ECERTA meeting, The University of Liverpool, 13th-15th September 2010

Stochastic Finite Element Model Updating and its Application in Aeroelasticity

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E C E R T A – Enabling Certification by Analysis

UNIVERSITY OF LIVERPOOL

Marie Curie Excellence Team

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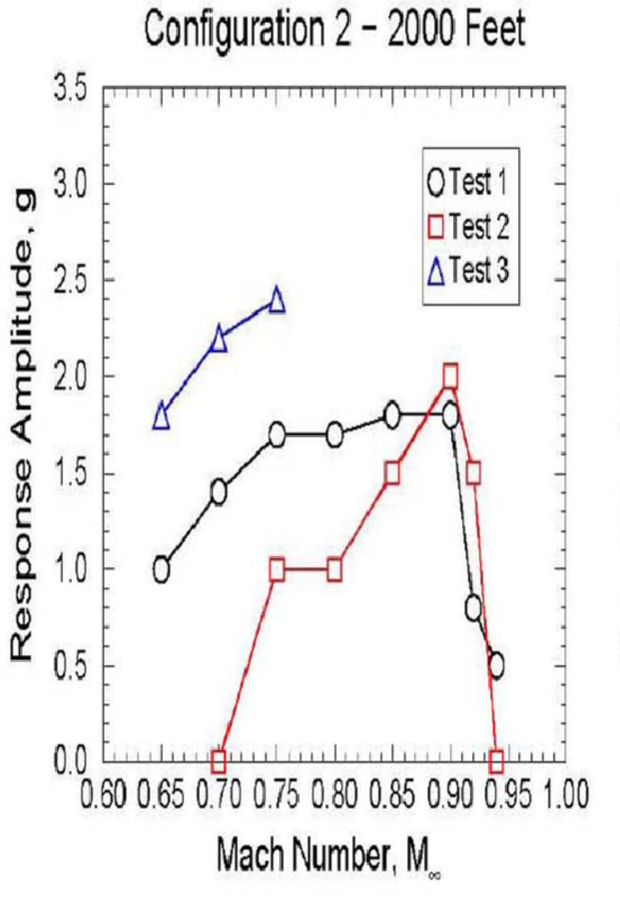
Structural uncertainty in aircrafts

Dowell, AFRL workshop, 2006

Thomas, AIAA-2005-1917



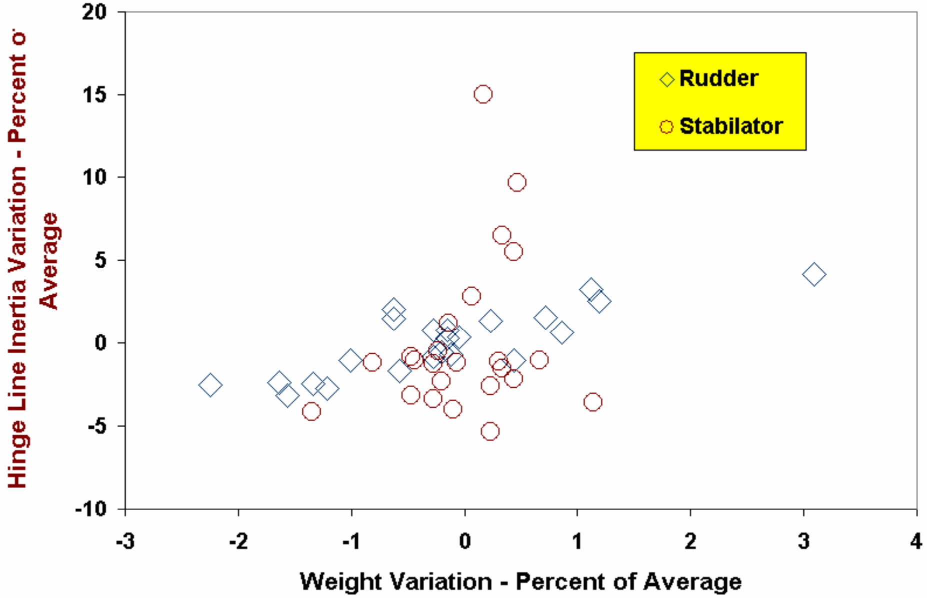
Pitt, AIAA-2008-2198

GVT: frequency variation 15%

Brooks et al, IFASD07

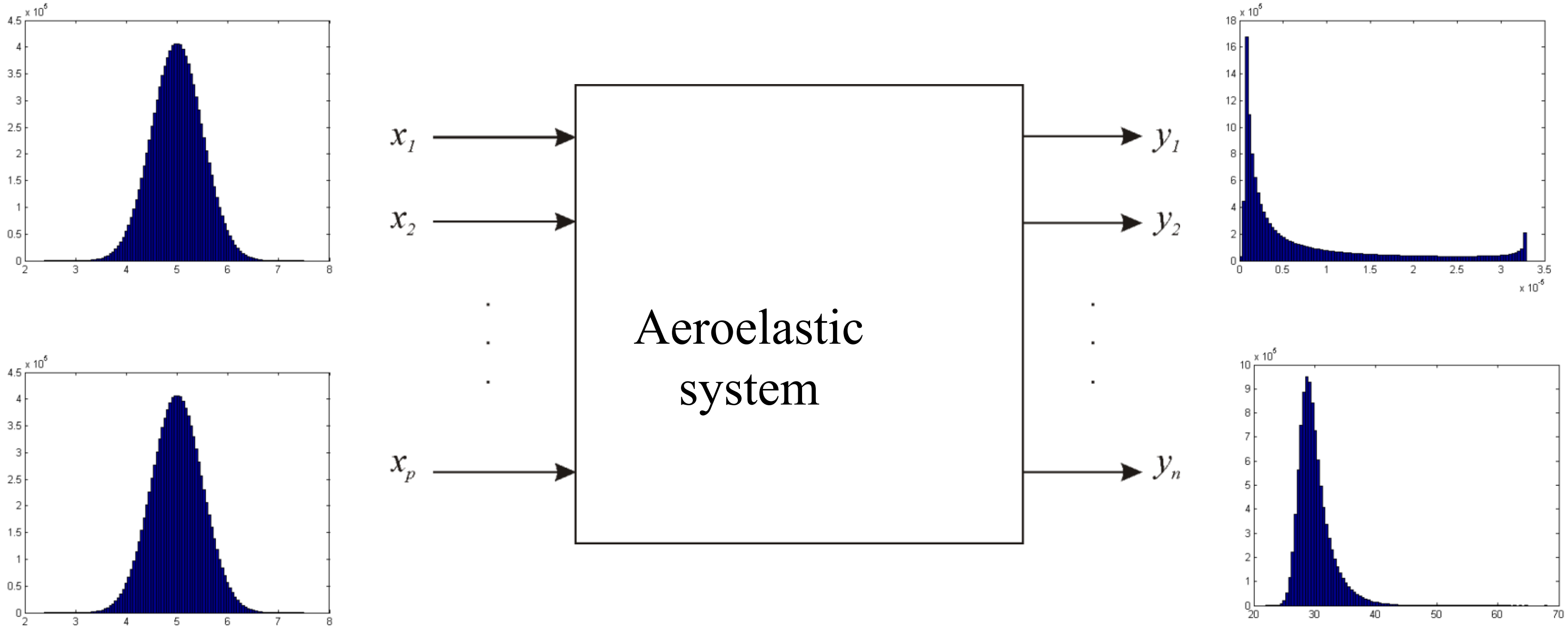
Tip attachment bolts - LCO

Fuselage fuel mass – $\Delta M = 0.05$



Uncertainty modelling and propagation

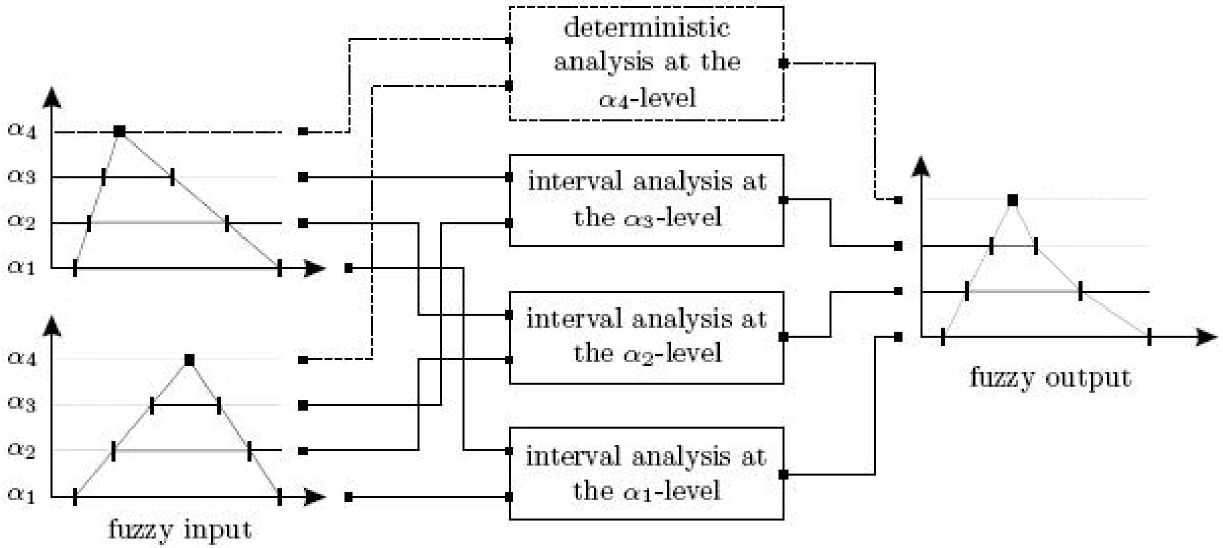
Probabilistic models



Uncertainty modelling and propagation

Non-Probabilistic models

Interval model
Fuzzy sets



Propagation by 4 α level, for a function of two triangular fuzzy parameters.
[D. Mones and D. Vandepitte JSV 288 (2005) 431-462]

Uncertainty Propagation method

- Monte Carlo Simulation
- Perturbation methods
- Asymptotic integral
- Fuzzy-logic
- Interval analysis
- Meta model

Probabilistic model updating

- The updating parameters are considered as random variables having presumed probability density functions.

- The statistical moments of updating parameters will be updated so that the scatter of measured data converges upon the scatter of analytical output data obtained from a randomised FE model.

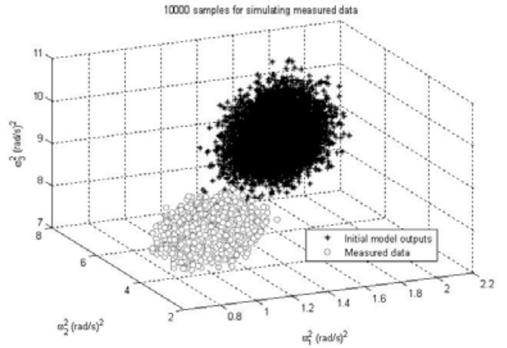
Interval model updating

- The updating parameters are defined to vary within an initial interval.

- The interval of updating parameters will be updated using the scatter of measured data.

