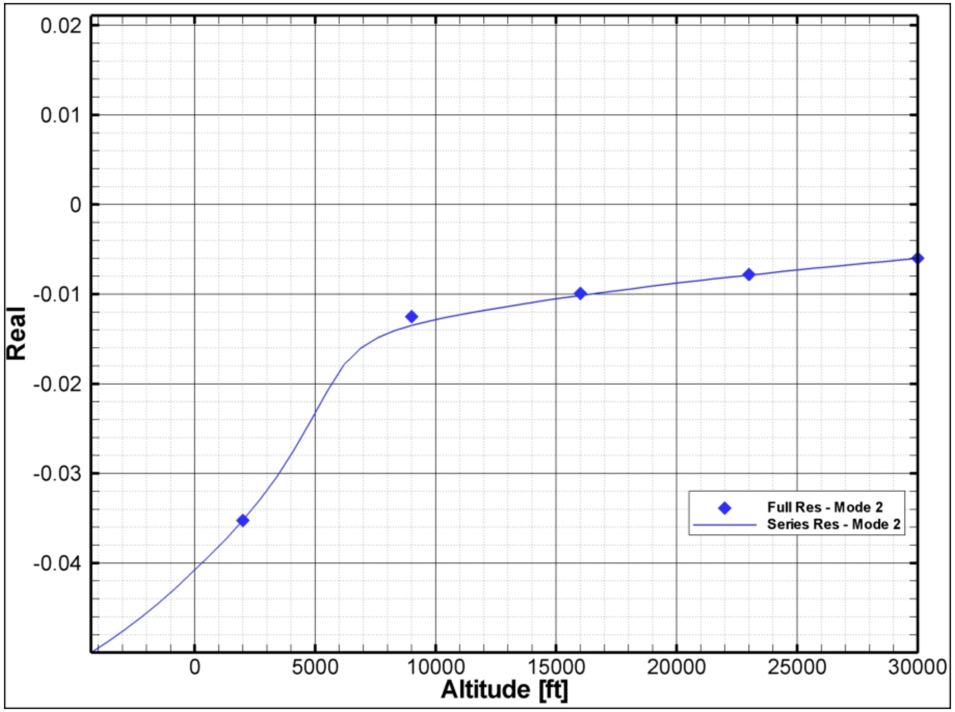

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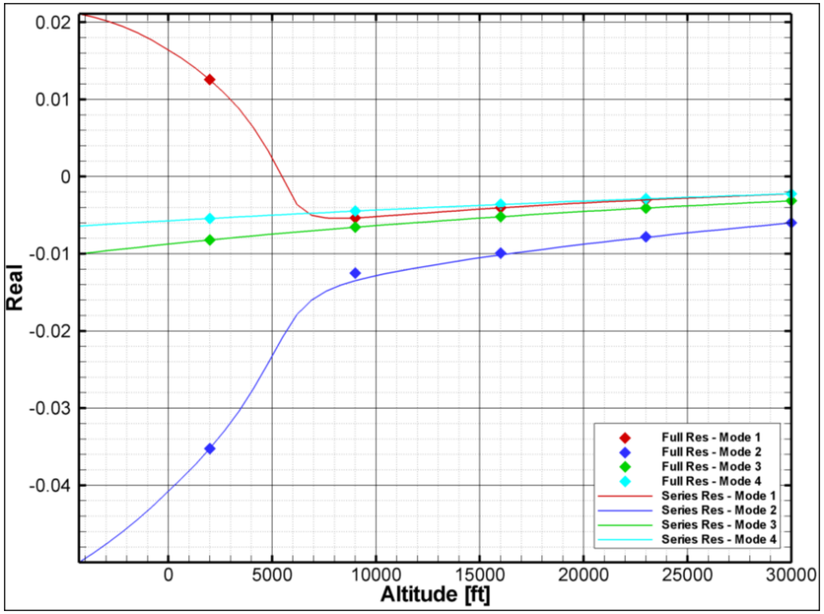
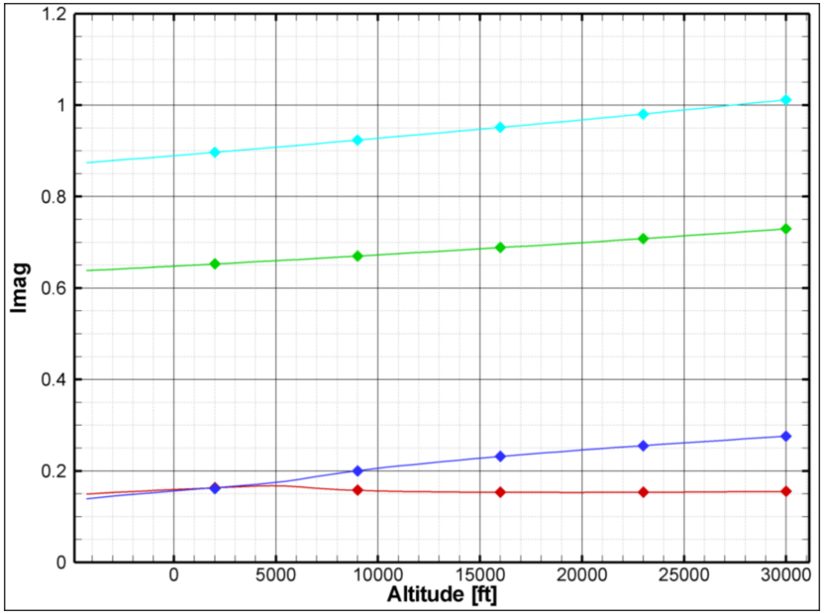
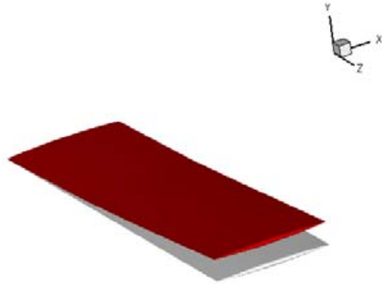


