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APPLICATION OF THE PERTURBATION METHOD WITH PARAMETER WEIGHTING MATRIX ASSIGNMENTS FOR ESTIMATING VARIABILITY IN A SET OF NOMINALLY IDENTICAL WELDED STRUCTURES

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ABSTRACT

A perturbation method is employed in this paper and the problem of model updating in the presence of uncertainty due to manufacturing variability is addressed. Statistical properties of experimental data are considered and updating parameters are treated as random variables. The perturbation equations are used for estimation of means and covariances of updating parameters. The perturbation formulation is included and two approaches of parameter weighting matrix assignments are explained. Results from one of the approaches demonstrate good correlation between the predicted mean natural frequencies and their measured data, but poor correlation is obtained between the predicted and measured covariances of the outputs. In another approach, different parameter weighting matrices are assigned to the means and covariances updating equations. Results from the latter approach are in very good agreement with the experimental data and excellent correlation between the predicted and measured covariances of the outputs is achieved.

NOMENCLATURE

b Overall width of structure.
d Diameter of weld.
h Overall height of structure.
l Overall length of structure.
u Eigenvectors.
w Total weight of structure.
z Predicted output measurements
z_m Measured output measurements
E Young's modulus.
G Shear modulus.
$\mathbf{K}$ Stiffness matrix.
$\mathbf{M}$ Mass matrix.
$\mathbf{S}$ Sensitivity matrix.
$\mathbf{T}$ Transformation matrix.
$\mathbf{W}_{\varepsilon\varepsilon}$ Weighting matrix of output measurements.
$\mathbf{W}_{\theta\theta}$ Weighting matrix of parameters.
λ Regularisation parameter.
Λ Eigenvalue.
θ Structural parameters.

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ρ Mass density of bulk material.

INTRODUCTION

Interest in uncertainty in the design of engineering structures are growing; owing to the fact that structural properties are normally uncertain and hence consequent uncertainty exist in the dynamic response. Properties of an individual structure normally change with time due to environmental erosion and damage [1], and also when the structure is being reassembled. It is also unavoidable to have manufacturing variability [2–4] that exists among nominally identical structures, built in the same way from the same materials, such as in a mass production of automotive body-in-white (BIW).

Uncertainty and variability are frequently used inconsistently in literature. Therefore, for the purpose of clarity, uncertainty and variability terminologies are used based on definitions presented in Ref. [5] in this paper. Variability is defined as a measure of heterogeneity or diversity in a population (for example, intrinsic randomness in a set of structures) and usually is irreducible through further study or measurement. Variability is classified as *aleatory uncertainty*. Uncertainty (or *epistemic uncertainty*), on the other hand, represents lack of knowledge, and can be reducible through further study or measurement. However, since a variability could also be a subject to lack of knowledge when information within its range is missing, consequently such variability becomes an uncertainty.

Issues relating to uncertainty and variability, such as safety and reliability, leads to increasing demands for improved computational methods that incorporate uncertainties in the structural properties. When these uncertainties are taken into account, a deterministic problem then changes to a non-deterministic (or stochastic) problem. It is highly appreciated that the ability to numerically predict the behaviour of a structure with uncertainties is very useful and of great scientific value.

Finite element (FE) model updating has become an active research topic in the past decades [6]. In the FE model updating, adjustment is made to the system parameters so that the difference between the measured and predicted modal parameters (i.e., natural frequencies, mode shapes, etc.) is minimised. Application of model updating is well established for deterministic problems, but due to uncertainties in real test structures, stochastic model updating has become more popular.

Stochastic model updating method [2] allows for manufacturing variability and modelling uncertainty to be incorporated so that numerical models with randomised parameters can be updated to match their experimental counterparts. As a result, robust and credible models are produced which in turn increase trust in design and analysis of such structures. Stochastic model updating problems are computationally expensive, mainly due to the randomised parameters, hence various assumptions and simplifications have to be made to ensure the efficiency of the methods, as investigated by Haddad Khodaparast and Mottershead in Ref. [7]. Two efficient methods in stochastic model updating, 1) a perturbation method, and 2) a method based upon the minimisation of an objective function, were developed and the first method was shown to be viable and the needs to compute second order sensitivities was removed, leading to considerable reduction in computational effort in practical engineering applications. Another study using the perturbation method is presented in Ref. [8]. This work demonstrates a method to adjust parameter means and covariance matrix from multiple sets of experimental modal data. The method is performed by updating mean parameters to minimise the difference between the measured and predicted outputs, followed by updating of parameter covariance matrix, where the difference between the measured and analytical output covariance matrices is minimised.

