Simulation of Aircraft Manoeuvres Based On Computational Fluid Dynamics

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The use of computational fluid dynamics to generate and test aerodynamic data tables for flight dynamics analysis is described in this paper. The test case used is the Ranger 2000 fighter trainer for which flight test data is available. The generation of the tables is done using sampling and reconstruction to allow a large number of table entries to be generated at low computational cost. The testing of the tables is done by replaying, through a time accurate CFD calculation which features the moving control surfaces, manoeuvres and comparing the forces and moments against the tabular values. The manoeuvres are generated using a time optimal prediction code with the feasible solutions based on the tabular aerodynamics. The generated manoeuvres are evaluated against flight data to show that they are qualitatively representative. Then the time accurate and tabular aerodynamics are compared, and as expected are in close agreement.

I. Introduction

The aerodynamic forces acting on aircraft need to be estimated for the analysis of the vehicle’s motion. The formulation of these forces date back to the early 1900s, when G.H. Bryan and W.E. Williams introduced the first stability-derivatives aerodynamic model, which is still found reliable for conventional aircraft in aerodynamically benign regions of the flight envelope. To meet the demands of accurately modeling the aerodynamics of manoeuvring aircraft, some improved methods are required for representing the nonlinear and unsteady flows around the aircraft. The source of nonlinearity and unsteadiness is mainly due to shock waves, separation and vortices at high angles of attack. The prediction of the flows over aircraft with high angle of attack and large amplitude manoeuvres is complicated by the fact that aerodynamic forces and moments of aircraft responding to sudden changes of flow not only depends on the instantaneous values but also the time history of the motion. There have only been a limited number of attempts to include the nonlinearity and unsteadiness. Examples include the work of Tobak and his colleagues for the non-linear indicial response methods, and Goman and Khrabov for the state-space model. The basic parameters of both methods are often identified based on the measured flight test data, but for a comprehensive model, a large number of training manoeuvres are needed that makes such a model very expensive.

Traditionally, aerodynamic data are obtained from wind tunnel testing or flight test data. The wind tunnel experiments can be expensive and suffer from scaling issues, along with difficulty in representing some dynamic motions. The flight test data are also very expensive and available late in the development. Today, CFD tools are becoming credible for simulating the physics of the aerodynamic flow. In principle,

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CFD can produce the aerodynamic inputs for the formulation of aircraft loads, also, simulate directly the
aerodynamic responses of a manoeuvring aircraft by coupling the time-dependent Reynolds-Averaged Navier-
Stokes (RANS) equations and the dynamic equations governing the aircraft motion. The moving rigid body
aerodynamics has been studied over the years, but its use was rather limited to the prediction of aircraft
dynamic derivatives\(^1\,^2\,^3\) or two-dimensional test cases.\(^4\,^5\,^6\,^7\) Recently, there is an increasing interest of the
coupled CFD-flight dynamics of a full geometry.\(^8\,^9\,^10\) Although the coupling is not likely to be a tool
for routine flight mechanics studies due to expensive computational cost, such a simulation could be very
powerful for investigating and understanding potential problem with the aerodynamic models.

The aerodynamic model considered in this paper is tabular in form. Tables, in contrast to stability
derivatives, are not linearized, and are consistent with quasi-steady aerodynamics for a wide range of flight
conditions. There are several difficulties associated with aerodynamic tables. First, a large number of table
entries must be filled. The brute-force calculation of the entire table using CFD is not feasible due to
computational cost. Sampling and data fusion methods have been proposed to overcome this.\(^21\) Secondly,
the pre-computed nature of tables lacks the ability to describe hysteresis of the aerodynamic phenomena.

The current study uses access to flight test data for the Ranger 2000 to evaluate CFD generated aero-
dynamics and manoeuvres. The four objectives of this paper are: (a) demonstrate the sampling/Kriging
approach to generating aerodynamic data tables; (b) test the realism of the manoeuvres generating using
an optimisation procedure; (c) compare the CFD predicted aerodynamics with the identified model; (d)
evaluate tables using time accurate replay. The paper first reviews the flow solver and how the manoeuvres
are computed. Then the test case is described and the aerodynamic predictions validated. The generation
of the aerodynamic tables and the approach to defining the time optimal manoeuvres is then given. The
evaluation of the aerodynamic tables for these manoeuvres is then made by replaying them directly through
an unsteady CFD calculation. Finally, conclusions are given.