Probabilistic model updating: perturbation method

Classical model updating: $\boldsymbol{\theta}_{j+1} = \boldsymbol{\theta}_j + \mathbf{T}_j(\mathbf{z}_m - \mathbf{z}_j)$

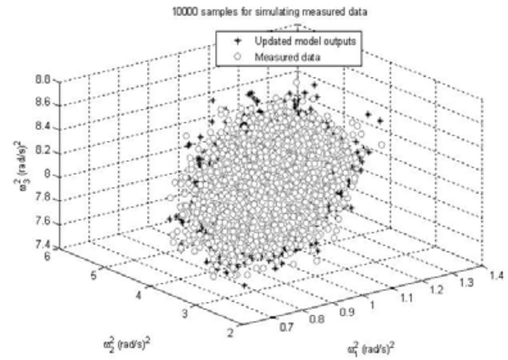


Measurement: $\mathbf{z}_m = \bar{\mathbf{z}}_m + \Delta\mathbf{z}_m$

Prediction: $\mathbf{z}_j = \bar{\mathbf{z}}_j + \Delta\mathbf{z}_j$

Parameters: $\boldsymbol{\theta} = \bar{\boldsymbol{\theta}} + \Delta\boldsymbol{\theta}$

Mean $\bar{\bullet}$
Variability Δ



Transformation matrix: $\mathbf{T} = \bar{\mathbf{T}} + \Delta\mathbf{T}_j$ $\qquad \Delta\mathbf{T}_j = \sum_{k=1}^{n} \frac{\partial \mathbf{T}_j}{\partial Z_{mk}} \Delta z_{mk}$

Stochastic model updating:

$$\bar{\boldsymbol{\theta}}_{j+1} + \Delta\boldsymbol{\theta}_{j+1} = \bar{\boldsymbol{\theta}}_j + \Delta\boldsymbol{\theta}_j + \left(\bar{\mathbf{T}}_j + \Delta\mathbf{T}_j\right)\left(\bar{\mathbf{z}}_m + \Delta\mathbf{z}_m - \bar{\mathbf{z}}_j - \Delta\mathbf{z}_j\right)$$

Perturbation method

$$\mathbf{O}\left(\mathbf{\Delta}^{0}\right): \qquad \bar{\boldsymbol{\theta}}_{j+1} = \bar{\boldsymbol{\theta}}_{j} + \bar{\mathbf{T}}_{j}\left(\bar{\mathbf{z}}_{m} - \bar{\mathbf{z}}_{j}\right)$$

- the mean values of the updating parameters

$$\mathbf{O}(\mathbf{\Delta}^{1}): \qquad \mathbf{\Delta\theta}_{j+1} = \mathbf{\Delta\theta}_{j} + \bar{\mathbf{T}}_{j}\left(\mathbf{\Delta z}_{m} - \mathbf{\Delta z}_{j}\right) + \left(\left(\sum_{k=1}^{n} \frac{\partial \bar{\mathbf{T}}_{j}}{\partial z_{mk}} \Delta z_{mk}\right)\left(\bar{\mathbf{z}}_{m} - \bar{\mathbf{z}}_{j}\right)\right)$$

- leads to an expression for the parameter covariances

Haddad Khodaparast, H, Mottershead, J.E., Friswell M. I., Perturbation methods for the estimation of parameter variability in stochastic model updating, Mechanical System and Signal Processing 22 (8) (2008) 1751-1773.

Parameter covariances

$$\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j+1},\Delta\boldsymbol{\theta}_{j+1}\right)=\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j}+\mathbf{A}_{j}\,\Delta\mathbf{z}_{m}+\overline{\mathbf{T}}_{j}\left(\Delta\mathbf{z}_{m}-\Delta\mathbf{z}_{j}\right),\ \Delta\boldsymbol{\theta}_{j}+\mathbf{A}_{j}\,\Delta\mathbf{z}_{m}+\overline{\mathbf{T}}_{j}\left(\Delta\mathbf{z}_{m}-\Delta\mathbf{z}_{j}\right)\right)$$

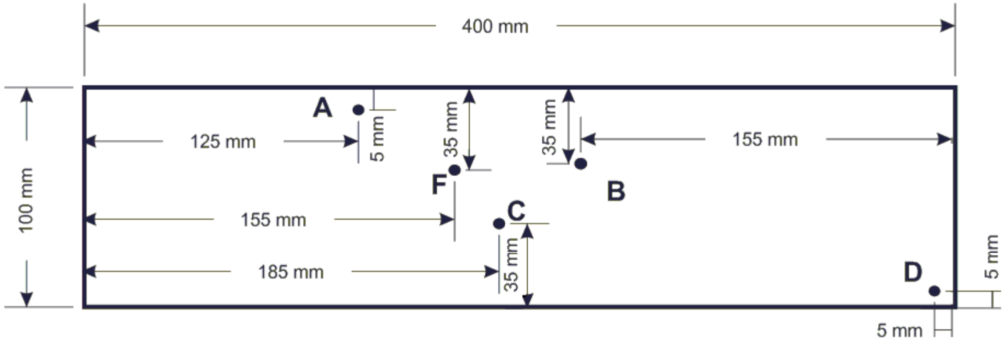
$$\begin{aligned}
&=\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\boldsymbol{\theta}_{j}\right)+\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{m}\right)\mathbf{A}_{j}^{T}+\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{m}\right)\overline{\mathbf{T}}_{j}^{T}-\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{j}\right)\overline{\mathbf{T}}_{j}^{T}\\
&\quad+\left(\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{m}\right)\mathbf{A}_{j}^{T}\right)^{T}+\mathbf{A}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{m}\right)\mathbf{A}_{j}^{T}+\mathbf{A}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{m}\right)\overline{\mathbf{T}}_{j}^{T}-\mathbf{A}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{j}\right)\overline{\mathbf{T}}_{j}^{T}\\
&\quad+\left(\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{m}\right)\overline{\mathbf{T}}_{j}^{T}\right)^{T}+\left(\mathbf{A}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{m}\right)\overline{\mathbf{T}}_{j}^{T}\right)^{T}+\overline{\mathbf{T}}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{m}\right)\overline{\mathbf{T}}_{j}^{T}-\overline{\mathbf{T}}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{j}\right)\overline{\mathbf{T}}_{j}^{T}\\
&\quad-\left(\mathrm{Cov}\left(\Delta\boldsymbol{\theta}_{j},\Delta\mathbf{z}_{j}\right)\overline{\mathbf{T}}_{j}^{T}\right)^{T}-\left(\mathbf{A}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{j}\right)\overline{\mathbf{T}}_{j}^{T}\right)^{T}-\left(\overline{\mathbf{T}}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{m},\Delta\mathbf{z}_{j}\right)\overline{\mathbf{T}}_{j}^{T}\right)^{T}+\overline{\mathbf{T}}_{j}\mathrm{Cov}\left(\Delta\mathbf{z}_{j},\Delta\mathbf{z}_{j}\right)\overline{\mathbf{T}}_{j}^{T}
\end{aligned}$$

$$\mathbf{A}=\left[\frac{\partial\overline{\mathbf{T}}_{j}}{\partial z_{m1}}\left(\overline{\mathbf{z}}_{m}-\overline{\mathbf{z}}_{j}\right)\ \ \frac{\partial\overline{\mathbf{T}}_{j}}{\partial z_{m2}}\left(\overline{\mathbf{z}}_{m}-\overline{\mathbf{z}}_{j}\right)\ \cdots\ \frac{\partial\overline{\mathbf{T}}_{j}}{\partial z_{mn}}\left(\overline{\mathbf{z}}_{m}-\overline{\mathbf{z}}_{j}\right)\right]\qquad \overline{\mathbf{T}}_{j}=\left(\overline{\mathbf{S}}_{j}^{T}\mathbf{W}_{1}\overline{\mathbf{S}}_{j}+\mathbf{W}_{2}\right)^{-1}\overline{\mathbf{S}}_{j}^{T}\mathbf{W}_{1}$$

$\mathrm{Cov}(\Delta\theta_j,\Delta\mathbf{z}_j)$and $\mathrm{Cov}(\Delta\mathbf{z}_j,\Delta\mathbf{z}_j)$ are determined by forward propagation

Experimental Case Study – Plate Thickness Variability

Arrangement of accelerometers (A, B, C, D) and driving point (F)



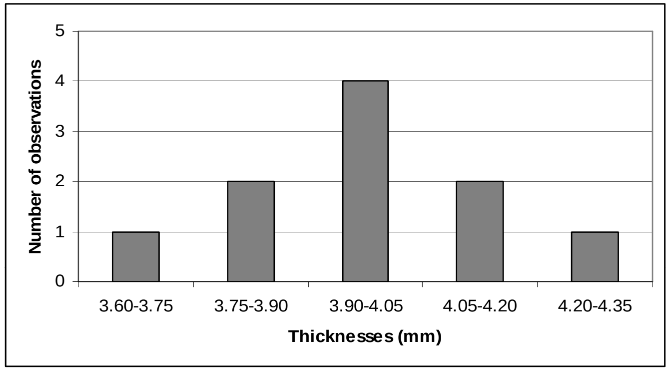
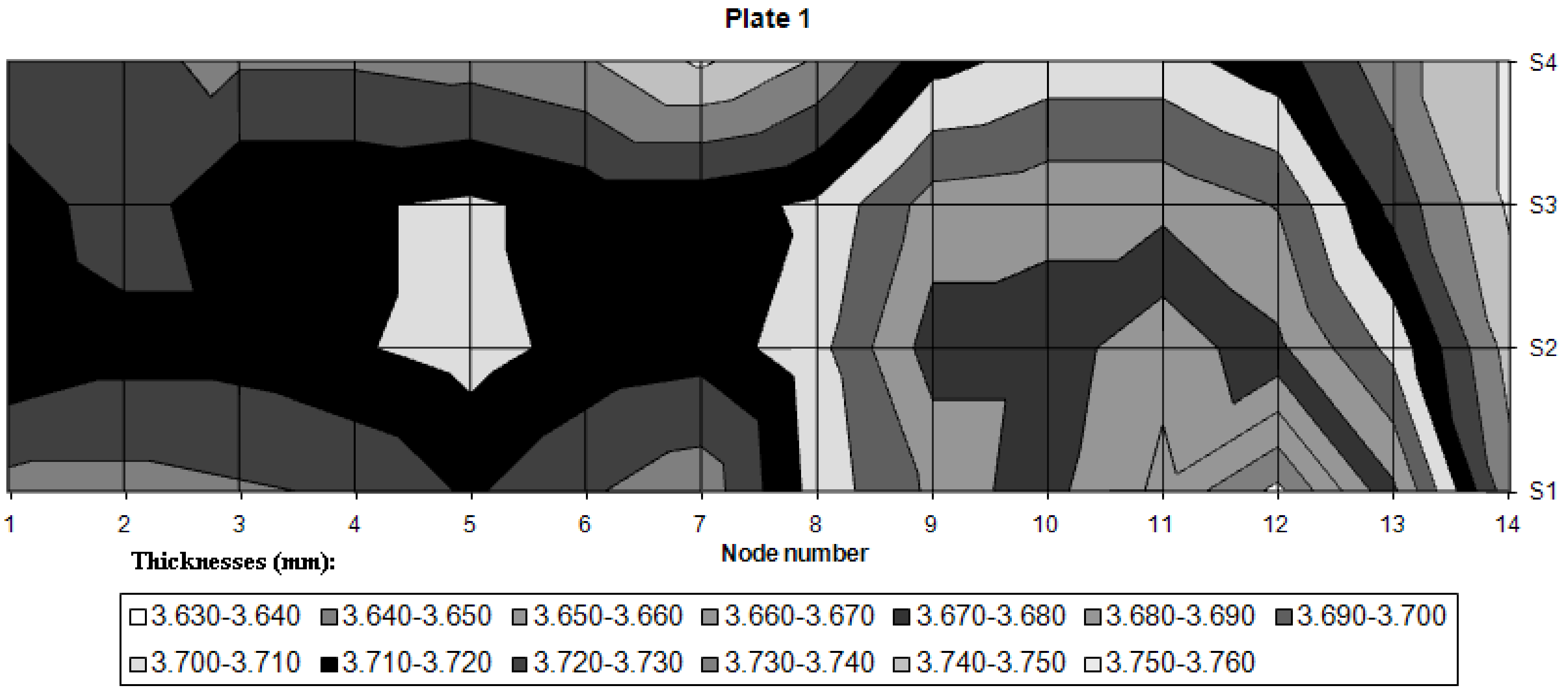
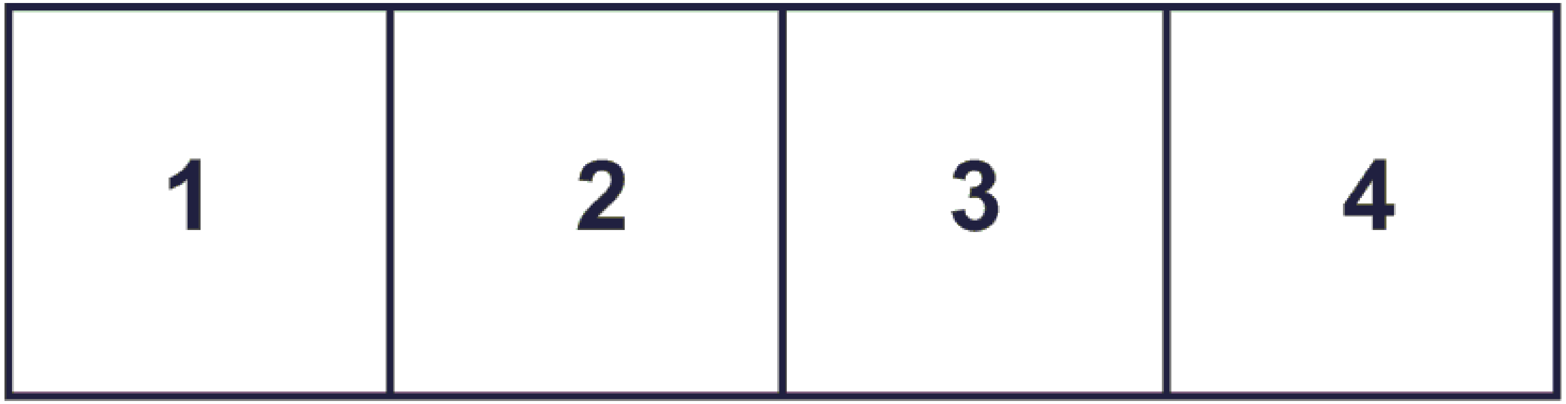