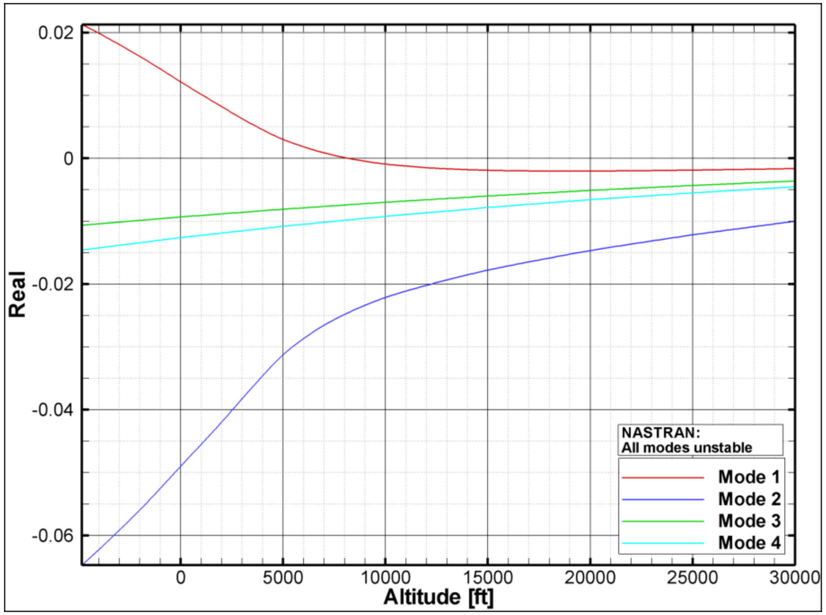


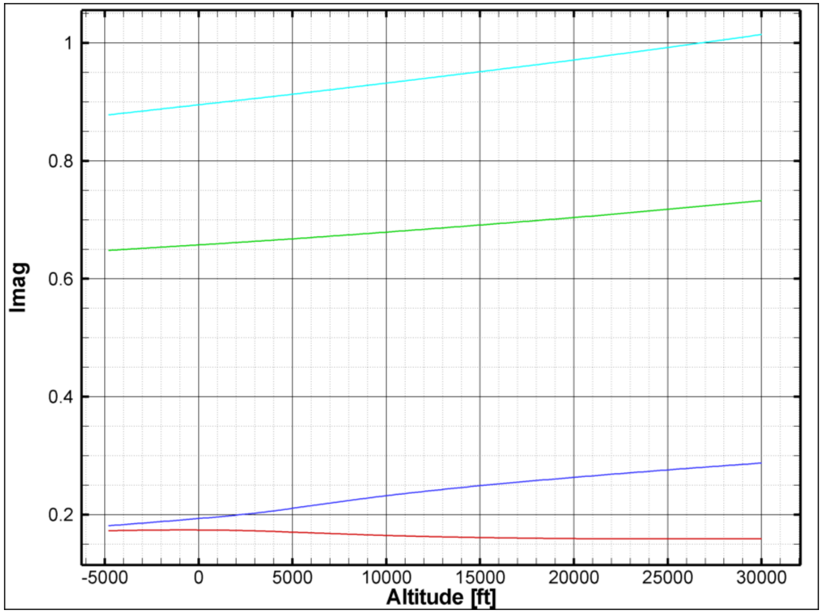
Mach 0.5



- 40k pts grid
- Tracking 4 modes
- 1 Workstation < 12 minutes;
  - Steady State – 1 min
  - $A_{sf} A_{ff}^{-1} A_{fs}$ and $A_{sf} A_{ff}^{-2} A_{fs}$ - 10 min
  - Envelope Sweep < 1min; 5 Full Evaluations- 25 min