II. Formulation

A. CFD

The flow solver used for this study is the University of Liverpool PMB (Parallel Multi-Block) solver. The
Euler and RANS equations are discretised on curvilinear multi-block body conforming grids using a cell-
centred finite volume method which converts the partial differential equations into a set of ordinary differ-
ential equations. The equations are solved on block structured grids using an implicit solver. A wide variety
of unsteady flow problems, including aeroelasticity, cavity flows, aerospike flows, delta wing aerodynamics,
rotorcraft problems and transonic buffet have been studied using this code. More details on the flow solver
can be found in Badcock et al\(^22\) and a validation against flight data for the F-16XL aircraft is made in
reference.\(^23\)

B. Table Generation

The aerodynamic model considered in this paper is tabular in form. The model consists of tables of forces
and moments for a set of aircraft states and controls (e.g. aircraft angle of attack, side-slip angle, control
deflections, etc.), which spans the flight envelope. This potentially entails a large number of calculations,
which will be a particular problem due to the computational cost if CFD is the source of the data. This issue
has been addressed by sampling and reconstruction based on Kriging interpolation and data fusion using
Co-Kriging as described in reference.\(^21\)

Two scenarios were considered, based on (1) a requirement for the generation of static table that include
the table of Mach number and angles of attack and side-slip and (2) for updating the static table for control
tables of elevator, rudder and aileron. Both approaches are available in the Computerized Environment for
Aircraft Synthesis and Integrated Optimization Methods, CEASIOM,\(^24\) computer code.

In the first scenario it is assumed that the static table is generated without user intervention. The
emphasis is on a sampling method which identifies the nonlinearities in the force and moment tables with
respect to the static aircraft state parameters. Approaches to the sampling based on the Mean Squared Error
(MSE) criterion of Kriging and the Expected Improvement Function (EIF) were considered in reference.\(^21\)

In the second scenario, it is assumed that the control tables are incremented from the static table, i.e.
the static table represents the main trends of aerodynamic forces and moments, while the controls result in
the increments of these trends. Data fusion based on Co-Kriging is then used to update the static table,
based on a small number of calculations of control surface deflections.

Using these techniques it was shown that tables which are practically useful could be generated in the order of 60 calculations under the first scenario and 10 calculations under the second scenario.

C. Time Optimal Manoeuvres

We use the optimal control approach,\textsuperscript{25,26} that finds the optimal controls that transfer a system from the initial state to the final state while minimizing (or maximizing) a specified cost function.\textsuperscript{27} The optimal control aims to find a state-control pair \(x^*(\cdot), u^*(\cdot)\) and possibly the final event time \(t_f\) that minimizes the cost function

\[
J[x(\cdot), u(\cdot), t_0, t_f] = E(x(t_0), u(t_f), t_0, t_f) + \int_{t_0}^{t_f} F(x(t), u(t), t) dt
\]

where \(x \in \mathbb{R}^n\) and \(u \in \mathbb{R}^m\). The function \(E\) and \(F\) are endpoint cost and Lagrangian (running cost), respectively.

There are many different methods of solving the optimal control problems, in the current paper, the commercially available code, DIDO\textsuperscript{28} and MATLAB\textsuperscript{29} are used. In DIDO, the total time history is divided into \(N\) segments, spaced using shifted Legendre-Gauss-Lobatto (LGL) rule.\textsuperscript{30–32} The boundaries of each time segment are called nodes. The code exploits pseudo-spectral (PS) methods for solving the optimal control problems.

For the problem of an aircraft optimal time manoeuvre, the general 6-degree-of-freedom aircraft equations of motion detailed in Etkin\textsuperscript{33} and Stevens and Lewis\textsuperscript{34} serve as one of the constraints. The aircraft state vector consists of the position of the aircraft \((x, y, z)\), the standard Euler angles \((\phi, \theta, \psi)\), the velocity components in terms of Mach number and flow angles\((M, \alpha, \beta)\), and the body-axis components of the angular velocity vector \((p, q, r)\). The initial and final state parameters are fixed with trimmed flight conditions, but the rest of the manoeuvre is out of trim conditions. The loads in the aircraft equations of motion are interpolated from the generated look-up tables.

D. Replay

The key functionality for the CFD solver in the current application is the ability to move the mesh. Two types of mesh movement are required. First, a rigid rotation and translation is required to follow the motion of the aircraft. Secondly, the control surfaces are deflecting throughout the motion. The control surfaces are blended into the geometry in the current work following the approach given in reference.\textsuperscript{35} After the surface grid point deflections are specified, transfinite interpolation is used to distribute these deflections to the volume grid.