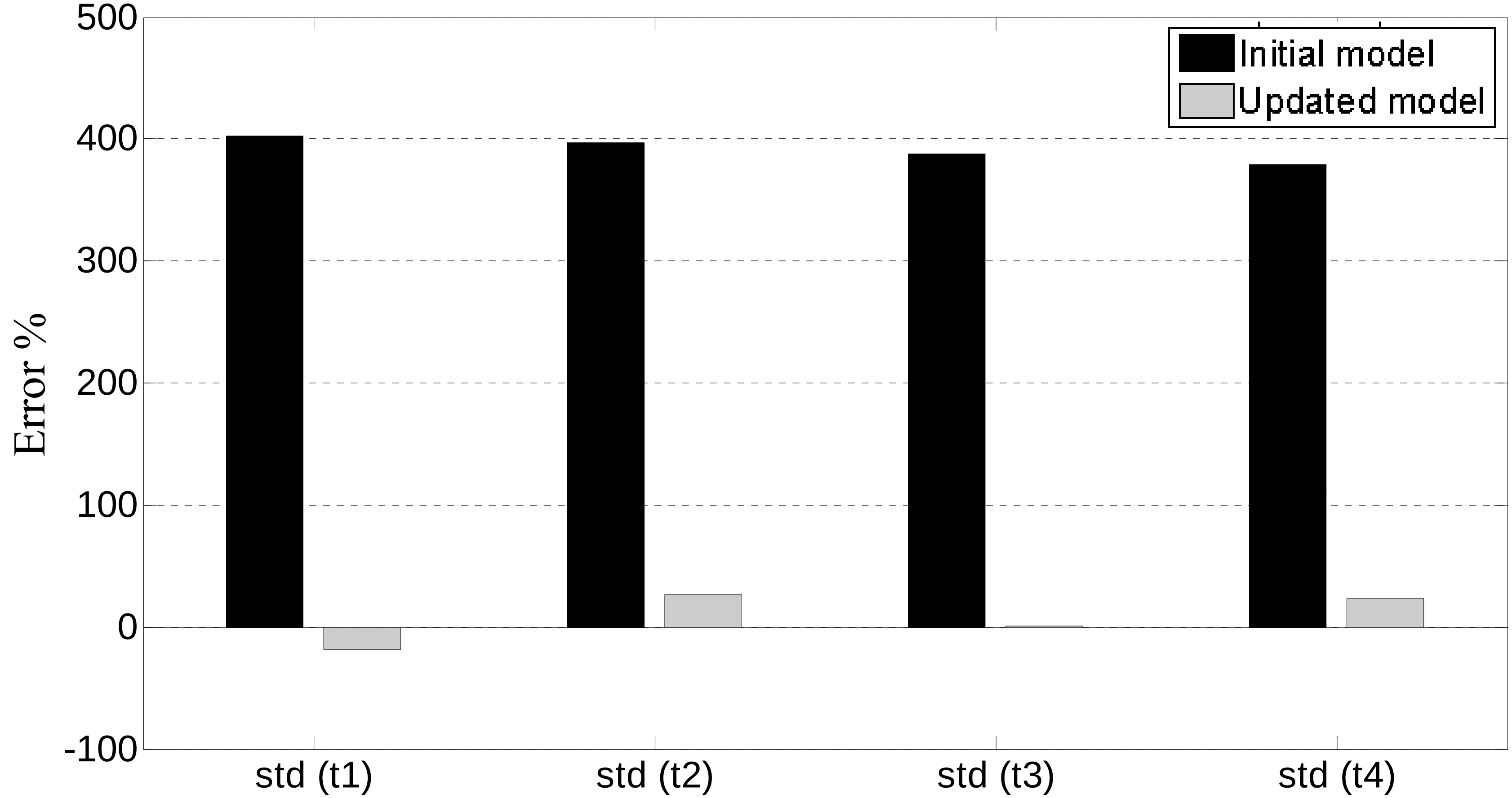
Plate thickness distribution

Model Parameterization



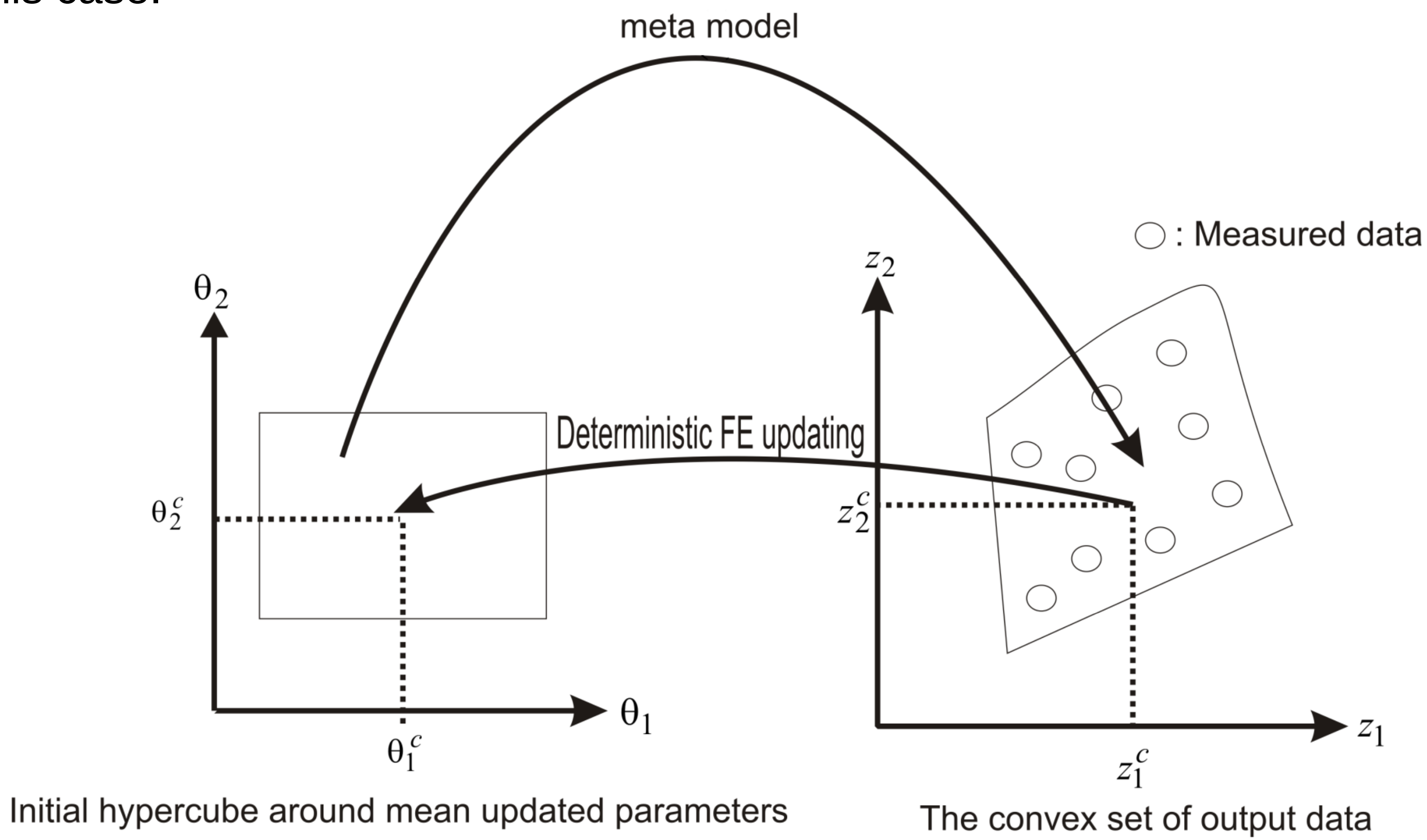


Errors % of initial and updated standard deviation of parameters



Interval model updating

Case2: updating parameters are geometric and output data are considered to be a vector including eigenvalues and eigenvectors of dynamic system. It can be mathematically shown that **the parameter vertex solution is not valid** in this case.



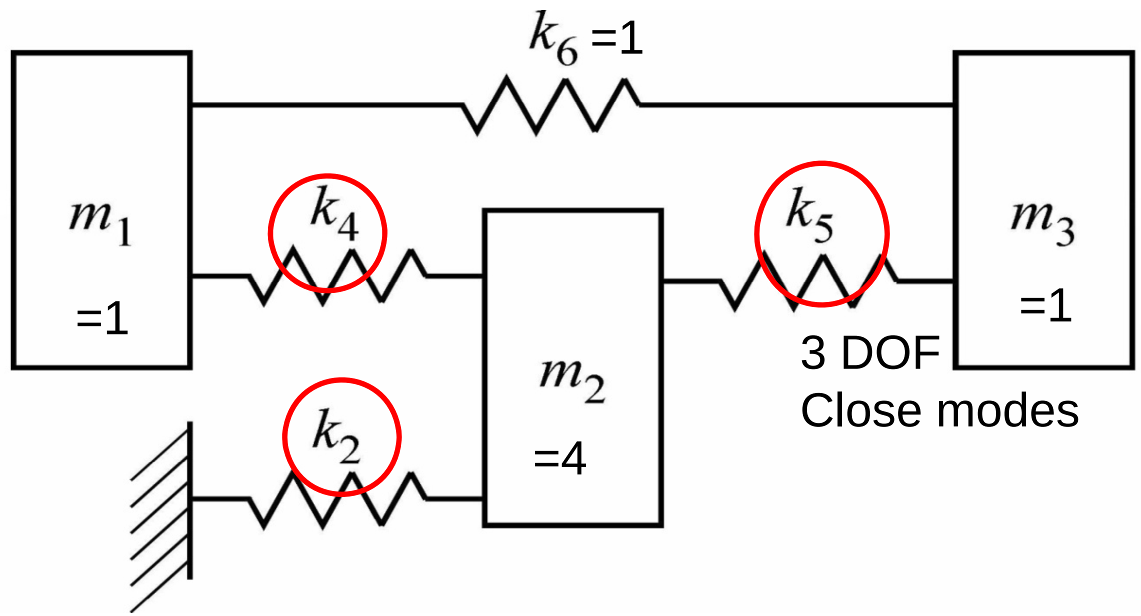
An iterative procedure can be defined as follows for the solution of above equation:

