Interval Flutter Analysis using the Transformation Method

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Contents

Definition of Robust Flutter Stability

- Dynamic aeroelastic problems attach great importance for new aircraft designs
- Consideration of all possible aircraft configuration including failure cases for certification
- Numerical simplification of aeroelastic models for simulation purposes caused by methological and economical constraints
- Verification of dynamic models by comparison with results from Ground Vibration Test GVT → *Deviations between model and experimental results*
- No available simulation models for small aircrafts, relying on experimental vibration data, measurement errors
  - → Robust flutter analysis propagates the effects of identified uncertainties towards aeroelastic stability of the aircraft to cover the uncertain-but-bounded parameter space.
  - → Robust stability is guaranteed when the uncertainties cannot destabilize the aeroelastic system.

Uncertainty Propagation in Modal Analysis

- Uncertain-but-bounded structural parameters cause perturbation in physical stiffness and mass matrix of aircraft
- Both are formulated as interval matrices with centrum and perturbation (radius)

$$\left[K^I\right]=\left[\left[K^C\right]-[\Delta K],\left[K^C\right]+[\Delta K]\right]$$

$$\left[M^I\right]=\left[\left[M^C\right]-[\Delta M],\left[M^C\right]+[\Delta M]\right]$$

- Interval eigenvalue problem:

$$\left(\left[K^C \pm \Delta K\right]-\left(\omega_r^I\right)^2\left[M^C \pm \Delta M\right]\right)\left\{\phi^I\right\}_r=\{0\}$$

- For solution several perturbation or interval eigenvalue solver are available
- Uncertainty leads to centered eigenfrequencies and modeshapes with perturbation: $\left\{\omega^I\right\}=\left\{\omega^C\right\}\pm\{\Delta\omega\},\quad\left[\Phi^I\right]=\left[\Phi^C\right]\pm[\Delta\Phi]$
- Similiar formulation can be found for experimental GVT-results

Uncertainty Propagation in Aerodynamic Loads

- Unsteady aerodynamic theory in subsonic range: ***Doublet-Lattice-Method***:
- Differential pressure of each aerodynamic box dependent on the downwash from normal modeshapes:

$$\{\Delta c_p\} = \left[AIC(M_\infty, k)\right]\{w\}$$

- Aerodynamic influence coefficient matrix AIC only depends on geometry, Mach number and reduced frequency
- Downwash w is calculated from structural mode shapes by multiplication of transformation matrices
- Modal aerodynamic loads are integrated pressures weigthed by modal deflections

$$[Q] = [\Phi]^T \underbrace{[T]^T [S]\left[AIC(M_\infty, k)\right]\left(\frac{\partial [T]}{\partial x} + \frac{ik}{c}[T]\right)}_{FIC}[\Phi]$$

- Propagation of interval mode shapes leads to interval generalized aerodynamic loads:

$$\left[Q^I\right] = \left[\Phi^c \pm \Delta\Phi\right]^T [FIC]\left[\Phi^c \pm \Delta\Phi\right]$$

$$= \underbrace{\left[\Phi^C\right]^T [FIC]\left[\Phi^C\right] + [\Delta\Phi]^T [FIC][\Delta\Phi]}_{\left[Q^C\right]} \pm \underbrace{\left[\Phi^C\right]^T [FIC][\Delta\Phi] \pm [\Delta\Phi]^T [FIC]\left[\Phi^C\right]}_{[\Delta Q]}$$

Direct Solution of Flutter Equations

- Flutter equations are formulated in Laplace domain $s = \sigma + i\omega$

$$\underbrace{\left([M]s^2 + [C]s + [K] - \frac{1}{2}\rho_\infty V_\infty^2 \left[Q(M_\infty, k)\right] \right)}_{\left[F\left(s,\rho_\infty,V_\infty,M_\infty,k\right)\right]} \{q\} = \{0\}$$