The rigid motion and the control deflections are both specified from a motion input file. For the rigid motion the location of a reference point on the aircraft is specified at each time step. In addition the rotation about this reference point is also defined. Mode shapes are defined for the control surface deflections.\textsuperscript{18} Each mode shape specifies the displacement of the grid points on the aircraft surface for a particular control surface. These are prepared as a preprocessing step using a utility that identifies the points on a control surface, defines the hinge, rotates the points about the hinge and works out their displacements. The motion input file then defines a scaling factor for each mode shape to achieve the desired control surface rotation.

The desired motion to be replayed through the unsteady CFD solution is specified in the motion input file. The aircraft reference point location, rotation angles and control surface scaling factors are needed. The rotation angles are obtained straight from the pitch, yaw and bank angles. The aircraft reference point velocity \(v_a\) is then calculated to achieve the required angles of attack and sideslip, and the forward speed. The velocity is then used to calculate the location. The CFD solver was originally written for steady external aerodynamics applications. The wind direction and Mach number are specified for a steady case. If the initial aircraft velocity is denoted as \(v_0\), then this vector is used to define the Mach number and the angle of attack for the steady input file. Then the instantaneous aircraft location for the motion file is defined from the relative velocity vector \(v_a - v_0\).

III. Test Case

The aircraft considered in this paper is the Ranger 2000. This is a mid-wing, tandem seat training aircraft powered by one turbofan engine with uninstalled thrust of 14190 N. The wing and fuselage are manufactured...
of composite material and the empennage is a metal T-tail design. A three-view of the vehicle is shown in Fig. 1. Also, the general dimensions and the mass/inertial properties for both maximum take-off weight (MTOW) and operating empty weight (OEW) are listed in tables 1 and 2.

![Figure 1. Three-Views of Ranger 2000](image)

<table>
<thead>
<tr>
<th>Table 1. General Dimensions of Ranger 2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length Overall (m)</td>
</tr>
<tr>
<td>Wing Span (m)</td>
</tr>
<tr>
<td>Height Overall (m)</td>
</tr>
<tr>
<td>Wing Area m$^2$</td>
</tr>
<tr>
<td>Mean Aerodynamic Chord (MAC) (m)</td>
</tr>
<tr>
<td>Wing Taper Ratio</td>
</tr>
</tbody>
</table>

The operational envelope in terms of the flight altitude and speed is shown in Fig. 2 for the specified weight of 3765 kg. The envelope has a maximum operational ceiling of 9449 m and the diving speed ($M_D$) and the maximum operating speed ($M_{MO}$) of 0.7 and 0.75, respectively.

The vehicle flight control system consists of three conventional control surfaces: The elevator at the tail, the rudder at the fin and left and right ailerons (Fig. 3). The position limits of each surface is given in table 3, with positive angles indicating the elevator trailing edge down, the left aileron trailing edge down and the rudder trailing edge toward the left wing (see Fig. 3).

The flight test data consists of all aerodynamic forces and moments with respect to the aircraft states of: angle of attack, side-slip angle, Mach number, rotational rates, acceleration rates, elevator, rudder, control and the altitude of flight. Table 4 summarizes the range of measured data. Various aerobatic manoeuvres were performed to demonstrate general aircraft handling qualities. These include Barrel Rolls, Clover Leafs, Immelmann Turns, inverted flight, Lazy Eights, Loops, and Split-S. The entry conditions and the time histories for each manoeuvre are provided by EADS military air systems.

A multiblock structured grid was generated using the commercial grid generation tool ICEMCFD. Both
Table 2. Mass/Inertias of Ranger 2000

<table>
<thead>
<tr>
<th></th>
<th>MTOW</th>
<th>OEW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{xx}$ (kgm$^2$)</td>
<td>9287.1</td>
<td>4600.9</td>
</tr>
<tr>
<td>$I_{yy}$ (kgm$^2$)</td>
<td>13584.1</td>
<td>13462.3</td>
</tr>
<tr>
<td>$I_{zz}$ (kgm$^2$)</td>
<td>21237.6</td>
<td>16474.6</td>
</tr>
<tr>
<td>MTWO (kg)</td>
<td>2586</td>
<td></td>
</tr>
<tr>
<td>MTWO(maximum) (kg)</td>
<td>3765</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Ranger 2000 Accelerated Climb Rate with 3765 kg Gross Mass

Table 3. Control surface deflection limits.