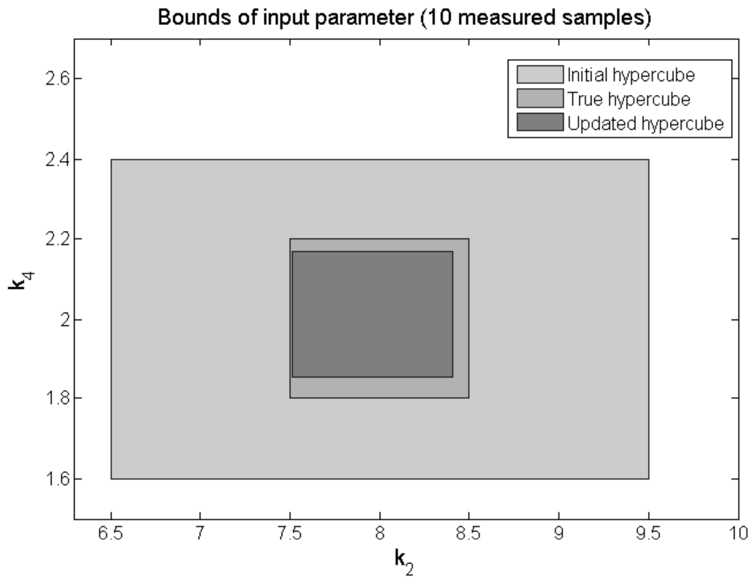
$$\boldsymbol{\theta}_{j+1} = \left[\mathbf{H}^T\mathbf{H} + \mathbf{B} + \mathbf{C} + \mathbf{D} - \mathbf{A}\right]^{-1}_{|\boldsymbol{\theta}=\boldsymbol{\theta}_j} \left\{\mathbf{f}(\boldsymbol{\theta}) + \mathbf{H}^T\boldsymbol{\mu} - \mathbf{H}^T\boldsymbol{\Lambda}\boldsymbol{\rho} - \mathbf{g}(\boldsymbol{\theta})\right\}_{|\boldsymbol{\theta}=\boldsymbol{\theta}_j}$$

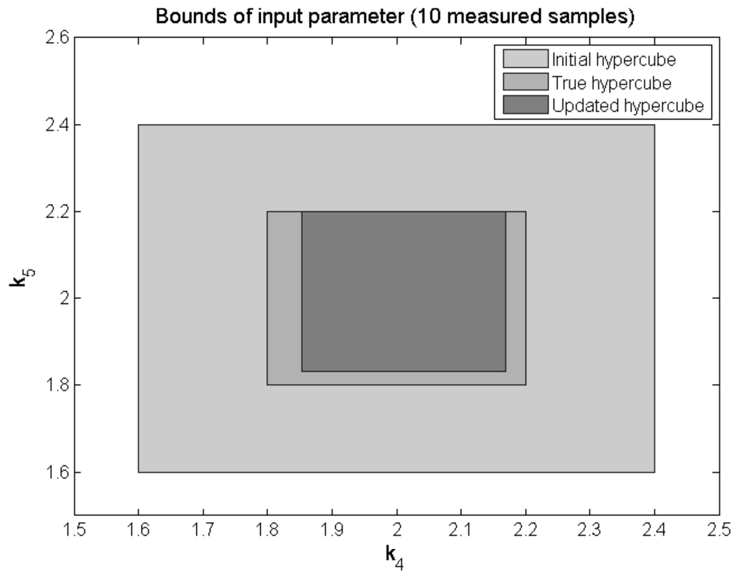
To treat the ill-conditioning of above system, a weighting function can be added to both side of above equation as:

$$\boldsymbol{\theta}_{j+1} = \left[\mathbf{H}^T\mathbf{H} + \mathbf{B} + \mathbf{C} + \mathbf{D} - \mathbf{A} + \mathbf{W}_{\boldsymbol{\theta}}\right]^{-1}_{|\boldsymbol{\theta}=\boldsymbol{\theta}_j} \times \left\{\mathbf{f}(\boldsymbol{\theta}) + \mathbf{H}^T\boldsymbol{\mu} - \mathbf{H}^T\boldsymbol{\Lambda}\boldsymbol{\rho} - \mathbf{g}(\boldsymbol{\theta}) + \mathbf{W}_{\boldsymbol{\theta}}\boldsymbol{\theta}\right\}_{|\boldsymbol{\theta}=\boldsymbol{\theta}_j}$$

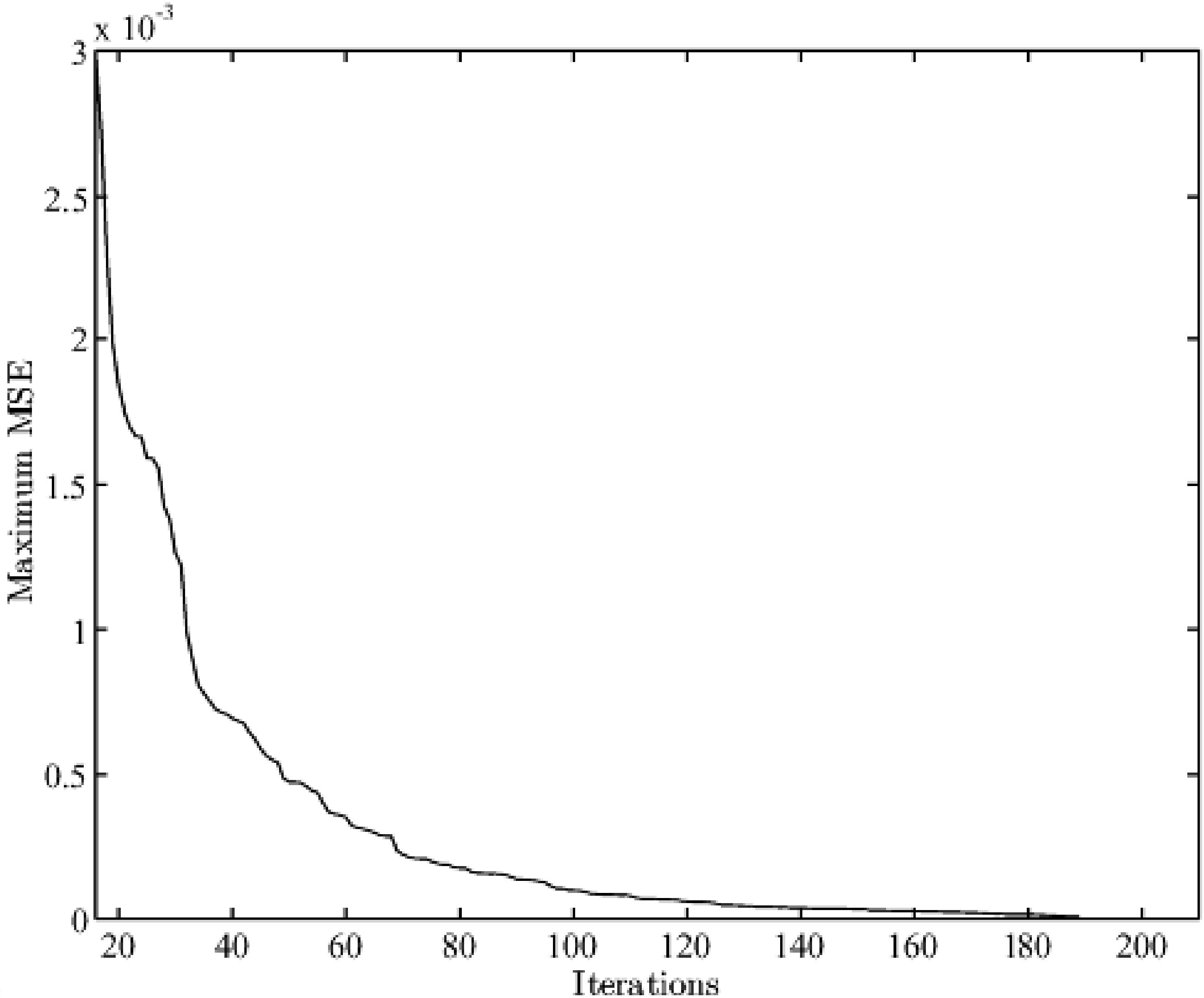
$\mathbf{W}_{\boldsymbol{\theta}}$ is chosen so that the condition number of whole matrix improves


k_6 =1
m_1
=1
k_4
k_5
m_3
=1
m_2
=4
k_2
3 DOF
Close modes

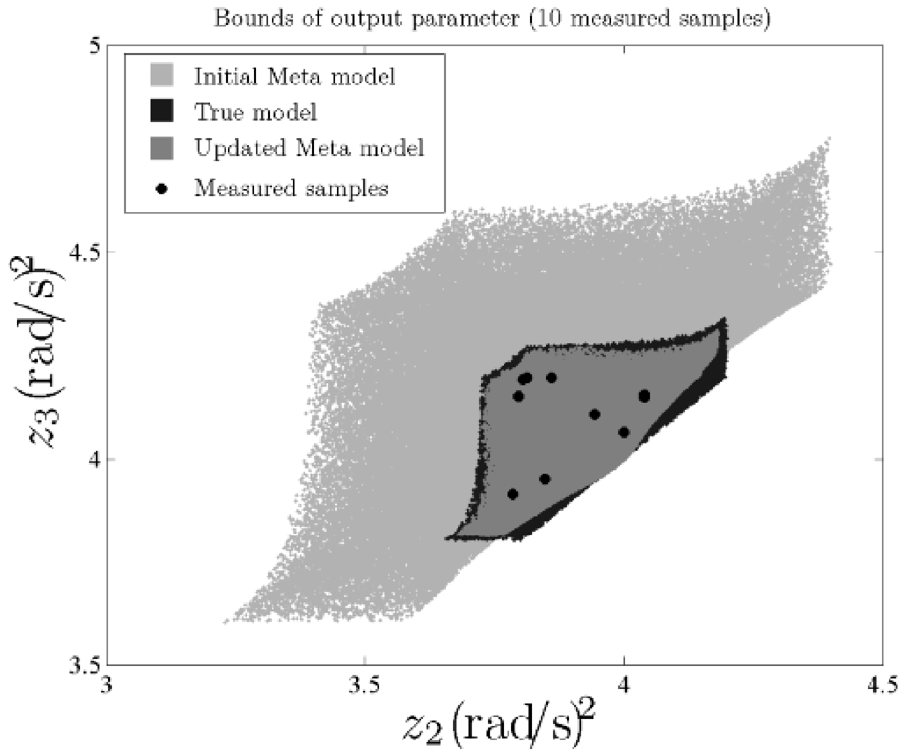

Bounds of input parameter (10 measured samples)
Initial hypercube
True hypercube
Updated hypercube
k4
k2

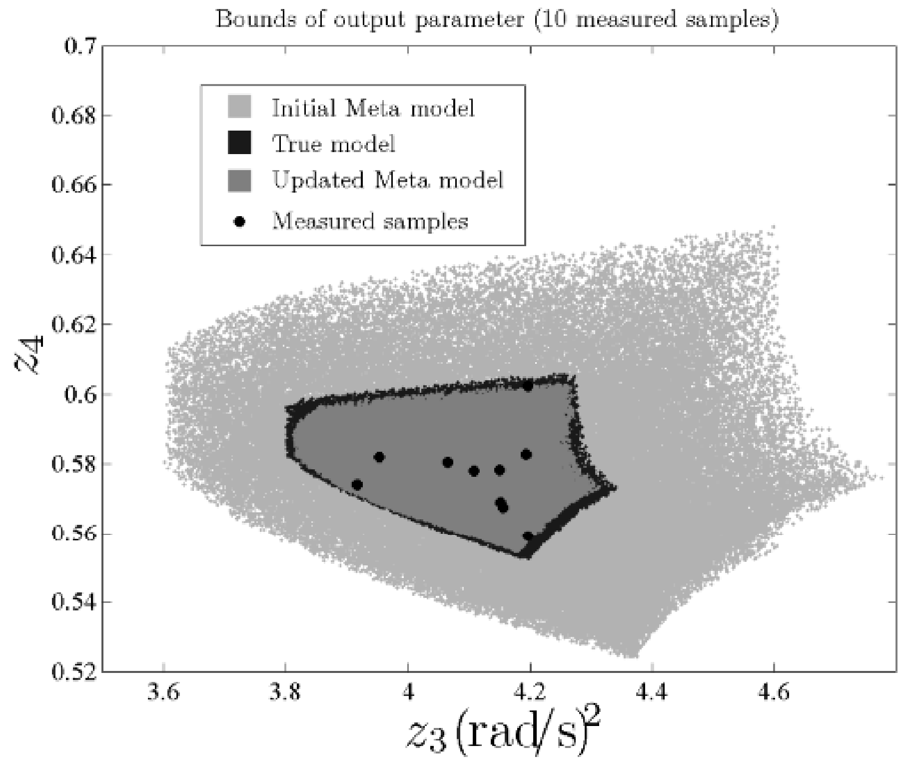

Bounds of input parameter (10 measured samples)
Initial hypercube
True hypercube
Updated hypercube
k5
k4