- Eigenvector $\{q\}$ is one non-unique solution of parameter-dependent flutter coefficient matrix $[F]$
- Determination of ***unique*** solution requires additional constraints:
  1. Normalization of complex eigenvector in value and phase
  2. Relationship eigenvalue - reduced frequency:
- System of expanded non-linear equations:

$$\{y(\{x\})\} = \left\{ \begin{array}{c} \left[F(s,\rho_\infty,V_\infty,M_\infty,k)\right]\{q\} \\ \{q\}^T\{q\} - 1 \\ \mathrm{Im}(s) - \dfrac{V_\infty}{c}k \end{array} \right\} = \{0\}$$

- Application of available Numerical Continuation Methods

$$\{x\}^T = \left\{\{q\}^T, s, k, V_\infty\right\}^T$$

- *Advantage in comparison to available solutions (e.g. p-k-methods):*

  eigenvalue + eigenvector are used to find new solutions for increasisng flight velocity

  → no commutation of solution branches (important for interval analysis)

Numerical Continuation Method

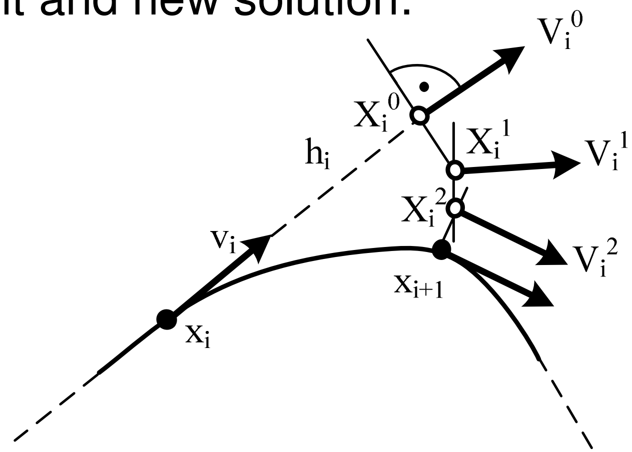
- Numerical Continuation: Method to find successively solutions along one solution branch with *predictor-corrector*-algorithm
  - MATLAB-Toolbox: MATCONT (www.matcont.ugent.be)
- ***Predictor step***:
  - starting from estimated solution $\{x\}_i$
  - extrapolation along normalized tangent $\{v\}_i = \frac{dy}{dp}$ with stepsize h

$$\{x\}^0 = \{x\}_i + h\{v\}_i$$

- ***Corrector step***: Newton-iteration finds the nearest point on the solution curve using orthogonality between tangent and new solution:

$$\begin{Bmatrix} y(\{x\}) \\ \left(\{x\} - \{x\}^0\right)^T \{v\}_i \end{Bmatrix} = \{0\}$$

$$\{x\}^{k+1} = \{x\}^k - \underbrace{\left(\left. \frac{d\left[y\left(\{x\}^k\right)\right]}{d\{x\}} \right|_{\{x\}^k} \right)^{-1}}_{Jacobian\,Matrix} \left[y\left(\{x\}^k\right)\right]$$



Determination of Tangent Vector / Jacobian Matrix

- All free parameter are fixed, except flight velocity
- Tangent vector of velocity is determined by Jacobian matrix

$$\underbrace{\begin{Bmatrix} \frac{d(\mathrm{Re}\{q\})}{dV_\infty} \\ \frac{d(\mathrm{Im}\{q\})}{dV_\infty} \\ \frac{d\sigma}{dV_\infty} \\ \frac{d\omega}{dV_\infty} \\ \frac{dk}{dV_\infty} \end{Bmatrix}}_{\frac{d\{x\}}{dV_\infty}} = \underbrace{\begin{bmatrix} \mathrm{Re}([F]) & -\mathrm{Im}([F]) & \mathrm{Re}\left(\frac{\partial[F]}{\partial s}\{q\}\right) & -\mathrm{Im}\left(\frac{\partial[F]}{\partial s}\{q\}\right) & \mathrm{Re}\left(\frac{\partial[F]}{\partial k}\{q\}\right) \\ \mathrm{Im}([F]) & \mathrm{Re}([F]) & \mathrm{Im}\left(\frac{\partial[F]}{\partial s}\{q\}\right) & \mathrm{Re}\left(\frac{\partial[F]}{\partial s}\{q\}\right) & \mathrm{Im}\left(\frac{\partial[F]}{\partial k}\{q\}\right) \\ \mathrm{Re}(\{q\})^T & -\mathrm{Im}(\{q\})^T & 0 & 0 & 0 \\ \mathrm{Im}(\{q\})^T & \mathrm{Re}(\{q\})^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{V_\infty}{c} \end{bmatrix}^{-1}}_{\left[\frac{\partial\{y\}}{\partial\{x\}}\right]^{-1}} \times \underbrace{\begin{Bmatrix} \mathrm{Re}\left(\frac{\partial[F]}{\partial V_\infty}\{q\}\right) \\ \mathrm{Im}\left(\frac{\partial[F]}{\partial V_\infty}\{q\}\right) \\ 0 \\ 0 \\ 0 \end{Bmatrix}}_{\frac{\partial(\{y\})}{\partial V_\infty}}$$