<table>
<thead>
<tr>
<th>Control Surface</th>
<th>Deflection Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elevator</td>
<td>$\delta_{\text{ele}}$ -25°/+15°</td>
</tr>
<tr>
<td>Right Aileron</td>
<td>$\delta_{\text{ail}}$ -25°/+15°</td>
</tr>
<tr>
<td>Left Aileron</td>
<td>$\delta_{\text{ail}}$ -25°/+15°</td>
</tr>
<tr>
<td>Rudder</td>
<td>$\delta_{\text{rud}}$ ±17.5°</td>
</tr>
</tbody>
</table>

Table 4. The boundary of experimental data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle of Attack</td>
<td>$\alpha$ -7°/+36°</td>
</tr>
<tr>
<td>Side-slip Angle</td>
<td>$\beta$ -20°/+20°</td>
</tr>
<tr>
<td>Mach number</td>
<td>$M$ 0/0.75</td>
</tr>
<tr>
<td>Altitude</td>
<td>$h$ 0/10 km</td>
</tr>
</tbody>
</table>
full and half configurations were generated, the latter to save on computing costs in the simulation of longitudinal flight dynamics. The lateral flight dynamics tables, on the other hand, required the full model flow prediction. The half configuration has 14.5 million points arranged in 2028 blocks.

Figure 4 (a) shows the overall view of the meshed geometry. Figures 4 (b) and (c) show the diamond shaped tip topology used to accommodate the cells around the main wing and tail-plane. The wing has a H-type topology around the leading edge to improve the cell quality in the wing-engine-fuselage junction while the horizontal tail-plane blocking consists of a C-type round its LE.

The aforementioned control surfaces are also included in the geometry (Fig. 3). These were the elevator, aileron and rudder which can be deflected for steady state calculations and during time accurate simulations. The half grid requires 1.5 hours on 128 processors on the United Kingdom’s academic supercomputer (Cray inc) to obtain a fully converged steady state solution.

Figure 3. Ranger control surfaces- The configuration shows the positive deflection angles.

IV. Results

A. Generation of Aerodynamic Tables

The format of the aero look-up table used in the CEASIOM code is given in table 5. The aerodata provides the aerodynamic forces and moments for each point corresponding to the aircraft states.

<table>
<thead>
<tr>
<th>θ</th>
<th>M</th>
<th>δ_e</th>
<th>δ_a</th>
<th>δ_r</th>
<th>C_L</th>
<th>C_D</th>
<th>C_m</th>
<th>C_Y</th>
<th>C_l</th>
<th>C_n</th>
</tr>
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<tbody>
<tr>
<td>x</td>
<td>x</td>
<td>x</td>
<td>-</td>
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</tr>
</tbody>
</table>

where θ is the angle of attack, M is the Mach number and δ is the side slip angle. The last six columns are the coefficients of lift, drag, pitching-moment, side-force, rolling moment and yawing moment. Based on this format, four three-parameter sub-tables need to be generated.

The table (angle of attack, Mach number and side-slip angle) was generated from scratch using sampling. A 500 entry table was first generated using the PMB solver for 65 calculations. The generated table describes the underlying behaviour of aerodynamic forces and moments. Note that the angles of attack and Mach number in the tables are limited to 12 degrees and 0.6, respectively. All flight test data are within these limits.
Next, the tables for varying angle of attack, Mach number and control surface deflections were generated. Twelve additional samples for each configuration at non-zero deflection angles were calculated. The values at these samples were then used to increment the static table values using Co-Kriging. For the purpose of generation of manoeuvres, instead of the actual values, the increments of aerodynamic forces and moments are stored in tables, i.e. \( \Delta C_{L\delta_{\text{ele}}} \), \( \Delta C_{D\delta_{\text{ele}}} \), and etc.

Wind tunnel and flight data for the aerodynamic coefficients are available at low and transonic speeds. The comparisons between the measurements and Euler predictions using PMB are shown in Figs 5-6 for two speeds. The lift coefficient results match the measurements well, with the exception of an overprediction at high angle of attack. At low speeds, the drag is shifted down when compared with the measurements, reflecting the lack of the skin friction contribution. Also, the pitching moment curve slope from the Euler results is more negative.