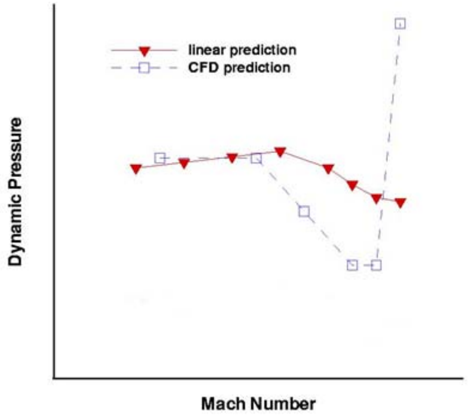
GOLAND WING



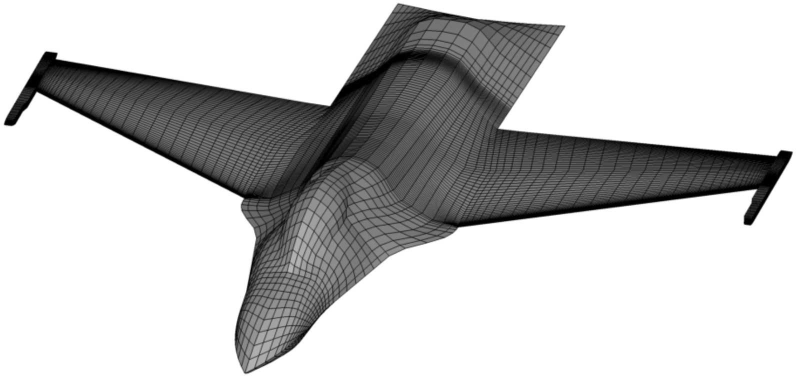


Mach 0.97

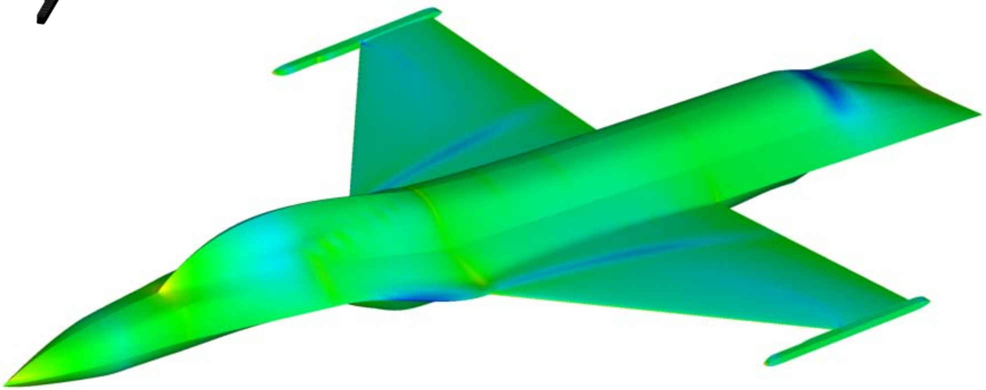
- Coarse grid – 40k pts
- Tracking 4 modes
- 1 Workstation <12 minutes;
  - Steady State – 1 min
  - $A_{sf} A_{ff}^{-1} A_{fs}$ and $A_{sf} A_{ff}^{-2} A_{fs}$ - 10 min
  - Envelope Sweep < 1min