$$[F] = \left([M]s^2 + [C]s + [K] - \frac{1}{2}\rho_\infty V_\infty^2 [Q(M_\infty, k)]\right)$$

$$\frac{\partial[F]}{\partial V_\infty} = \left(\rho_\infty V_\infty [Q] + \frac{1}{2}\frac{\rho_\infty V_\infty^2}{a_\infty}\frac{\partial[Q]}{\partial M_\infty}\right)$$

$$\frac{\partial[F]}{\partial s} = \left(2s[M] + [C]\right)$$

$$\frac{\partial[F]}{\partial k} = -\frac{1}{2}\rho_\infty V_\infty^2 \frac{\partial[Q]}{\partial k}$$

- Same procedure can be applied for tangent vector of each interval parameter with fixed velocity

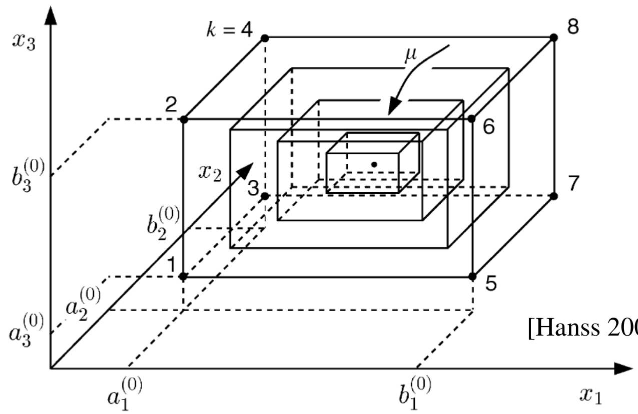
Solution of Interval Flutter Problem

- Formulation of flutter equations as non-linear system of equations including additional interval parameters:

$$[F]\{q\}=\{0\} \quad \Rightarrow \quad \left[F^C\left(s,\rho_\infty,V_\infty,M_\infty,k\right)\pm\Delta F\left(\tilde{u}_1,\tilde{u}_2,\ldots\right)\right]\{q\}=\{0\}$$

$$\left(diag\left(m_i^C\pm\Delta m_i\right)s^2+diag\left(\left(m_i^C\pm\Delta m_i\right)\left(\omega_i^C\pm\Delta\omega_i\right)^2\right)-\frac{1}{2}\rho_\infty V_\infty^2\left[Q^C\pm\Delta Q\right]\right)\{q\}=\{0\}$$