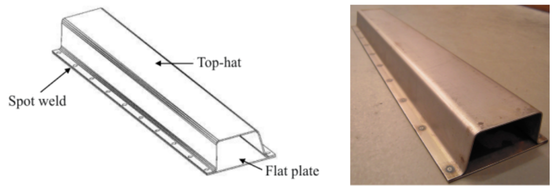

Figure 1. THE LASER SPOT WELDED STRUCTURE

In this work, the perturbation method used by Haddad Khodaparast et al. [4, 7] is employed. Variability that exists between a set of nominally identical test structures is investigated. The variability is quantified using the perturbation method and propagated using the Monte Carlo simulation method. Experimental modal analysis (or modal testing) [9, 10] is conducted to obtain the measured means and covariances of the outputs, which are then used in the stochastic model updating to estimate the means and covariances of the structural parameters. Two approaches of parameter weighting matrix assignments are employed and explained. Results from both approaches are discussed and compared.

DESCRIPTION OF STRUCTURES AND EXPERIMENTAL PROCEDURE

A set of nine laser spot welded structures (see Fig. 1), which are simplification of substructures normally used in automotive BIW, are investigated in the study. The structures are made by following a manufacturing specification by Mottershead et al. [3] in order to reduce the manufacturing variability in the test structures. The physical and geometrical properties of the structures are shown in Table 1 and the thickness of the metal sheets used to construct the welded structures is 1.5 mm.

Modal testing [9, 10] was performed by conducting three main aspects of experimental modal analysis [9], which are (1) excitation of the structure, (2) measurement of response, and

Table 1. MATERIAL AND GEOMETRIC PROPERTIES OF THE WELDED STRUCTURES

Properties	Values
Overall length (l)	564 mm
Overall width (b)	110 mm
Overall height (h)	40 mm
Mass density (ρ)	7860 kgm^{-3}
Young's modulus (E)	209 GPa
Shear modulus (G)	83 GPa
Total weight (w)	1.8 kg

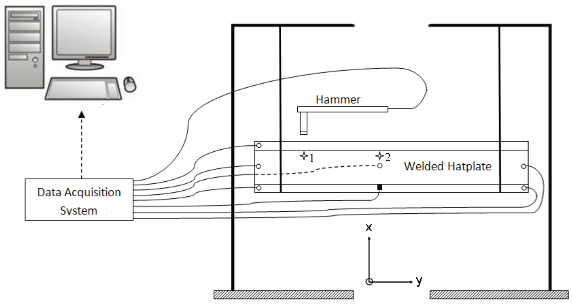

Figure 2. THE EXPERIMENTAL SETUP

(3) data acquisition and processing. The welded structures were tested with free-free hammer tests using two hammer points and seven measurement points (as depicted in Fig. 2) to determine the first five natural frequencies. Hammer point 1 was hit in two directions (i.e., the x- and z-directions), while hammer point 2 was hit only in the z-direction. Multiple hammer points were chosen to excite certain modes that apparently could not be excited when a single hammer point was employed. Seven Kistler accelerometers were used, with six of them placed on the flat plate where most deformations occur and only one was placed on the sidewall of the hat. Both hammer and measurement points were chosen with care so that they are not near any nodal points. The responses were measured by using a 12-channel LMS system and extracted using an LMS PolyMAX curve-fitting procedure. The first five measured natural frequencies of the nine structures, together with their means and standard deviations, are given in Table 2.

An FE model (shown in Fig. 3) developed in previous work as reported in Ref. [11] is used to represent the structures. The model was developed using MSC NASTRAN with approximately 3500 CQUAD4 elements and 20 CWELD elements. The problem can also be modelled in a more detailed approach, such as using solid elements and a finer mesh, but that will result in

Table 2. MEASURED NATURAL FREQUENCIES (in Hz) OF THE WELDED STRUCTURES

	Modes				
Samples	1	2	3	4	5
1	509.33	557.16	578.25	634.42	646.65
2	511.08	554.14	577.00	626.59	640.17
3	508.67	554.53	575.92	626.59	645.54
4	501.33	541.26	567.79	616.64	634.45
5	512.18	558.39	580.89	630.84	646.44
6	509.32	552.90	578.38	627.65	646.08
7	507.04	550.40	572.83	625.47	643.49
8	508.03	558.13	573.71	630.04	645.74
9	506.15	556.29	573.74	628.78	644.40
Mean	508.12	553.69	575.39	627.45	643.66
Std.	3.15	5.33	3.87	4.88	4.00

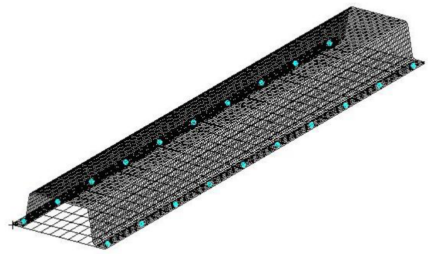

Figure 3. FE MODEL OF THE STRUCTURES

highly expensive computational effort. The FE model did not incorporate the Kistler accelerometers used in the experiments as they were considerably lighter (approximately 1.6 grams each) than the structure under investigation.