The drag predictions at transonic speed become closer as the angle of attack increases, reflecting the increasing dominance of wave drag as the shock wave increases in strength. The formed shock on the upper surface of the wing at eight degree angle of attack is presented in Fig. 7. As the angle of attack increases at transonic speed, the shock becomes stronger and this results in a forward shift of the aerodynamic centre and consequently changes the slope of pitching moment (see the pitching-moment curve in Fig. 6). Inspecting the CFD solutions shows that the Euler solver does predict the shock formation and movement. The Euler solutions also predict the trends of the lateral coefficients, however, the results do not match the measurements due to the lack of the skin friction contribution.

Wind tunnel data for the aerodynamic coefficients are available at low speed. The trend of control power with angle of attack from CFD and experimental data is compared in Figs. 8(c)-(e). The flight speed is M=0.25 with zero side-slip angle for all the figures. The results of two elevator surface deflections are presented, while the angle of deflection for rudder and aileron is +9 degree. The results are in agreement with measured data in particular at low angles of surface deflection. Difference are possibly due to using the blended approach for the treatment of surface deflections.

In this paper, the dynamic effects are modeled by the stability derivatives that represent the influence of the aircraft rates around the three body axes on the aerodynamic forces and moments. A well-established
Figure 5. Validation of static table at M=0.25. The lift, drag and pitching moment correspond to zero side-slip angle, while the side-slip angle in lateral coefficients have is $-8^\circ$. The circles denote experiments. All moment coefficients are in the body axis.
Figure 6. Validation of static table at M=0.6. The lift, drag and pitching moment correspond to zero side-slip angle, while the side-slip angle in lateral coefficients have is $-8^\circ$. The circles denote experiments. All moment coefficients are in the body axis.
framework\textsuperscript{12} for the prediction of dynamic derivatives was defined using the PMB solver. The dynamic derivatives were estimated by processing the solution in the time domain using a developed FFT algorithm. A linear regression model was also implemented, providing the very similar values to the frequency domain technique.

B. Generation and Evaluation of Manoeuvres

The flight data available for the Ranger2000 is used for two purposes in this paper. The predicted aerodynamic forces and moments are validated against the identified model in the usual way. Secondly, the manoeuvres generated by the time optimal prediction code are tested against the flight manoeuvres to show that they are qualitatively similar. This is important so that the time accurate replay of the optimal manoeuvres is for realistic motions. To generate the optimal manoeuvres the following assumptions and approach is taken.

- The propulsive moment terms are zero in the aircraft equations of motion.
- The variation of the engine force with altitude and flight speed is estimated using semi-empirical equations given in Roskam.\textsuperscript{36}
- No ground trajectory is reported in the test data, so these values are defined by an upper and lower value.
- Moments of inertia are not given for each manoeuvre. The maximum take-off data are used for all cases.
- The vertical movement of the centre of gravity is not available and therefore, not considered.

Three manoeuvres are generated. The first is the lazy eight (Fig. 9-a). The aircraft performs a climbing and rolling followed by a diving turn until the final aircraft heading is 180\degree changed from the initial heading.

A manoeuvre is defined in DIDO using the available experimental tables. The aircraft is allowed to roll from 0\degree to 90\degree. The manoeuvre is initiated from a straight and wing level position, then starts a climb while increasing the roll and yaw angles at the same time. Near the maximum altitude, the roll angle has reached 90\degree and then the aircraft starts to dive with rolling towards the left wing. The heading is still increasing until it reaches 180\degree and the aircraft is back to the initial altitude, velocity and the roll angle.
Figure 8. Control tables at $M=0.25$. The deflection angle for aileron and rudder is nine degrees. The circles denote experiments.
Figure 9. Manoeuvres generated using DIDO and experimental aero data. The aircraft to the flight path scale in the barrel roll and lazy eight is 100:1. For the Immelmann Turn this is 50:1.
Figure 10. Time histories of states and controls of Lazy eight simulated in DIDO. The results correspond to 60 DIDO nodes, using CG=24.077% of mean aerodynamic chord and the aircraft mass of 3498 kg. The lines with filled circles show the flight test data.
An optimal solution is found and the results are shown against flight test data in Fig. 10. The simulation results closely follow the trend of flight test data.