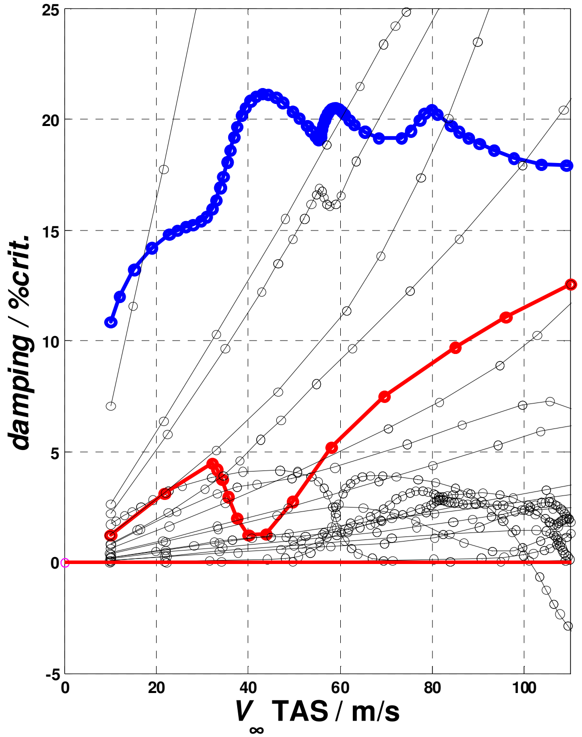
- Solution process using predictor-corrector steps :
  1. Continuation of one modal dof of the central flutter equations for increasing flight speed to find a new nominal solution (all interval parameters are fixed)
  2. Application of <u>*Transformation Method*</u> (Hanss: Applied Fuzzy Arithmetic, 2005) to evaluate all combinations of lower and upper bounds of interval matrix to scan hypercube corners
  3. Direct solution of perturbed flutter equations for fixed velocity with Newton-method from corrector step
  4. Searching the identified set of eigenvalues for minimum and maximum of damping and frequency

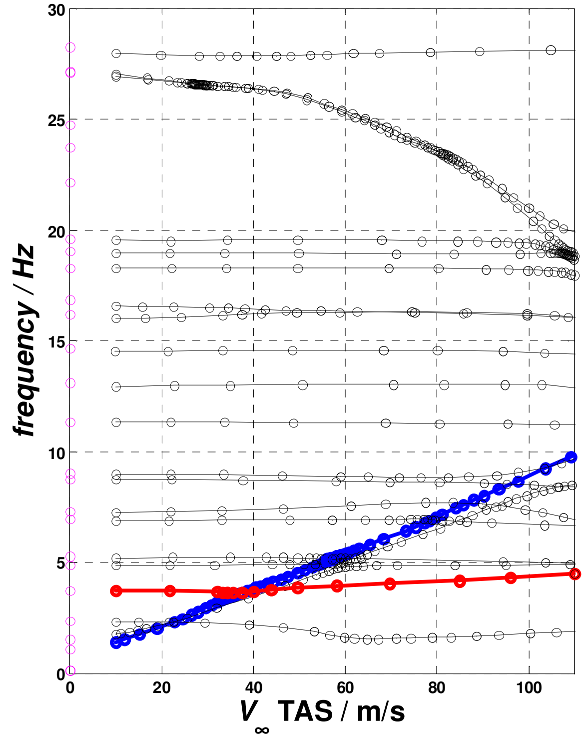


[Hanss 2005]

Continuation Method: V,g- V,f – Diagrams

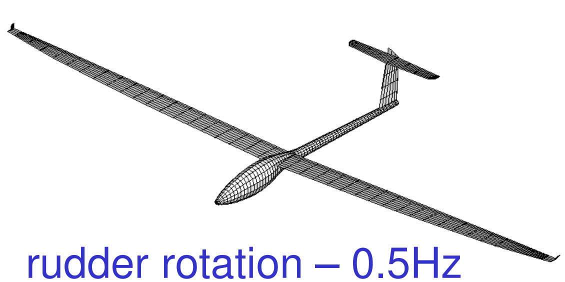
- Application to flutter analysis of glider aircraft
- damping and frequency curves for 30 modal dof
- adaptive stepsize small steps for
  - strong curvature
  - solutions neighboured in frequency