FE models of the components, i.e., the flat plate and the top-hat, has been updated prior to being used in modelling the welded structures to isolate any uncertainties from the models. Therefore, it is assumed that the main uncertainties in the FE model comes from the weld parameters. Nominal value is used for the thickness (i.e., 1.5 mm) and the values for the material properties are assigned as tabulated in Table 1. Initial values of the weld and patch parameters follow the guidelines given by Ref. [11], as shown in Table 3.

METHODOLOGY

Structural parameters are normally assumed to be known in forward problem, but unknown for inverse problem such as in

Table 3. MATERIAL AND GEOMETRIC PROPERTIES OF THE WELDS

Properties	Values
Diameter of welds (d)	5.5 mm
Young's modulus of welds (E_{weld})	220 GPa
Young's modulus of patch (E_{patch})	650 GPa

model updating [6]. It is often easier to measure the structural response (such as modal properties and frequency response functions (FRFs)) than measuring the parameters. The statistical estimates from the response measurements may then be used to deduce the statistical estimates of the parameters. This inverse problem of estimating the parameter distributions from the response measurements is called uncertainty identification or quantification [12], and is the subject of this paper.

Statistical methods have been widely used in model updating and methods for dealing with the estimation of parameter variability in the stochastic model updating has evolved over the past years. One of the most widely used methods is the perturbation method, which is based on the Taylor series expansion and uses the sensitivities expressing the influence of the stochastic input parameters on the output quantity. The perturbation method generally aims at calculating the first two statistical moments (i.e., mean and standard deviations) of the parameters, which is further explained in the following subsection. Driven by its popularity, the perturbation approach remains under continuous development.

The Perturbation Method

Conventional, deterministic model updating methods are based on the simple first-order Taylor series expansion and the general form of this expansion is

$$z_{\boldsymbol{m}} = z_j + \mathbf{S}_j \left(\theta_{j+1} - \theta_j\right) \tag{1}$$

In equation (1), $\mathbf{S}_j$ is an mxn sensitivity matrix at j^{th} iteration, which denotes the rates of change of the structural eigenvalues ($\delta\Lambda_j$) with respect to changes in parameters ($\delta\theta$), which can be expressed as [13]

$$\mathbf{S}_j = \frac{\delta\Lambda_j}{\delta\theta} = \boldsymbol{u}_j^{\text{T}} \left[\frac{\delta\mathbf{K}}{\delta\theta} - \Lambda_j \frac{\delta\mathbf{M}}{\delta\theta}\right] \boldsymbol{u}_j \tag{2}$$

Rearranging Eq. 1 and introducing a weighting matrix of measurements ($\mathbf{W}_{\varepsilon\varepsilon}$) and a weighting matrix of parameters ($\mathbf{W}_{\theta\theta}$) into the equation gives

$$\theta_{j+1} = \theta_j + \mathbf{T}_j \left(z_{\boldsymbol{m}} - z_j\right) \tag{3}$$

where $\theta \in \mathbb{R}^{nx1}$ is the vector of structural parameters, $z_{\boldsymbol{m}} \in \mathbb{R}^{mx1}$ is the vector of measured data and $z_j \in \mathbb{R}^{mx1}$ is the vector of predicted outputs. $\mathbf{T}_j$ is a transformation matrix, which can be written as

$$\mathbf{T}_j = (\mathbf{S}_j^{\text{T}} \mathbf{W}_{\varepsilon\varepsilon} \mathbf{S}_j + \mathbf{W}_{\theta\theta})^{-1} \mathbf{S}_j^{\text{T}} \mathbf{W}_{\theta\theta} \tag{4}$$

$\mathbf{W}_{\varepsilon\varepsilon}$ and $\mathbf{W}_{\theta\theta}$ are positive definite weighting matrices. $\mathbf{W}_{\varepsilon\varepsilon}$ is usually given by the reciprocals of the measurements variance, while $\mathbf{W}_{\theta\theta}$ must be chosen so that only uncertain parameters will change more during the updating procedure than the other parameters. The choice of $\mathbf{W}_{\varepsilon\varepsilon} = \mathbf{I}$ and $\mathbf{W}_{\theta\theta} = \mathbf{0}$ would results in pseudo-inverse [6]. For an ill-conditioned model updating problem, $\mathbf{W}_{\theta\theta} = \lambda * \mathbf{I}$ where λ is a regularisation parameter found by plotting an L-curve [14]. The weighting matrix assignment is explained and discuss further in the next subsection.