Next, for a barrel roll (Fig. 9-b) the aircraft performs a complete rotation around its longitudinal axis. For a barrel roll to the left, the maneuver is initiated by a pitch up and a roll turn to the left. During this part of the flight, the left wing is the lowest wing, while the aircraft yaws to the left. At the maximum altitude, the aircraft is nearly upside down. During the second half of the roll, the right wing is lowest and the aircraft tends to yaw to the right.

Again DIDO was used to re-generate the maneuver. The roll test angles are in the range of \([-180^\circ, 180^\circ]\). The aircraft is initially straight and wing level, then starts a roll turn, while climbing at the same time. Near the maximum altitude, the roll angle has reached \(-180^\circ\), i.e. the aircraft is upside down. Next, the aileron is gradually relaxed to complete the turn by bringing the roll angle again to zero.

The starting speed and altitude of the minimum time maneuver are set to 158 m/s and 6076 m, respectively. Also, the angles of attack should not exceed the upper limit of 12 degrees. The elevator, aileron and rudder deflection values are also limited to the upper and lower values given in flight test data. An optimal solution was found and the results are shown against flight test data in Fig. 11. The results show that the optimal solution resembles the flight test data. The main differences are probably due to setting take-off moments of inertia for this maneuver.

The Immelmann turn, a combined roll and loop maneuver, is initiated with a pitch up flight at the entry, and then continues to pull back on the controls such that the aircraft draws a loop in the longitudinal plane. The rudder and ailerons are deflected in a way that the half-loop becomes straight when viewed from the ground. At the end of this loop, the aircraft is upside down, then the aileron(s) is applied to execute a half-roll turn to correct the aircraft orientation. The maneuver is terminated at a higher altitude and a 180 degrees reversion of heading. The initial flight speed plays an essential role to accomplish this maneuver.

Based on available flight test data, an Immelmann turn maneuver is defined. The simulated maneuver changes the heading of the aircraft from 0 to 180°, while the final values of the velocity and latitude (x) are fixed with those at the initial time. The starting speed and altitude of the minimum time maneuver are set to 163.57 m/s and 2986 m, respectively. Also the angles of attack should not exceed the upper limit of 12 degrees. The elevator, aileron and rudder deflection values are also limited to the upper and lower values given in flight test data. The final maneuver is shown in Fig. 12 plotted against flight test data.

The results show that predicted values of altitude, aircraft flight speed and Euler angles match well with flight test data. Although the general trends of angle of attack and elevator deflection are also closely captured using DIDO time-optimal solution, the elevator oscillates at initial steps and there is a discrepancy at final values. High frequency values are seen for the aileron and rudder deflections.

The Immelmann turn contains an inherent discontinuity point. During the upward trajectory and before the pitch angle of the aircraft hits the maximum allowed angle of 90°, the heading and bank angles remain virtually constant. Just after the maximum pitch angle point, the aircraft orientation is suddenly reversed, resulting in a sharp jump of the heading and bank angle values at this point. The presence of discontinuities incurs serious theoretical and numerical difficulties for the optimal control problem.\(^7\)\(^7\) The implementation of standard direct pseudospectral(PS) method for the trajectory optimization problems with the jump discontinuities in the states exhibit an over-shoot of predictions at the jump, named Gibbs phenomenon. In such a case, the nodes refinement might lead into inefficiencies and an ill-conditioned problem.

The step discontinuity is seen around \(t = 20\) seconds. The lateral predictions are influenced by the presence of this discontinuity. Very small amplitude Gibbs oscillations are recorded at the yaw and roll angles, however the impacts are much larger for predicted values of side-slip angle and surface deflection of rudder and aileron.

C. Replay of Maneuvers

The unsteady Euler solver was applied to the DIDO solution in order to evaluate direct aerodynamic coefficients. The predictions were compared against aerodynamic forces from the tabular model. The comparisons test the CFD formulation of the maneuver replay, which is done in a time accurate fashion with control surface deflections. The lateral force and moment coefficients are small for the studied maneuvers, here, only longitudinal forces and moment are compared.