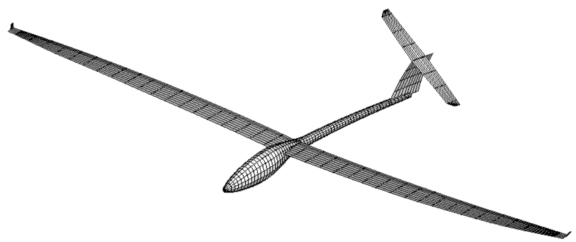


- coupling of rudder and ant. fuselage bending
- suspicious for

„hump mode flutter“



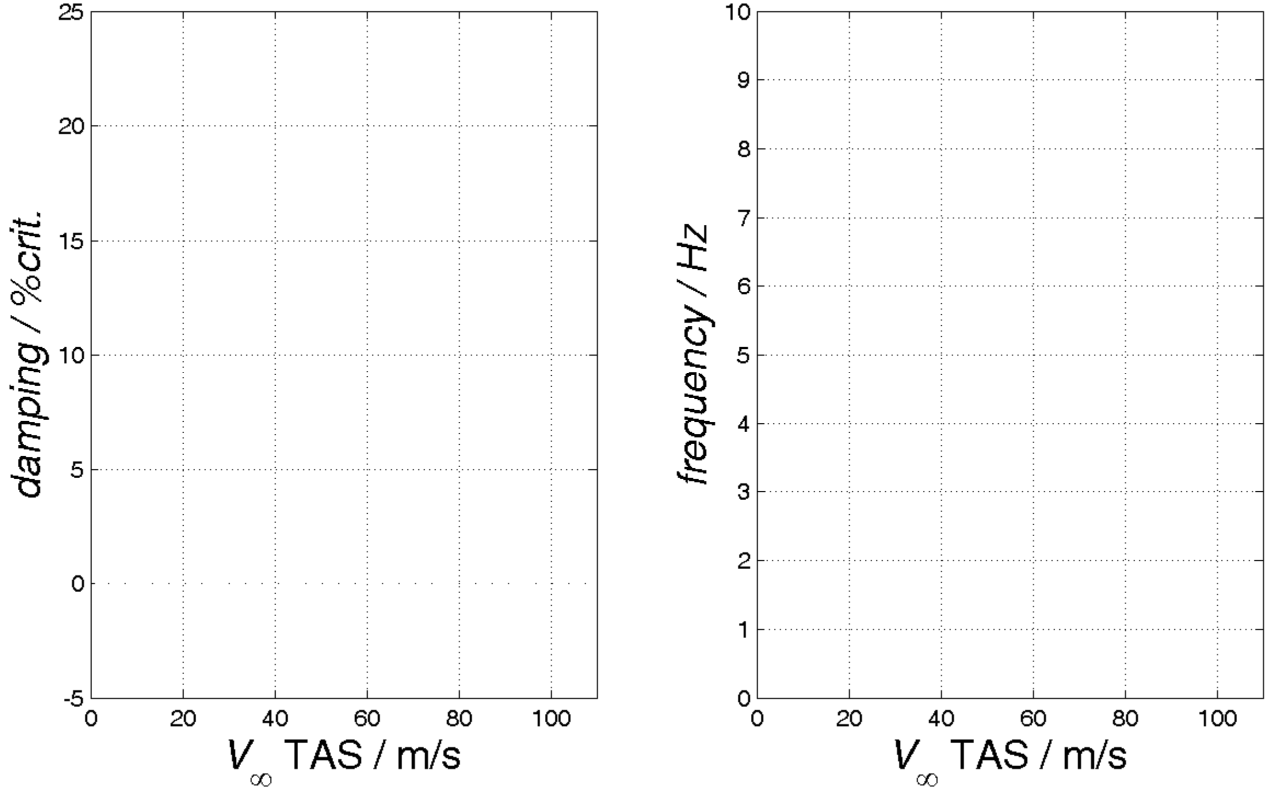
rudder rotation – 0.5Hz



fuselage bend. – 3.7Hz

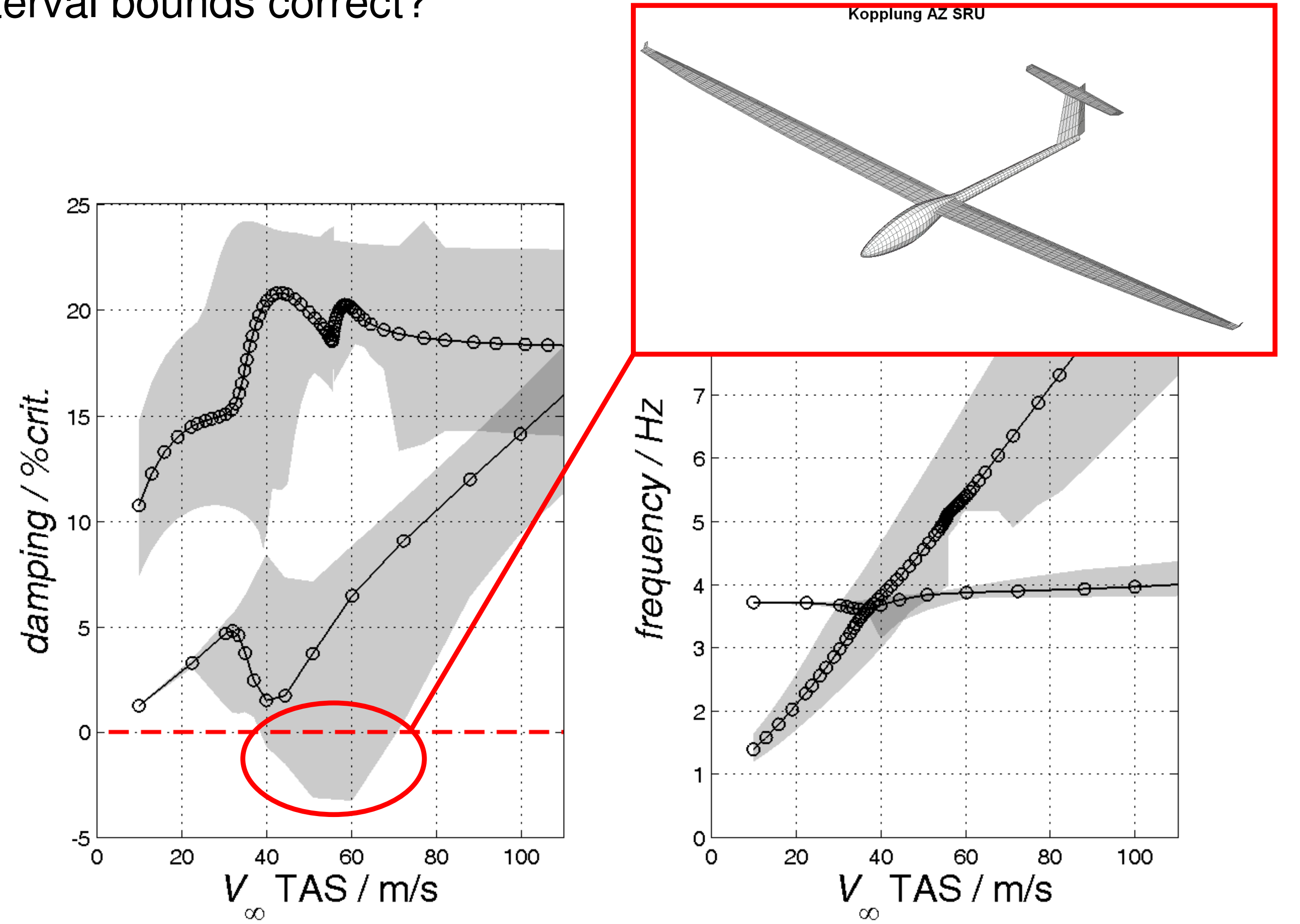
Interval Flutter Analysis using Continuation/Transformation Method

- Uncertainty in rudder mass +/-20% + aerodyn. hinge moment +/-20%
- 1. Step: continuation of centered flutter equations for velocity
- 2. Step: continuation of interval flutter equations for interval uncertainties