Including the variability in measurements,

$$z_{\boldsymbol{m}} = \bar{z_{\boldsymbol{m}}} + \Delta z_{\boldsymbol{m}} \quad \text{and} \quad z_j = \bar{z}_j + \Delta z_j \tag{5}$$

where $\bar{z}$ denotes the mean values of the measurements and Δz represents the vectors of random variables.

Similarly, the variability of the structural parameters at j^{th} iteration is defined as,

$$\theta_j = \bar{\theta}_j + \Delta\theta \tag{6}$$

where $\bar{\theta}$ denotes the mean values of the parameters and $\Delta\theta$ represents the vectors of random variables.

The transformation matrix is now represented by

$$\mathbf{T}_j = \bar{\mathbf{T}}_j + \Delta\mathbf{T}_j \tag{7}$$

where

$$\Delta\mathbf{T}_j = \sum_{k=1}^{n} \frac{\partial\mathbf{T}_j}{\partial z_{m_k}} \Delta z_{m_k} \tag{8}$$

Δz_{m_k} denotes the k^{th} element of Δz_m. Substituting Eqs. 5 to 7 into the deterministic problem (Eq. 3) produces the stochastic model updating equation, as follows.

$$\bar{\theta}_{j+1} + \Delta\theta_{j+1} = \bar{\theta}_j + \Delta\theta_j + (\bar{\mathbf{T}}_j + \Delta\mathbf{T}_j)\,(\bar{z_m} + \Delta z_m - \bar{z}_j - \Delta z_j) \tag{9}$$

Separating the zeroth-order and first-order terms from Eq. 9 gives,

$$\Delta^0\colon\ \bar{\theta}_{j+1} = \bar{\theta}_j + \bar{\mathbf{T}}_j\,(\bar{z_m} - \bar{z}_j) \tag{10}$$

$$\Delta^1\colon\ \Delta\theta_{j+1} = \Delta\theta_j + \Delta\mathbf{T}_j\,(\Delta z_m - \Delta z_j) \tag{11}$$

Eqs. 10 and 11 are used to determine the parameter means and the parameter covariance matrix, respectively, in the perturbation method [4]. The parameter covariance matrix equation can be written as,

$$\mathbf{C}_{\theta\theta_{j+1}} = \mathbf{C}_{\theta\theta_j} - \mathbf{C}_{\theta Z_j}\bar{\mathbf{T}}_j^{\mathrm{T}} + \bar{\mathbf{T}}_j\mathbf{C}_{EE}\bar{\mathbf{T}}_j^{\mathrm{T}} - \bar{\mathbf{T}}_j\mathbf{C}_{Z\theta_j} + \bar{\mathbf{T}}_j\mathbf{C}_{ZZ_j}\bar{\mathbf{T}}_j^{\mathrm{T}} \tag{12}$$

with the parameters covariance matrix,

$$\mathbf{C}_{\theta\theta} = \mathrm{Cov}(\Delta\theta, \Delta\theta)$$

the covariance matrix of the measured outputs,

$$\mathbf{C}_{EE} = \mathrm{Cov}(\Delta z_m, \Delta z_m)$$

the covariance matrix of the predicted outputs,

$$\mathbf{C}_{ZZ_j} = \mathrm{Cov}(\Delta z_j, \Delta z_j)$$

the covariance matrix of the parameters and the predicted outputs,

$$\mathbf{C}_{\theta Z_j} = \mathrm{Cov}(\Delta\theta, \Delta\theta)_j \times \mathbf{S}_j^{\mathrm{T}}$$

and the covariance matrix of the predicted outputs and the parameters,

$$\mathbf{C}_{Z\theta_j} = \mathbf{S}_j \times \mathrm{Cov}(\Delta\theta, \Delta\theta)_j$$

where ‘Cov’ represents the covariance between two random variables, which are computed using mean-centred first order perturbation method. A significant advantage of the perturbation method [4,7] used in this paper over another similar perturbation method by Hua et al. [15] is that only the first-order sensitivity matrix is needed in Eq. 12, hence big reduction in terms of computational effort is achieved.