Optimal sampling in three dimensional problem

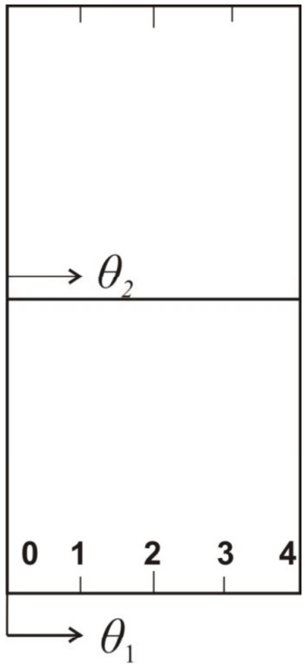


Updated output parameters

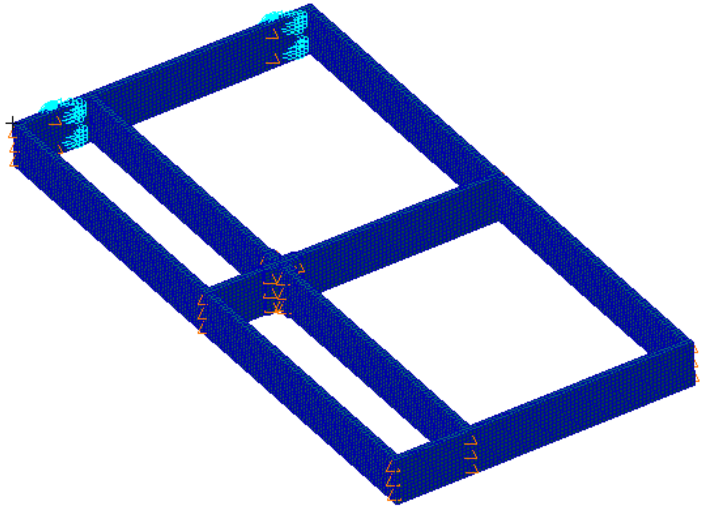




Experimental Case Study – Frame structure



Detailed FE model



Haddad Khodaparast, H, Mottershead, J.E., Badcock K.J., Interval Model Updating with Irreducible Uncertainty Using the Kriging Predictor, Mechanical System and Signal Processing under review.

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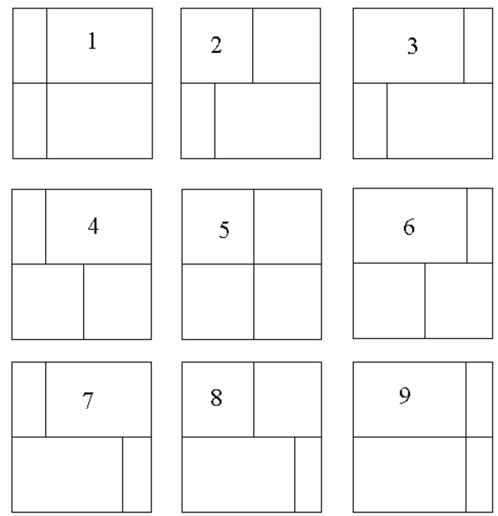
Measured and FE predictions of natural frequencies



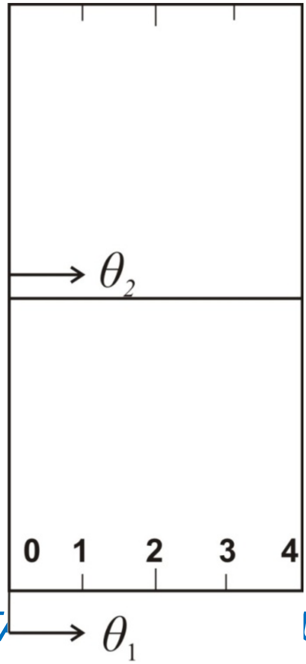
Frame - Free Free

Mode No.	EXP.	FE.	err. %
1	69.3	70.94	2.37
2	79.5	80.27	0.97
3	93.2	92.07	-1.21
4	199.1	200.58	0.74
5	235.6	236.17	0.24
6	259.8	259.33	-0.18
7	286.3	288.73	0.85
8	297.1	296.4	-0.24
9	299.1	303.03	1.31
10	318.6	327.58	2.82

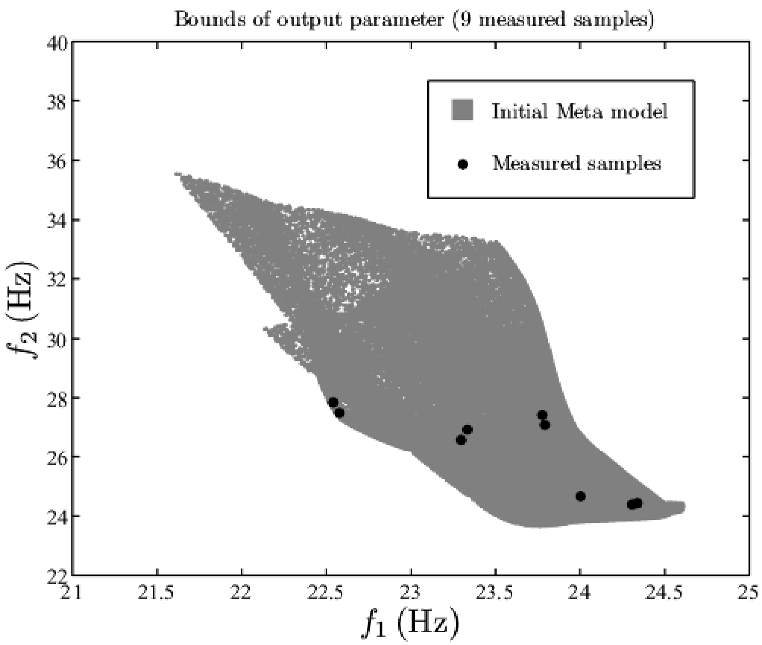
Clamped- Case 1			
Mode	Exp. Hz	FE Hz	Error %
1	22.541	22.59	0.22
2	27.844	27.27	-2.04
3	47.628	48.14	1.08
4	81.191	80.89	-0.37
5	201.35	201.55	0.10
6	233.708	233.41	-0.13
7	256.398	259.05	1.03
8	257.68	256.54	-0.44
9	283.094	283.35	0.09
10	298.46	305.34	2.30
11	312.387	316.49	1.31

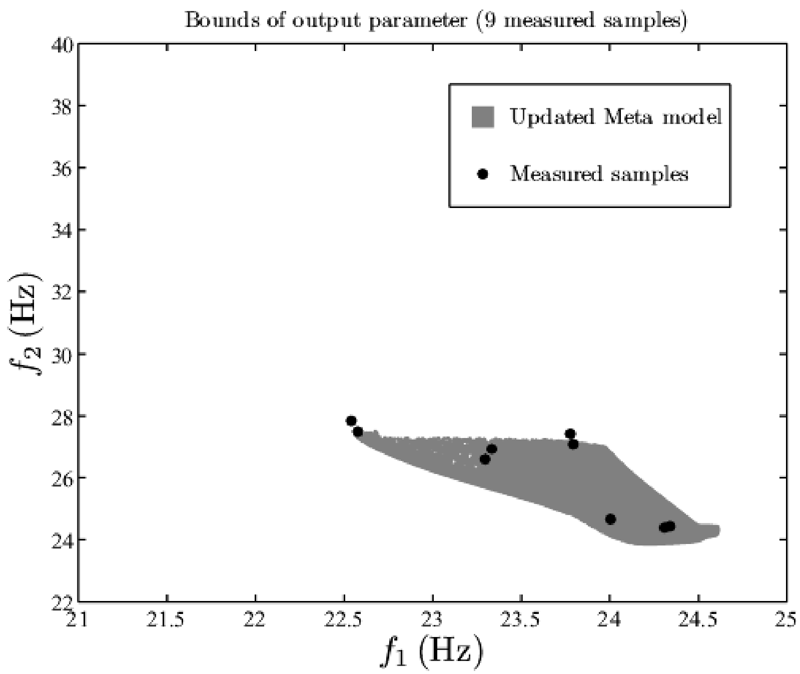
Deterministic model updating of beam locations

True parameters		Initial parameters		Updated parameters		Initial error %		Updated error %	
θ1	θ2	θ1	θ2	θ1	θ2	θ1	θ2	θ1	θ2
1.0	1.0	1.6	1.6	1.04	1.02	**60.00**	**60.00**	**3.73**	**2.00**
1.0	2.0	1.6	2.4	1.00	2.15	**60.00**	**20.00**	**-0.21**	**7.56**
1.0	3.0	1.6	2.4	1.00	3.08	**60.00**	**-20.00**	**0.20**	**2.76**
2.0	1.0	1.6	1.6	2.04	0.90	**-20.00**	**60.00**	**1.81**	**-9.78**
2.0	2.0	2.4	2.4	2.13	2.00	**20.00**	**20.00**	**6.48**	**-0.12**
2.0	3.0	2.4	2.4	1.95	3.09	**20.00**	**-20.00**	**-2.36**	**3.06**
3.0	1.0	2.4	1.6	2.98	0.89	**-20.00**	**60.00**	**-0.58**	**-11.00**
3.0	2.0	2.4	1.6	2.99	1.83	**-20.00**	**-20.00**	**-0.31**	**-8.36**
3.0	3.0	2.4	2.4	2.93	2.98	**-20.00**	**-20.00**	**-2.18**	**-0.58**