GENERIC FIGHTER

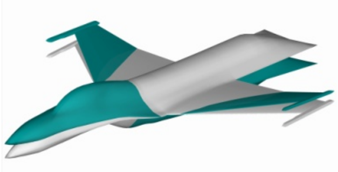


1.1M points

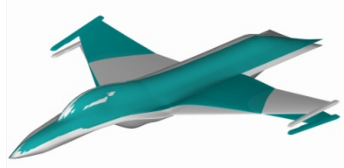


Mach 0.85; AoA 2.12°

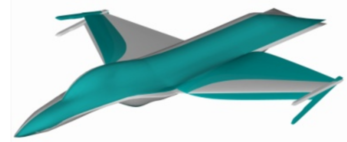
GENERIC FIGHTER



3.920 Hz



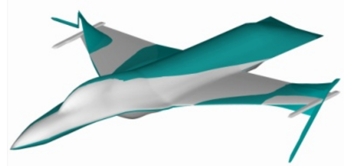
9.191 Hz



9.964 Hz

22.452Hz

22.608 Hz

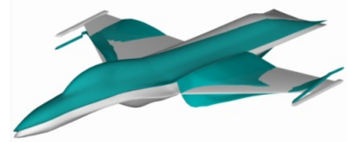


24.020 Hz

26.772 Hz

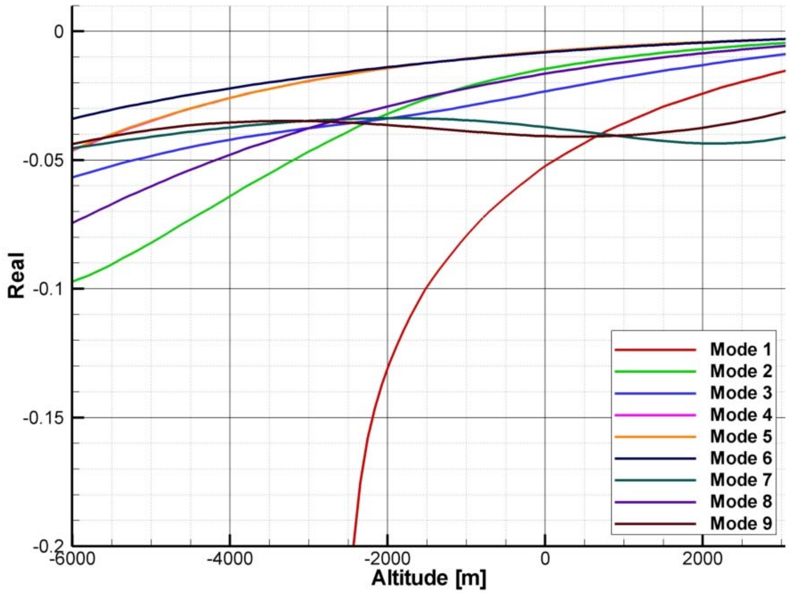
31.292 Hz

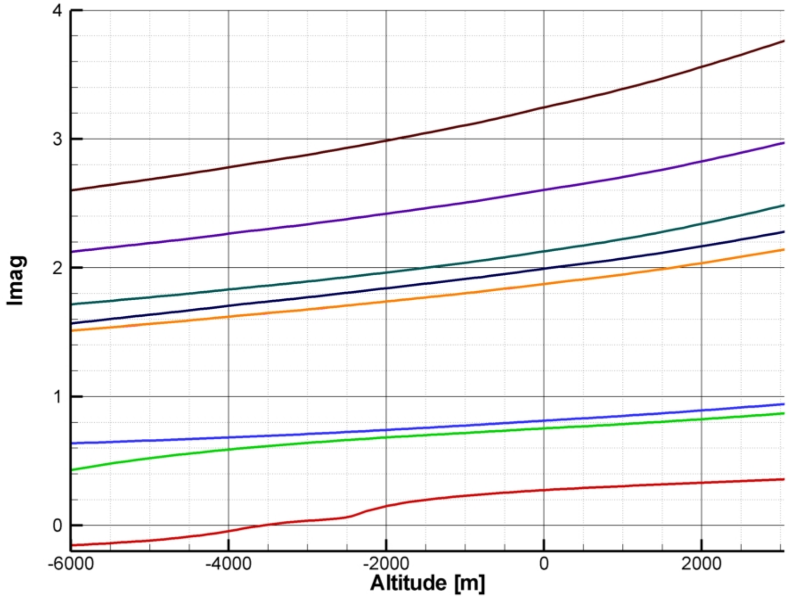
40.04 Hz



41.695 Hz

GENERIC FIGHTER

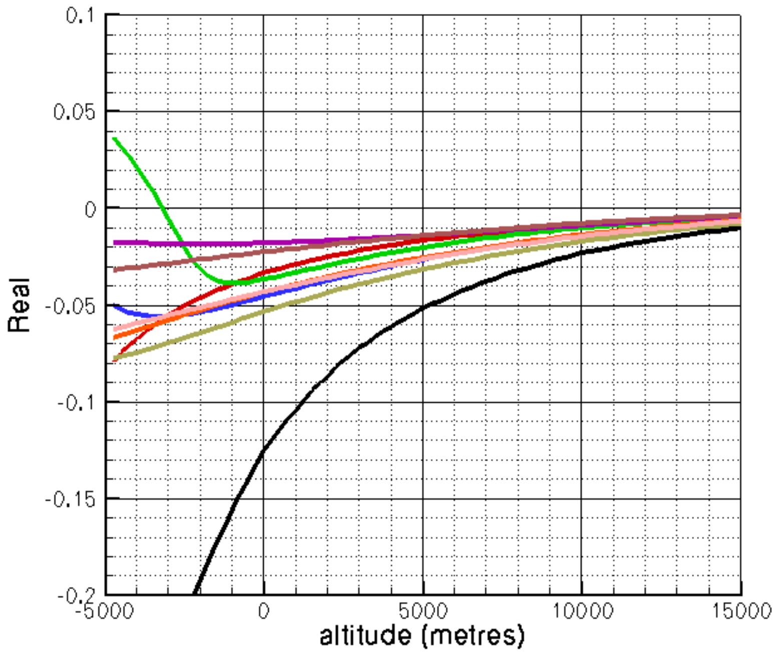


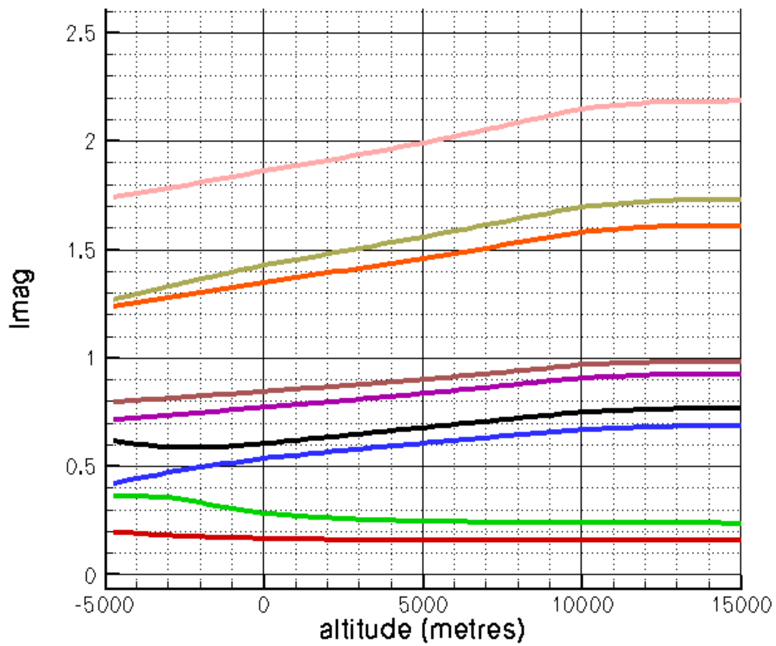


Mach 0.85; AoA 0°

- 32 Processores
    - Steady State – 15 min
    - $A_{sf} A_{ff}^{-1} A_{fs}$ and $A_{sf} A_{ff}^{-2} A_{fs}$ for 10 Modes - 12 Hours