The outlined procedure in Ref. [4] is followed to determine the statistical data (i.e., means and standard deviations) of the structural parameters that converge: (1) the mean predicted modal data ($\bar{z}$) on the mean measured modal data ($\bar{z_m}$), and (2) the covariance matrix of the predicted modal data ($\mathbf{C}_{ZZ}$) on the covariance matrix of the measured modal data ($\mathbf{C}_{EE}$). The values of the means and standard deviations of the structural parameters can then be propagated in the numerical model by using the multivariate normal distribution Monte Carlo simulation. In this work, 500 samples are generated according to the statistical data obtained by the perturbation method and the respective analytical model outputs (i.e., natural frequencies) are computed.

Modifications of the weighting assignment

The stochastic model updating in the case of variability in the experimental data where the output means and their covariances are known requires two steps to be carried out:

1. Adjustment of the mean parameters, where the difference between the measured and predicted outputs are minimised using a weighted least squares method. This is performed by using Eq. 10.
2. Adjustment of the parameter covariance matrix, where the difference between the measured and analytical output covariance matrix are minimised using the Frobenius norm. This is performed by using Eq. 12.

In this work, the perturbation formulation for the stochastic model updating is applied by using the reciprocals of the measurements variance as the measurement weighting matrix ($\mathbf{W}_{\varepsilon\varepsilon}$) and assigning two different approaches for the parameters weighting matrix ($\mathbf{W}_{\theta\theta}$). The first approach considers only the main uncertain parameters, hence equal weighting is employed when estimating the means and covariances of the parameters. On the other hand, more parameters are considered in the second approach regardless of their levels of variability. Two different weightings have to be introduced, one for each step of updating. Both approaches are explained as follows.

Approach 1 In this approach, the matrix of the parameters weighting is

$$\mathbf{W}_{\theta\theta} = \lambda * \mathbf{I}$$

Regularisation parameter (λ) of 400 is used and the weighting is assigned for both steps of the stochastic model updating. As mentioned beforehand, the main variability comes from the weld

Table 4. MEAN NATURAL FREQUENCIES ESTIMATED USING APPROACH 1

Mode	Experiment (Hz)	Initial FE (Hz)	Error (%)	Updated FE (Hz)	Error (%)
1	508.12	487.55	4.05	500.68	1.46
2	553.69	517.86	6.47	567.61	2.51
3	575.39	555.03	3.54	573.42	0.34
4	627.45	575.14	8.34	631.56	0.66
5	643.66	582.64	9.48	635.44	1.28

Table 5. IDENTIFIED MEAN PARAMETERS ESTIMATED USING APPROACH 1

Parameter	Initial value	Identified value
d	5.5 mm	6.0 mm
E_{weld}	220 GPa	230 GPa
E_{patch}	650 GPa	1595 GPa
std. (d)	0.05 mm	0.16 mm
std. (E_{weld})	2.00 GPa	5.57 GPa
std. (E_{patch})	6.50 GPa	10.49 GPa

parameters (i.e., d, E_{weld} and E_{patch}), hence this approach uses only the three weld parameters tabulated in Table 3 for the updating procedure.

Approach 2 In contrast to Approach 1, Approach 2 includes eight parameters (i.e., five from the components and three from the welds). The parameters of the components have low level of variability in comparison with the weld parameters, therefore, different assignments of weighting are used. The weighting matrix used to update the mean parameters (step 1) is

$$\mathbf{W}_{\theta\theta} = \lambda * \text{diag}(1000, 1000, 1000, 1000, 1000, 1, 1, 1)$$

where bigger weighting is given to the components parameters to limit their changes during updating. The weld parameters are not weighted as much to allow them to change more.

In the second updating step (i.e., to update the parameter covariance matrix), similar weighting as in Approach 1 is used. The variances of the parameters are assumed to have the same level of variability due to the fact that the components may have changed slightly during the welding process. This will increase the level of variability in the welded structures as a whole. Assigning different weighting matrices for each updating step will ensure that the mean parameter values reflects the physical parameters of the structures, allowing for uncertainties after the components are welded together to be considered.

RESULTS AND DISCUSSIONS

Results obtained by the perturbation method with $\mathbf{W}_{\theta\theta} = \lambda * \mathbf{I}$ are discussed first, followed by the results obtained when modification of weighting is introduced. Firstly, it can be seen from Table 4 that the updated mean natural frequencies obtained using the first approach are close to the mean measured frequencies.

The initial means and standard deviations of all weld parameters are shown in Table 5. The initial mean values are chosen from deterministic study done prior to this work [11], while the initial standard deviations are deliberately set at 1% of the mean values. The results of the updated means and standard deviations are not in good agreement with the deterministic values. Considerable changes can be observed on the mean parameters, as shown in Table 5, and the estimated standard deviations are generally bigger than the initial estimates.

