Taylor Series Expansion and Least Squares-based Lattice Boltzmann Method

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The development of the lattice Boltzmann method (LBM) as an alternative computational fluid dynamics approach has attracted more and more attentions in recent years [1-8]. However, because of the essential restriction of the standard lattice Boltzmann equation (LBE) to the lattice-uniformity, the broad application of the LBM in engineering has been greatly hampered. For many practical problems, an irregular grid or a meshless structure is always preferable due to the fact that curved boundaries can be described more accurately, and that computational resources can be used more efficiently with it.

The drawback of the standard LBE restricting to the lattice-uniformity comes from its precursor—the lattice gas cellular automata (LGCA) [9-10]. In the LGCA, the symmetry of lattice, which guarantees the isotropy of the fourth tensor consisting of particle velocities, is an essential condition to obtain the Navier-Stokes equations. By this condition, a particle at one lattice node must move to its neighboring node in one time step. This is the condition of lattice-uniformity. Although the LBE [11-12] with Bhatnagar-Gross-Krook (BGK) [13] model has made great improvements over the LGCA, it also inherits the feature of lattice-uniformity, which makes it macroscopically similar to a uniform Cartesian-grid solver.

Theoretically, the feature of lattice-uniformity is not necessary to be kept because the distribution functions are continuous in physical space. Currently, there are two ways to improve the standard LBM so that it can be applied to complex problems. One is the interpolation-supplemented LBM (ISLBM) proposed by He and his colleagues [14-16]. The other is based on the solution of a differential lattice Boltzmann equation (LBE). For complex problems, the differential LBE can be solved by the finite difference (FDLBE) method with the help of coordinate transformation [17] or by the finite volume (FVLBE) approach [18-21]. Numerical experiences have shown that these methods have good capability in real applications. However, the ISLBE has an extra computational effort for interpolation at every time step and it also has a strict restriction on selection of interpolation points, which requires upwind nine points for two-dimensional problems and upwind twenty-seven points for three-dimensional problems if a structured mesh is used. For the FDLBE and FVLBE methods, one needs to select efficient approaches such as upwind schemes to do numerical discretization in order to get the stable solution. As a consequence, the computational efficiency greatly depends on the selected numerical scheme. In addition, the numerical diffusion may affect the accuracy of results, especially in the region where the flow gradient is large.

In order to implement the LBE more efficiently for flows with arbitrary geometry, we propose in this work a new version of LBM, which is based the standard LBM, the well-known Taylor series expansion, the idea of developing Runge-Kutta method [22], and the least squares approach [23]. The final form of our method is an algebraic formulation, in which the coefficients only depend on the coordinates of mesh points and lattice velocity, and are computed in advance. The new method is also free of lattice models. To validate the proposed method, some theoretical analysis and a generalized hydrodynamic analysis are presented. Numerical simulations include "no flow" in the closed square cavity, a lid-driven flow in a square cavity and a polar cavity flow. All simulations use a non-uniform mesh with mesh points strongly clustering to the boundary. The obtained numerical results are very accurate. Numerical experiences showed that the present method is an efficient and flexible approach for practical application.

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