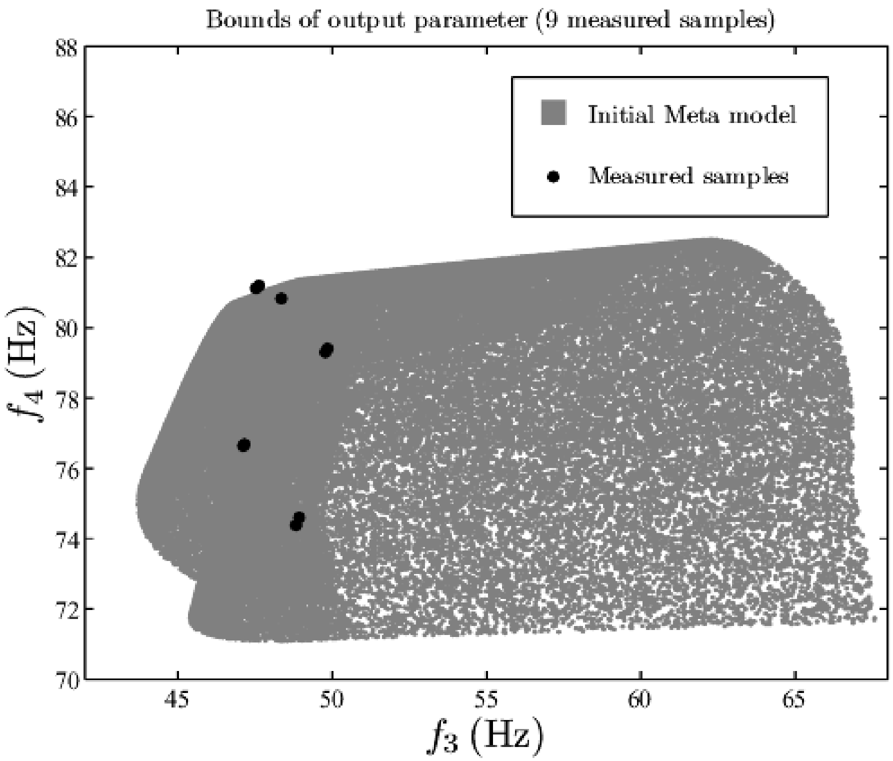


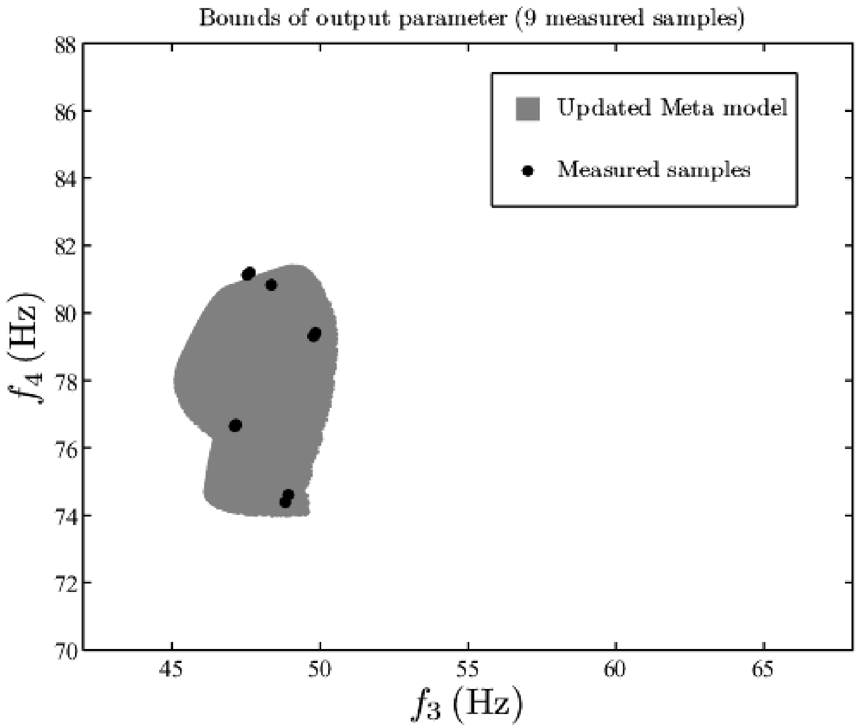
Initial and updated spaces of predicted data (100,000 points) based upon 9 measurement samples



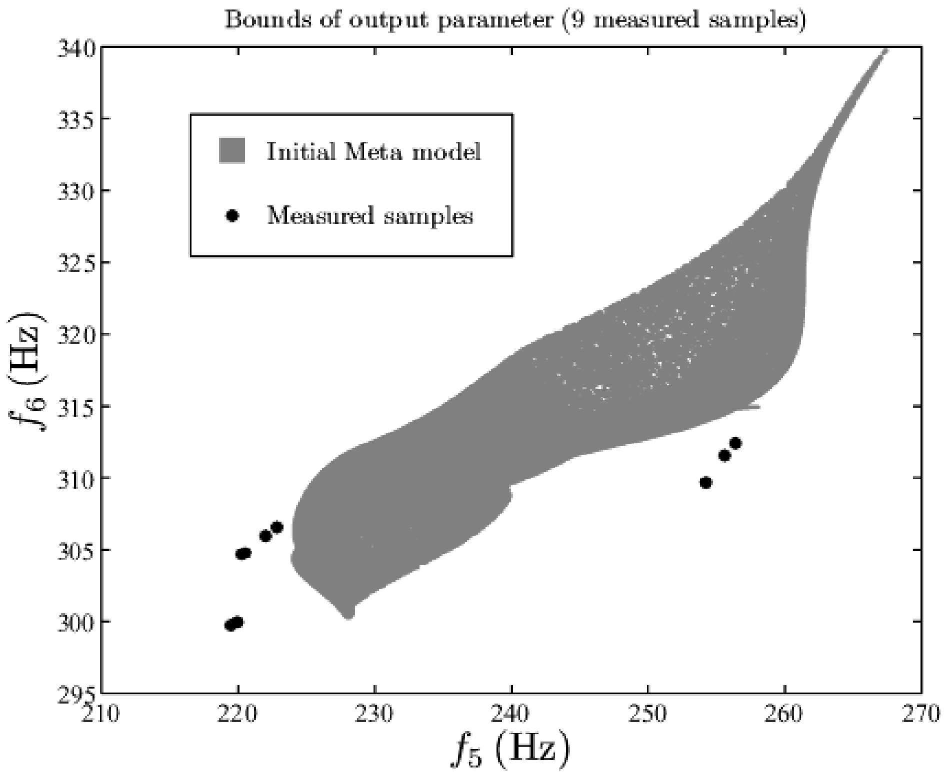


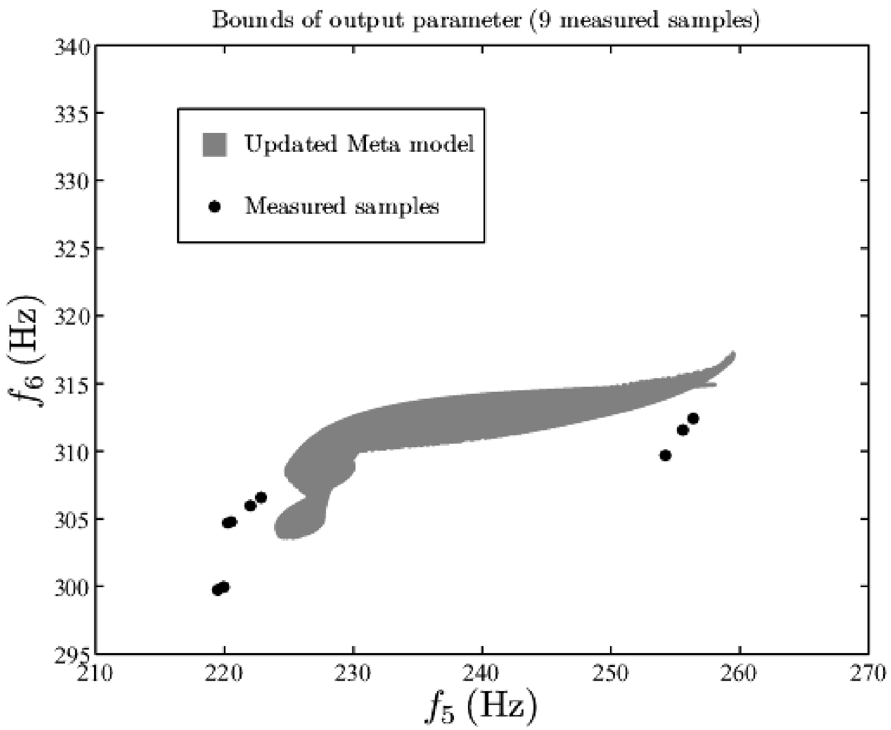
Initial and updated spaces of predicted data (100,000 points) based upon 9 measurement samples





Initial and updated spaces of predicted data (100,000 points) based upon 9 measurement samples





Application of forward propagation methods in Aroelasticity

$$\dot{\mathbf{u}} - \hat{\mathbf{A}}\mathbf{u} = \mathbf{0}$$

The aeroelastic responses can be estimated using following meta model within the region of variation of uncertain parameters:

$$y(\boldsymbol{\theta}) = \beta_0 + \sum_{i=1}^{m} \beta_i \theta_i + \sum_{i=1}^{m} \beta_{ii} \theta_i^2 + \sum_{i<j} \sum_{j=2}^{m} \beta_{ij} \theta_i \theta_j + \varepsilon = \beta_0 + \mathbf{b}^T \boldsymbol{\theta} + \boldsymbol{\theta}^T \mathbf{B} \boldsymbol{\theta} + \varepsilon$$

where:

$$\mathbf{b} = [\beta_1 \ \beta_2 \ \ldots \beta_m]^T$$

$$\mathbf{B} = \begin{bmatrix} \beta_{11} & \frac{\beta_{12}}{2} & \ldots & \frac{\beta_{1m}}{2} \\ & \beta_{22} & \ldots & \frac{\beta_{2m}}{2} \\ & & . & . \\ & & . & . \\ sym. & & . & . \\ & & & \beta_{mm} \end{bmatrix}$$

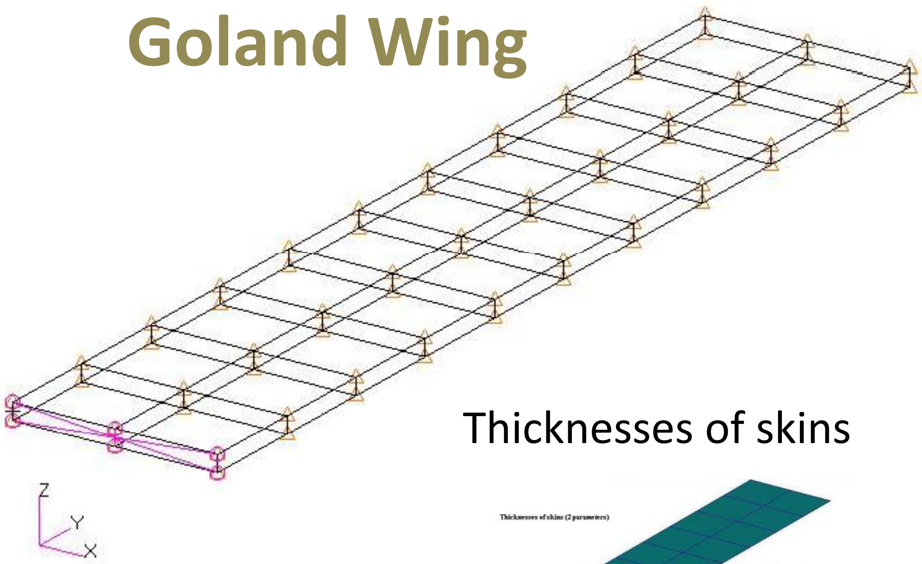
Regression coefficients

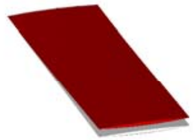
$$\underline{\boldsymbol{\theta}} \leq \boldsymbol{\theta} \leq \overline{\boldsymbol{\theta}}$$

$$\mathbf{g}(\boldsymbol{\theta}) = \mathbf{b} + 2\mathbf{B}\boldsymbol{\theta}$$ Gradient vector

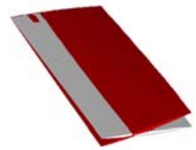
$$\mathbf{G}(\boldsymbol{\theta}) = 2\mathbf{B}$$ Hessian matrix