The covariance matrix of the measured outputs $(\mathbf{C}_{EE})$ obtained from the modal testing is

$$\mathbf{C}_{EE} = \begin{bmatrix} 9.95 & 13.31 & 11.53 & 11.51 & 8.70 \\ & 28.42 & 15.71 & 24.32 & 18.47 \\ & & 15.01 & 14.75 & 11.52 \\ & sym. & & 23.85 & 17.30 \\ & & & & 16.00 \end{bmatrix}$$

while the covariance matrix of the predicted outputs $(\mathbf{C}_{ZZ})$ computed using the identified parameter estimates (see Table 5) is given as follows.

$$\mathbf{C}_{ZZ} = \begin{bmatrix} 0.91 & 3.16 & 0.56 & 3.54 & 3.23 \\ & 12.92 & 4.11 & 14.35 & 12.58 \\ & & 6.78 & 2.49 & 4.69 \\ & sym. & & 18.09 & 14.19 \\ & & & & 14.10 \end{bmatrix}$$

The percentage of error between $\mathbf{C}_{EE}$ and $\mathbf{C}_{ZZ}$ is shown in Fig. 4. Generally, the errors are very big when using the first approach, hence it can be concluded that the approach fails to produce a good estimate of the $\mathbf{C}_{ZZ}$.

The updated mean natural frequencies using the second approach is tabulated in Table 6. It can be seen from the table that the identified and measured natural frequencies achieved by using the second approach are in good agreement. The identified means and standard deviations using the second approach are given in Table 7. As can be observed from the table, the changes in the identified means and standard deviations of the parameters with respect to the deterministic estimates are reasonable.

The identified estimates shown in Tables 6 and 7 give a $\mathbf{C}_{ZZ}$

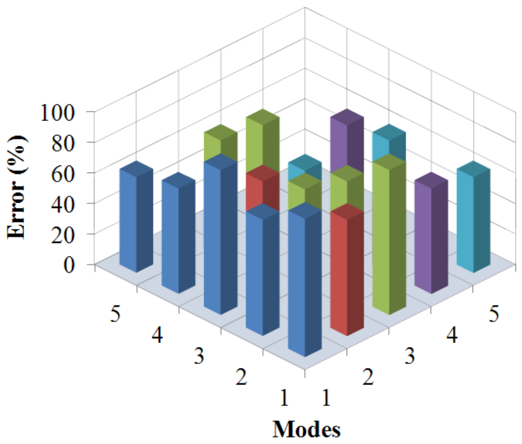

Figure 4. Error after updating between the measured and predicted measurements covariance using Approach 1

Table 6. MEAN NATURAL FREQUENCIES ESTIMATED USING APPROACH 2

Mode	Experiment (Hz)	Initial FE (Hz)	Error (%)	Updated FE (Hz)	Error (%)
1	508.12	487.55	4.05	498.10	1.97
2	553.69	517.86	6.47	568.14	2.61
3	575.39	555.03	3.54	570.35	0.88
4	627.45	575.14	8.34	632.85	0.86
5	643.66	582.64	9.48	636.40	1.13

as follows,

$$\mathbf{C}_{ZZ} = \begin{bmatrix} 10.65 & 13.30 & 11.60 & 11.84 & 9.99 \\ & 25.39 & 13.78 & 22.39 & 18.57 \\ & & 13.37 & 13.22 & 11.32 \\ & sym. & & 22.35 & 18.36 \\ & & & & 16.84 \end{bmatrix}$$

and the error as shown in Fig. 5. The errors of each element appear significantly small than the ones produced by the first approach. This shows that the estimated means and standard deviations of the second approach are reasonable and the modification made to the weighting matrices are valid.

Convergence of the normalised parameter estimates produced by the second approach is shown in Fig. 6. It can be seen from the figure that convergence is achieved in ten iterations. Fig. 7 and 8 show the convergence of the predictions upon the experimental data in the space of the first three natural frequencies using the second approach. Five hundreds samples are propagated by the Monte Carlo simulation and it is clearly enough to obtain

Table 7. IDENTIFIED MEAN PARAMETERS ESTIMATED USING APPROACH 2

Parameter	Initial value	Identified value
d	5.5 mm	5.6 mm
E_{weld}	220 GPa	222 GPa
E_{patch}	650 GPa	737 GPa
std. (d)	0.05 mm	0.06 mm
std. (E_{weld})	2.00 GPa	2.65 GPa
std. (E_{patch})	6.50 GPa	3.17 GPa