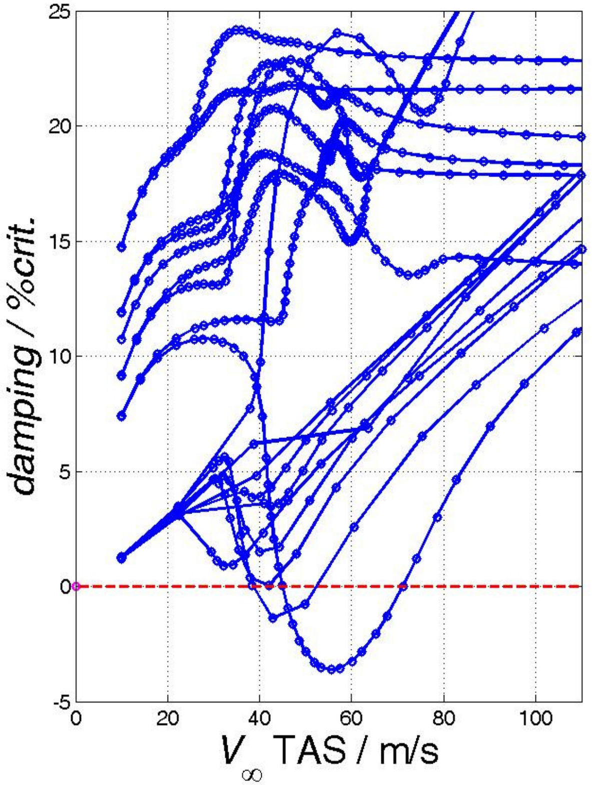
Hump-Mode Flutter

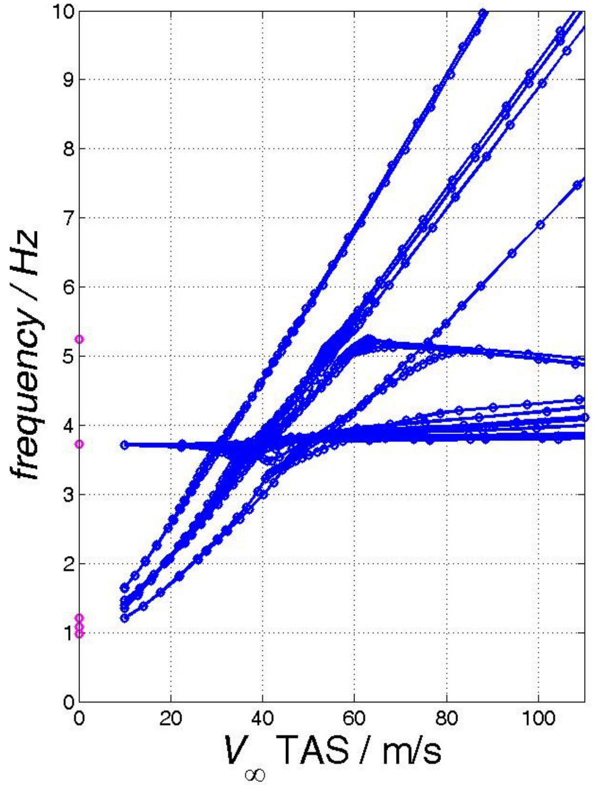
- Flutter might occur for heavy rudder + reduced aerodyn. efficiency
- Are the interval bounds correct?



Hump-Mode Flutter

- Flutter might occur for heavy rudder reduced aerodyn. efficiency
- Are the interval bounds correct?
  - Evaluation of all deterministic combinations shows exact agreement





Summary

- Uncertain-but-bounded parameters in structural aircraft model cause intervals of eigenfrequencies and modeshapes
- These modal structural uncertainties must be propagated through the flutter analysis process
- The solution with *Numerical Continuation Method* finds solution branches of modal dof for parameter dependent flutter equations
- No commutation of solution branches, because both complex eigenvalue and vector are used for continuation.
- Interval flutter analysis is performed in parallel to central flutter solution by application of *Transformation Method*.
- The lower and upper bounds of complex eigenvalues are evaluated exactly (no extrapolation).

  → V,g and V,f diagrams may include uncertainty bounds in addition

References

- *Eigenvalues of interval matrices*

  Deif, A.S.: The Interval Eigenvalue Problem, ZAMM Zeitschrift angew. Math. Mech. 71 (1991), pp. 61-64.

- *Continuation method*

  Cardani, C.; Mantegazza, P.: Continuation and Direct Solution of the Flutter Equation. Computers & Structures, Vol. 8 (1978) No. 3, pp. 185-192.

  Meyer, E. E.: Application of a New Continuation Method to Flutter Equations. 29th Structures, Structural Dynamics and Materials Conference, April 18-20, 1988, Williamsburg, VA, Part 3, pp.1118-1123.

- *Transformation method*

  Hanss, M.: Applied Fuzzy Arithmetic. Springer Verlag, 2005.

Thank you!