The Role of the Kutta-Joukowski Condition in the Numerical Solution of Euler Equations for a Symmetrical Airfoil.

M.Z.Dauhoo, Dept. of Math, University of Mauritius, Rep. of Mauritius.
E-mail : m.dauhoo@uom.ac.mu

Abstract

The solutions of the Euler equations are the approximate solutions of the Naviers-Stokes equations in the limit of vanishing viscosity (viscosity $\mu \to 0$ but $\mu \neq 0$). These solution are used to predict the lift experienced by airfoils and wings within the framework of inviscid flow, at a certain angle of attack. The classical Kutta-Joukowski hypothesis enables us to determine these solutions by imposing the Kutta-Joukowski condition at the sharp trailing edge of the airfoil.

In this work, we study the question of how the circulation required for lift is produced when time marching Euler calculations are performed for an airfoil. We discuss the vorticity production, within the framework of inviscid calculation, and its role in the generation of the lift within the framework of Euler codes used in CFD.

Theory

The curl of the momentum equation gives,

$$\frac{\partial\, \omega}{\partial\, t} + u_1 \frac{\partial \omega}{\partial x} + u_2 \frac{\partial \omega}{\partial y} = -\frac{1}{\rho^2} \left[\nabla \mathbf{p} \wedge \nabla(\rho) \right] \qquad (1)$$

The left hand side of Eq.(1) has the time rate of change of ω and the term giving vorticity advection. The right hand side of Eq.(1) vanishes when $\mathbf{p}$ and ρ are isentropically related. If $\nabla \mathbf{p}$ is not parallel to $\nabla \rho$, then this term produces vorticity. Such a production of vorticity takes place through what is termed as the *baroclinic mechanism.* For inviscid flows with the Kutta-Joukowski condition prevailing at the trailing edge, there is a sudden discontinuity in the wall boundary condition. As a result, sharp velocity gradients are developed and a non-isentropic change thereby takes place in the flow field in that location. Thus, strong gradients in pressure and density are set up and they in turn produce vorticity.

Results

In order to study the vorticity distribution at steady state, we have chosen a subsonic flow around NACA 0012 airfoil with $M_\infty = 0.63, \alpha = 2^0$. The two dimensional distributions of vorticity at steady state, for relatively coarse (120x60), medium (160x90) and fine (320x60) grid respectively, are shown in Fig.(1). Also, we computed the flow field for a fine grid of 28800 nodes (240x120) at low Mach number that is, $M_\infty = 0.16, \alpha = 2^o$.

For low Mach number computations, that is, in the incompressible limit, our numerical investigations show that the baroclinic mechanism is present (Fig.(2) and Fig(3)). In other words, the flow is always compressible in the neighbourhood of the trailing edge.

Our results suggest that as the grid is progressively refined, the vorticity will tend to a Dirac function, that is, it will be very large at the sharp trailing edge and comparatively zero everywhere in the domain.

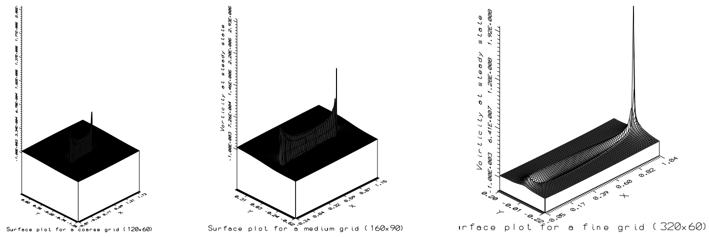

Figure 1: Surface Plots of Vorticity, at steady state, for different grids. ($M_\infty = 0.63, \alpha = 2°$)

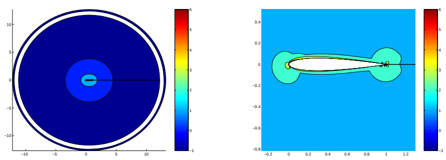

Figure 2: The Baroclinic term for the whole computational domain and close to the airfoil respectively on fine grid . ($M_\infty = 0.16, \alpha = 2°$)

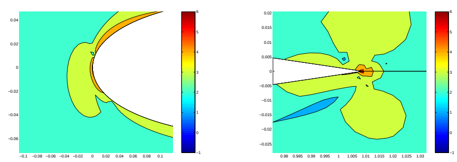

Figure 3: The Baroclinic term at the leading and trailing edge respectively on fine grid. ($M_\infty = 0.16, \alpha = 2°$)