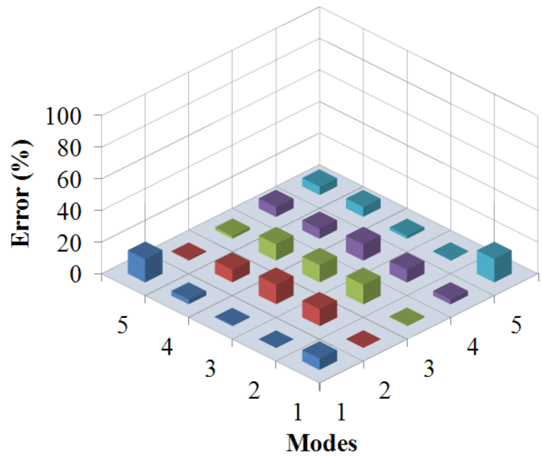

Figure 5. Error after updating between the measured and predicted measurements covariance using Approach 2

an accurate estimate of the parameter variability. General trends of the updated results are similar to that of the initials. The findings demonstrate that modification to the weighting assignment is capable of bringing the numerical results to convergence.

CONCLUSIONS

This paper studies a stochastic model updating problem for a set of welded structures with parameter variability. A perturbation method is employed and two approaches of assigning parameter covariance weighting matrices to the perturbation method have been introduced. The first approach considers only the main uncertain parameters, hence equal weighting has been employed when estimating the means and covariances of the parameters. On the other hand, more parameters are accounted for in the second approach regardless of their levels of variability. Two different weightings have to be introduced, one for each step of updating. The second approach has significantly improved the results over the first approach, and the predicted space has converged

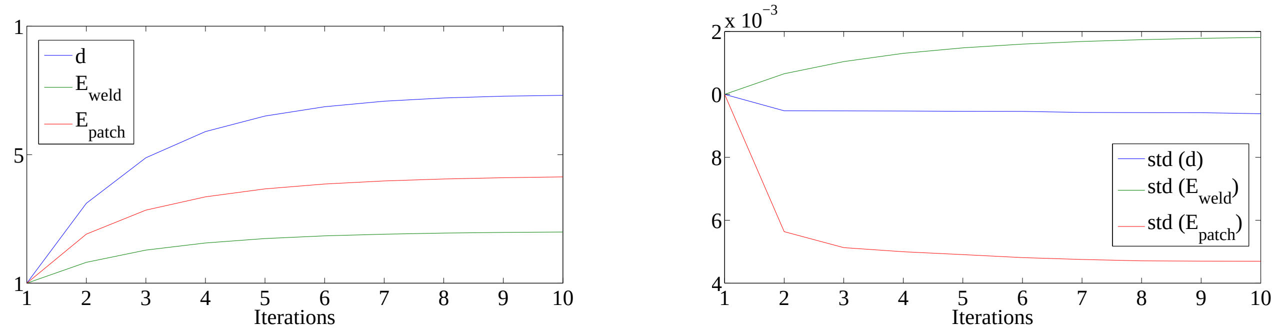

Figure 6. Convergence of parameter estimates

upon the measured space.

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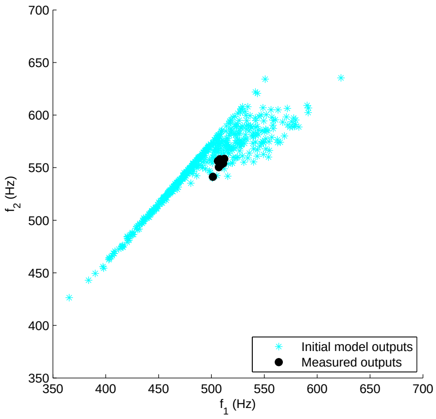
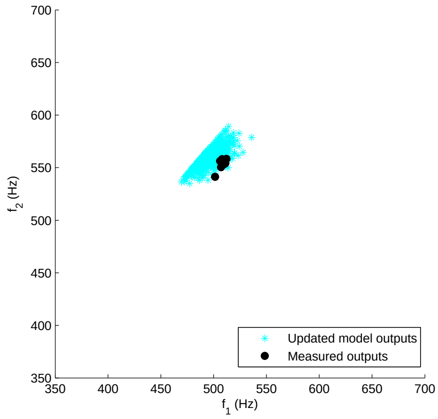
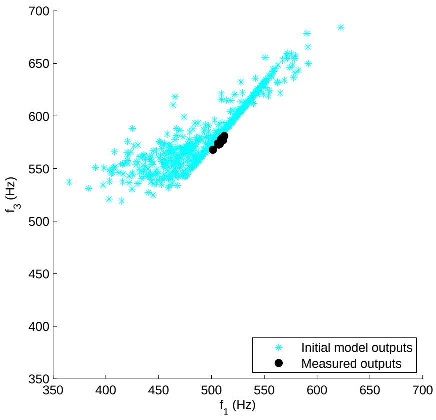