Goland Wing

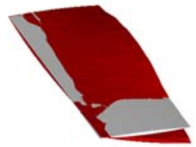




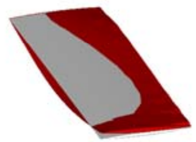
1.72 Hz



3.05 Hz

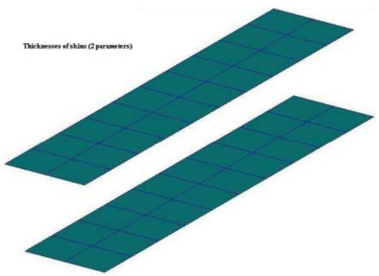


9.18 Hz



11.10 Hz

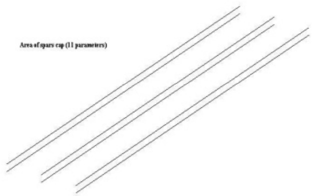
Thicknesses of skins



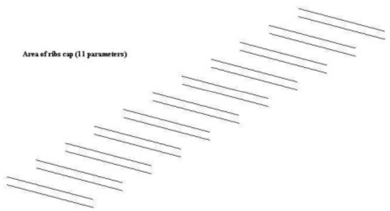
Thicknesses of spars

Thicknesses of ribs

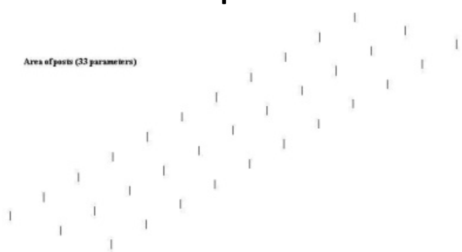
Area of spar caps



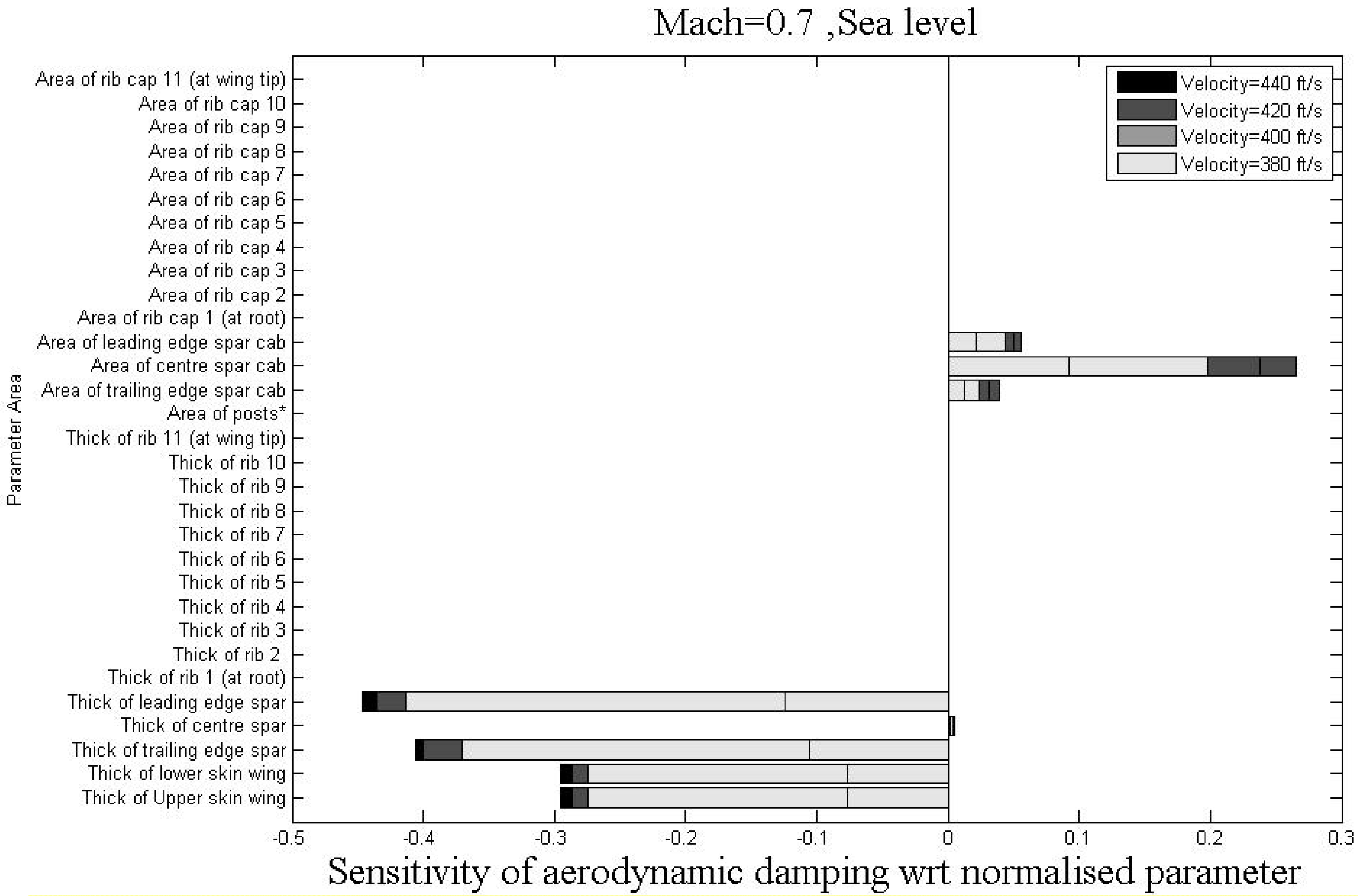
Area of rib caps



Area of posts

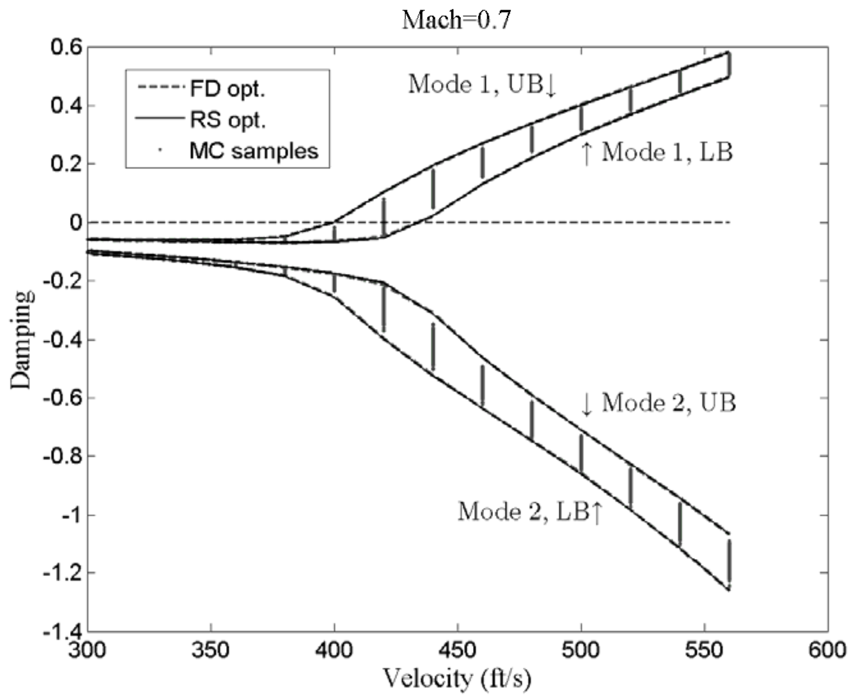


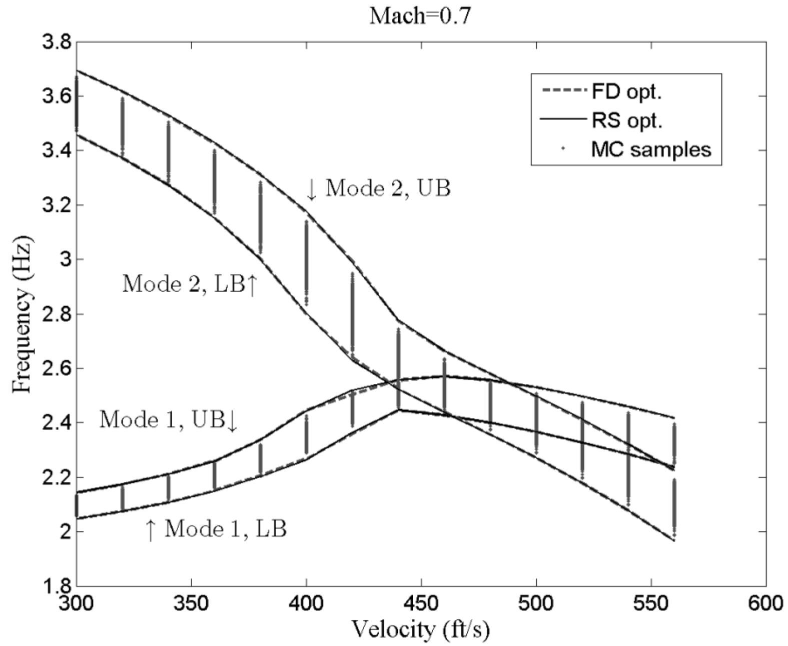
Sensitivity Analysis



7 Structural parameters significantly influence flutter – vector θ

Results : Interval and MCS

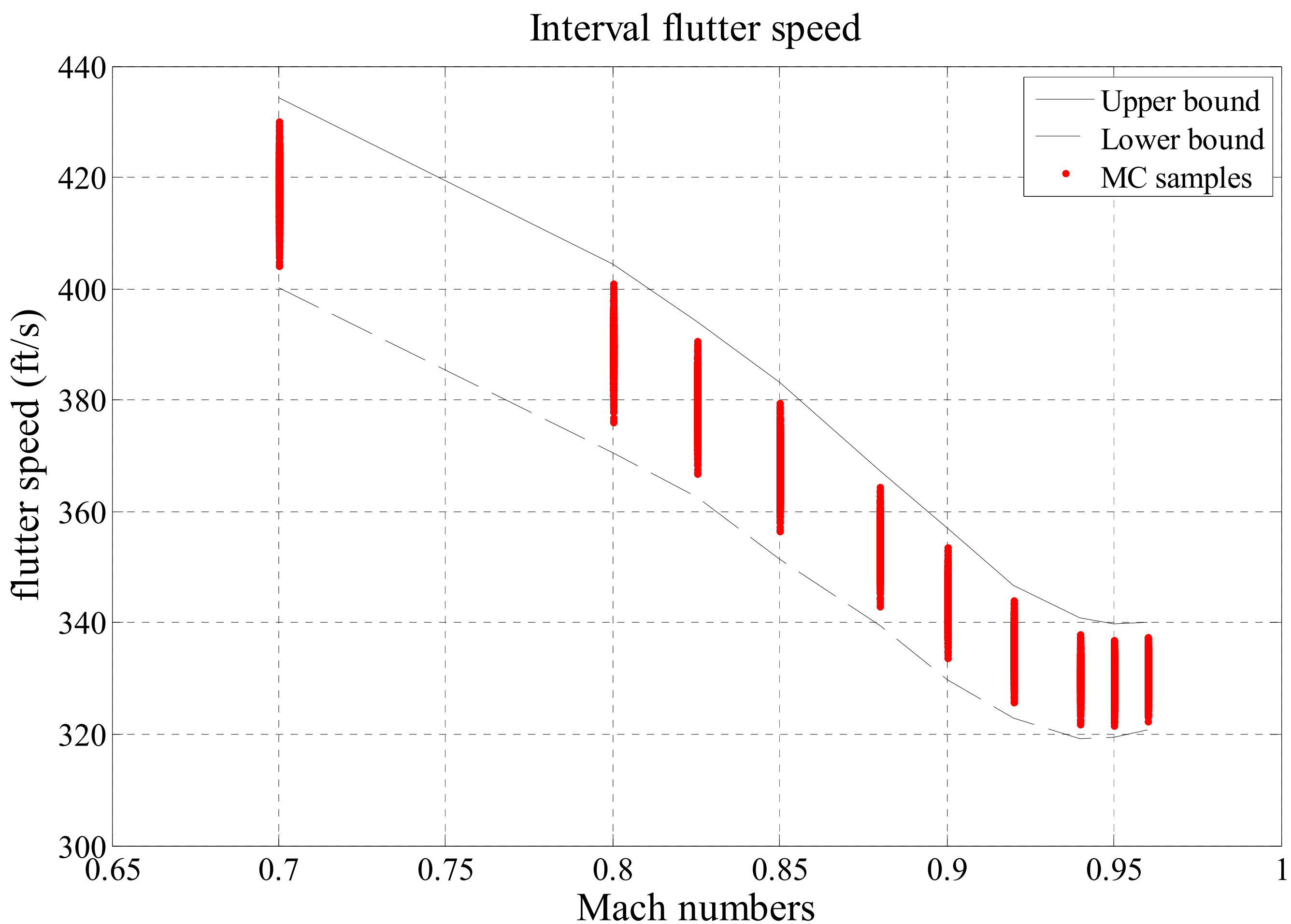




Haddad Khodaparast, H, Mottershead, J.E., Badcock K.J., Propagation of Structural Uncertainty to Linear Aeroelastic Stability. Computers and Structures Volume 88, Issues 3-4, February 2010, Pages 223-236.