In all cases, the full aircraft geometry was used with the grid scaling of 1.0. A converged steady-state solution was used as the starting condition. These conditions correspond to the initial point \((t = 0)\) of each maneuver. The physical time step correspond to the number of nodes used in the optimal code, DIDO.
Figure 11. Time histories of states and controls of Barrel roll simulated in DIDO. The results correspond to 60 DIDO nodes, using CG = 24.297% of mean aerodynamic chord and the aircraft mass of 3653 kg. The lines with filled circles show the flight test data.
Figure 12. Time histories of states and controls of Immelmann Turn simulated in DIDO. The results correspond to 60 DIDO nodes, using CG=24.265% of mean aerodynamic chord and the aircraft mass of 3629 kg. The lines with filled circles show the flight test data.
The CFD solver time is found from scaling the physical time based on the grid reference length and the reference velocity. The number of the time step was set to $n_s = 5000$ that has a uniform stepping from $t_1$ to $t_{n_s}$. The aircraft reference point velocity at each time step was defined from the initial conditions and the interpolation of DIDO solution. The surface-volume solutions and the body-axis forces and moments computed at each CFD time step are available.

The first manoeuvre presented is the Lazy-Eight. Based on the time histories of the DIDO solution, the values of aircraft reference point velocity, $v_a$, at each time step was defined. The initial steady state velocity, $v_0$, was set to 143 m/s. The steady state solution has 1.6 degree angle of attack obtained from the initial point of the manoeuvre. The tabular and replay values for the lift, drag and pitching moment are compared in Fig. 13. The results show that the tables are in perfect agreement with the replay solution. During the manoeuvre the angle of attack remains below six degrees with a maximum Mach number of 0.44. This aircraft has no time hysteresis at such a conditions and as expected the tables perfectly match the direct solution from the replay.

![Figure 13. Lazy-Eight replay solution. The solid line shows the replay simulation.](image)

The second manoeuvre considered is the Barrel-roll. The initial steady conditions are: $v_0 = 159$ m/s and an angle of attack of 1.9 degrees. From the DIDO solution, the grid point locations and grid point velocities are set. The tabular and replay values for the lift, drag and pitching moment are compared in Fig. 14. The results show good agreement with the replay solution. The angle of attack during manoeuvre time remains below eight degrees, while a good agreement is expected between the tables and the replay.

Finally, the results of the Immelmann turn are presented in Fig. 15. The initial steady conditions are: $v_0 = 164$ m/s and AOA=0.06. The trends of lift, drag and pitching moment closely follow the replay with some localized differences between the replay and tabular values. There are no history effects from the state parameters since the aircraft flies in aerodynamically benign regions of the flight envelope. The differences are mainly due to dynamic effects from the very rapid motion of the control surfaces. As discussed earlier, the Immelmann turn contains an inherent discontinuity point and results in the time optimal solution having
Figure 14. Barrel Roll replay solution. The solid line shows the replay simulation.
high frequency and large amplitude motion of the control surfaces. The effects of rapid motion of the control surface have been discussed in a paper by Ghoreyshi et al.\textsuperscript{20}

![Graphs of aerodynamic coefficients](image)

**Figure 15.** The Immelmann turn replay solution. The solid line shows the replay simulation.

V. Conclusions

The paper investigate the validity of the tabular approach for the maneuvering aircraft by testing the aerodynamic model through replaying manoeuvres using an unsteady CFD calculation to check the consistency of the aerodynamic forces and moments. The CFD solver uses two types of mesh movement, namely rigid motion, and transfinite interpolation for the control surfaces.

The Euler equations were used to demonstrate the replay framework. Sampling and Data fusion was used to allow the generation of the aerodynamic tables in a feasible number of CFD calculations. The validation against available experimental data showed the solutions are credible for the shock effects, but they fail to predict the drag force and lateral coefficients due to lack of skin friction.

Three aerobatic manoeuvres were generated using the time-optimal solver, DIDO. The results were compared with available flight test data. The trajectories of both simulation and test data match well for all cases. In terms of control values, the Immelmann turn simulation includes high frequency motion of the surface deflections, due to the presence of a discontinuity point in the roll angle. The time-optimal solver for the trajectory optimization experiences problems with the jump discontinuities in the states due to an over-shoot of predictions at the jump from the Gibbs phenomenon. This results in the formation of spikes in the replay solutions due to dynamic effects from the very rapid motion of the control surfaces.

The results of tabular predictions were compared against the coefficients from the replaying the manoeuvre using an unsteady CFD calculation. For all considered manoeuvres the aircraft flies in aerodynamically benign regions of the flight envelope and the tables perfectly match the replay solution. The results of
this paper demonstrated the validity of using tables for such conditions. Overall, the paper illustrates the validity of the CFD framework for generating aerodynamic tables and replaying manoeuvres to test them. This framework, when applied to more demanding manoeuvres can show the limitations in the tabular formulation.

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