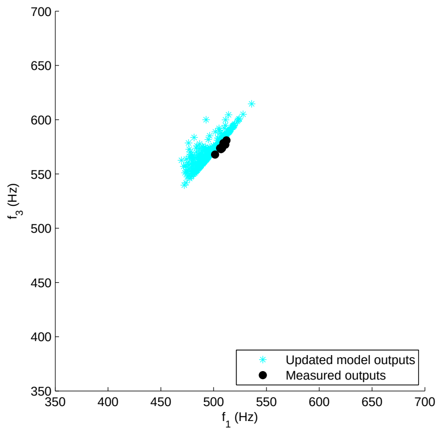
(a) Initial scatter plot for the first and second natural frequencies



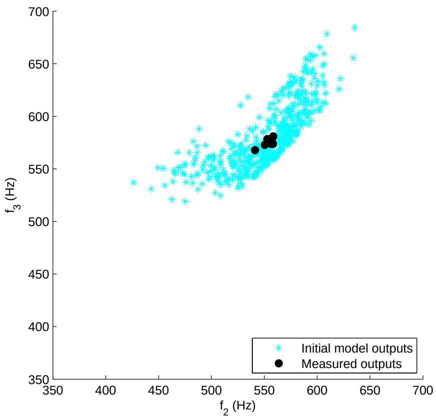
(b) Updated scatter plot for the first and second natural frequencies



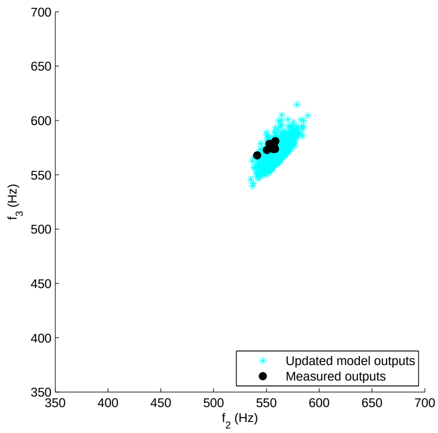
(c) Initial scatter plot for the first and third natural frequencies



(d) Updated scatter plot for the first and third natural frequencies

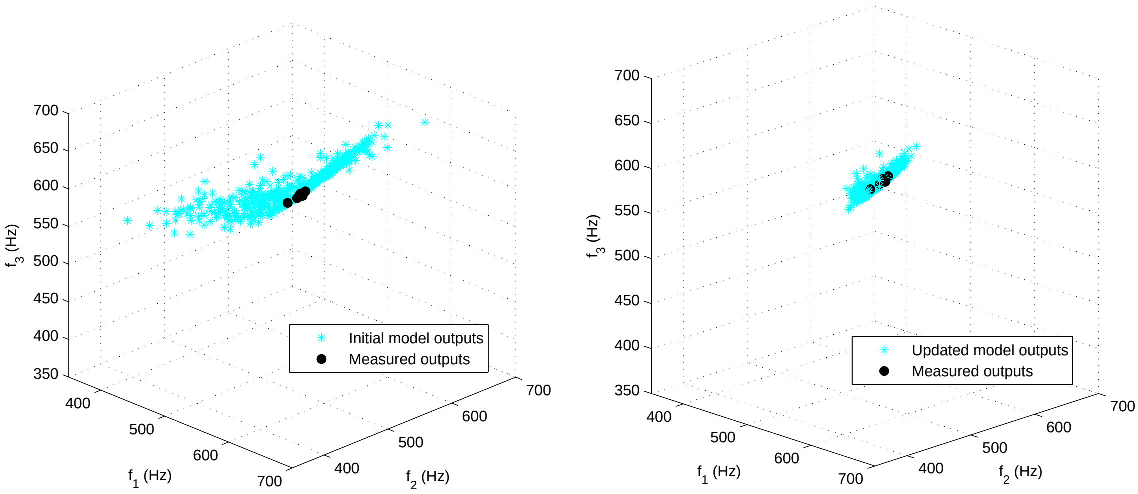


(e) Initial scatter plot for the second and third natural frequencies



(f) Updated scatter plot for the second and third natural frequencies

Figure 7. Initial and updated scatter plots for the first, second and third natural frequencies



(a) Initial scatter plot for the first three natural frequencies

(b) Updated scatter plot for the first three natural frequencies

Figure 8. Initial and updated scatter plots for the first three natural frequencies