GENERIC FIGHTER





Mach 0.85; AoA 0°

- 32 Processores
    - Steady State – 15 min
    - $A_{sf} A_{ff}^{-1} A_{fs}$ and $A_{sf} A_{ff}^{-2} A_{fs}$ for 8 Modes - 10 Hours

CONCLUSION

- A very fast method to calculate flutter boundary has been developed
  - The method is easilly parallelised
  - It allows for mode tracking at all conditions
  - Series approximation efficient and accurate

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- A very fast method to calculate flutter boundary has been developed
  - The method is easilly parallelised
  - It allows for mode tracking at all conditions
  - Series approximation efficient and accurate

- A realistic test case has been constructed and evaluated
  - Initial FE model improved considerably, to match experimental data
  - Detailed information about mode shapes and interactions obtained

FUTURE WORK

- Expand Generic Fighter Flutter Envelope
- Effects of Structural Uncertainty on Flutter
  - Compare Monte Carlo simulation with Stochastic Methods

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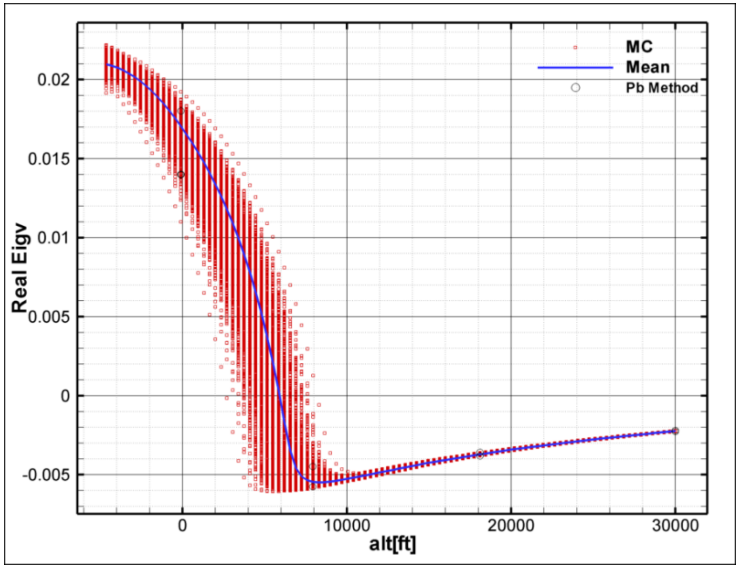
- Expand Generic Fighter Flutter Envelope

- Effects of Structural Uncertainty on Flutter
  - Compare Monte Carlo simulation with Stochastic Methods
  - 7 Parameters ►1000 cases



Q & A

Thank you for your attention.

Any Questions?