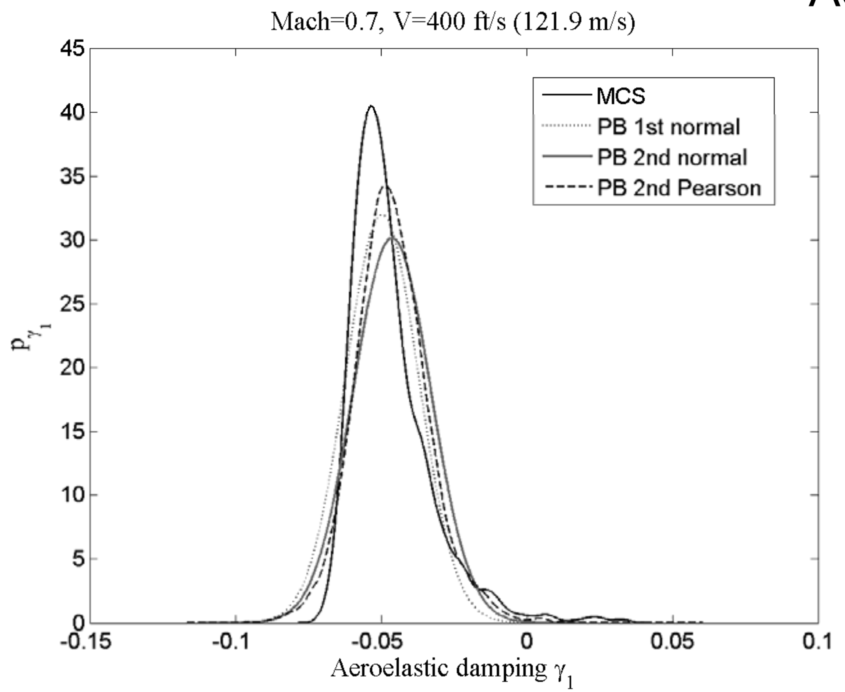
Results: Interval Analysis

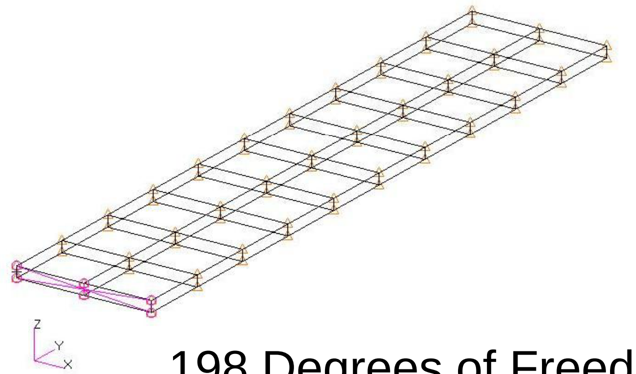
Interval analysis – COV=0.05



Results: Probabilistic and Fuzzy

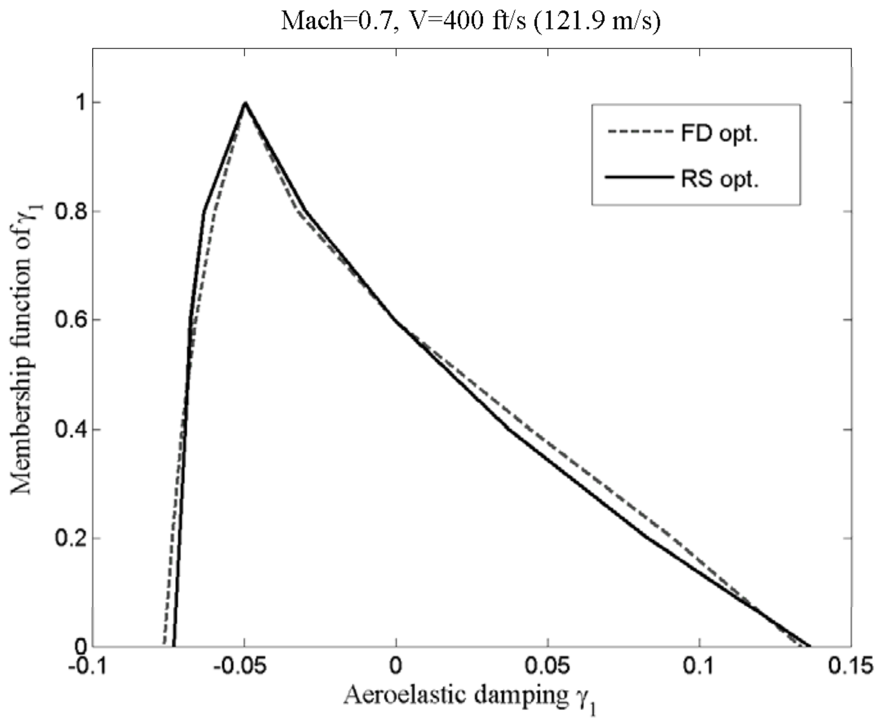
- Aeroelastic problem using linear aerodynamics



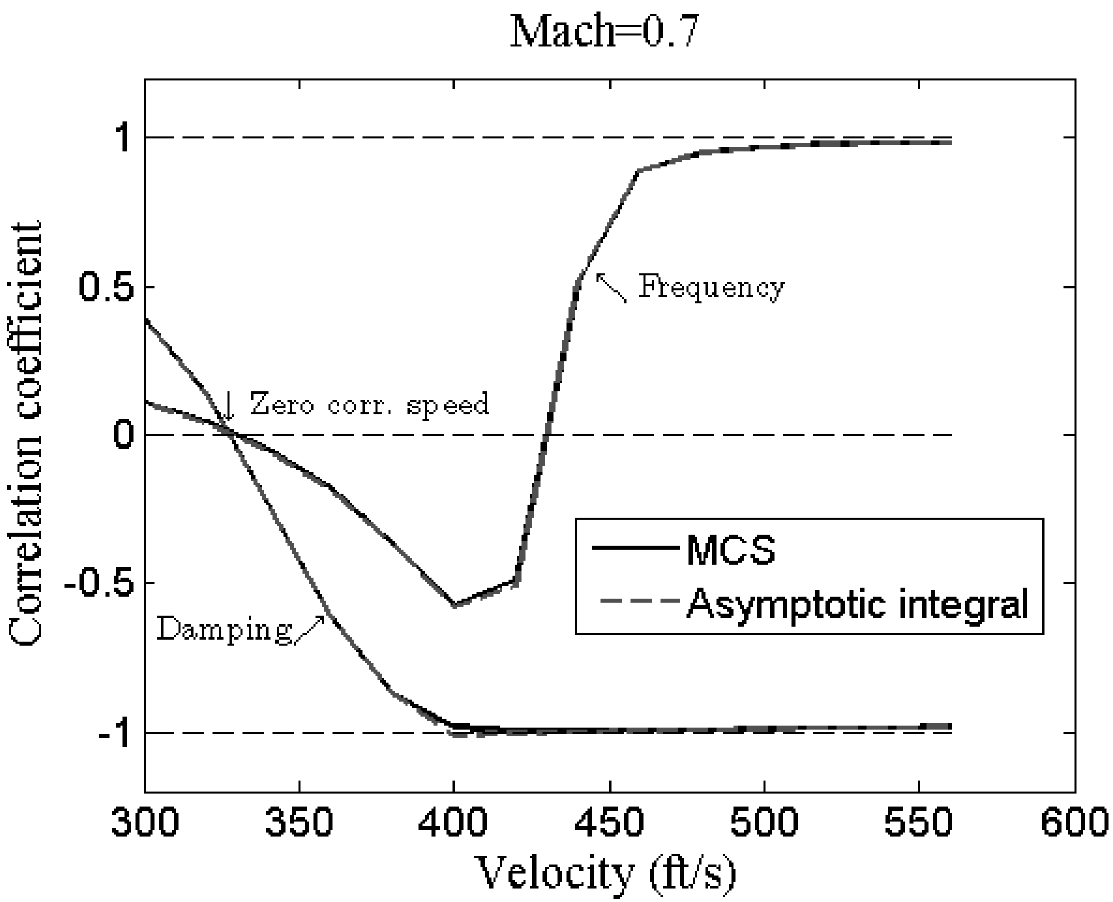


198 Degrees of Freedom

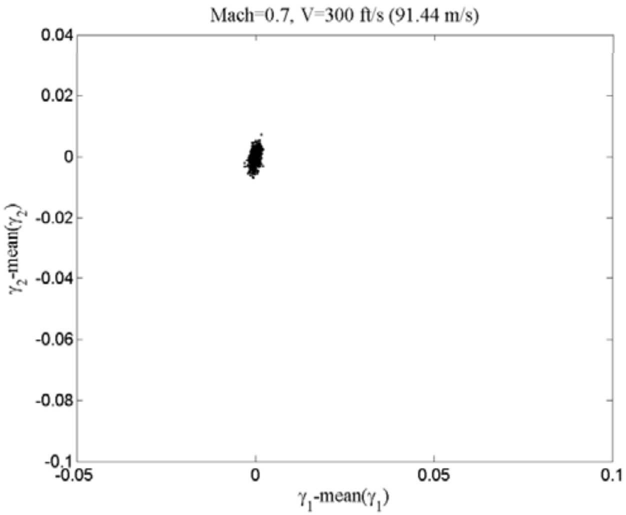
Meta model (Response Surface method) can be used to evaluate gradient and Hessian matrix and to speed up the procedure

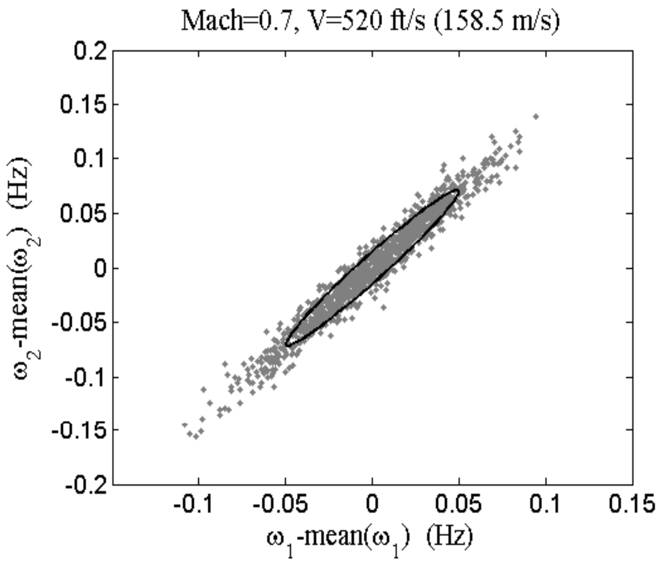


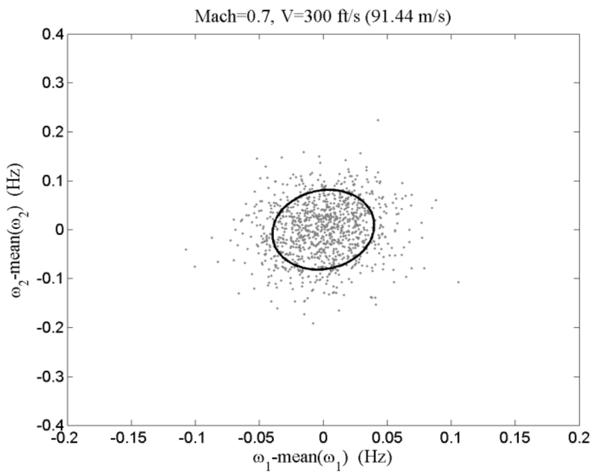
Correlation Coefficient: Crossing Modes

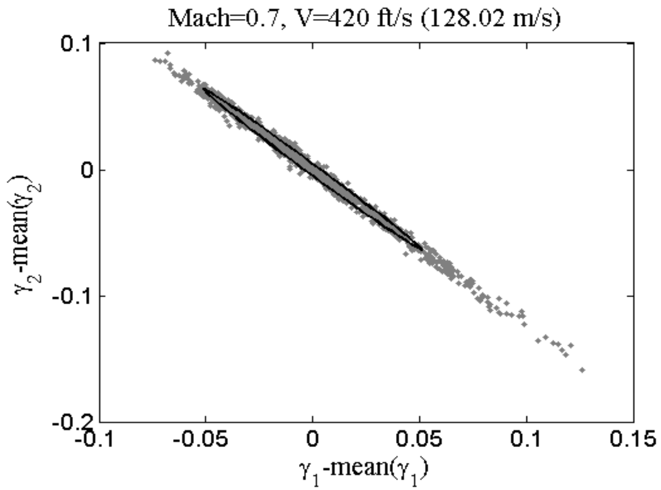


Correlation Coefficient: Crossing Modes

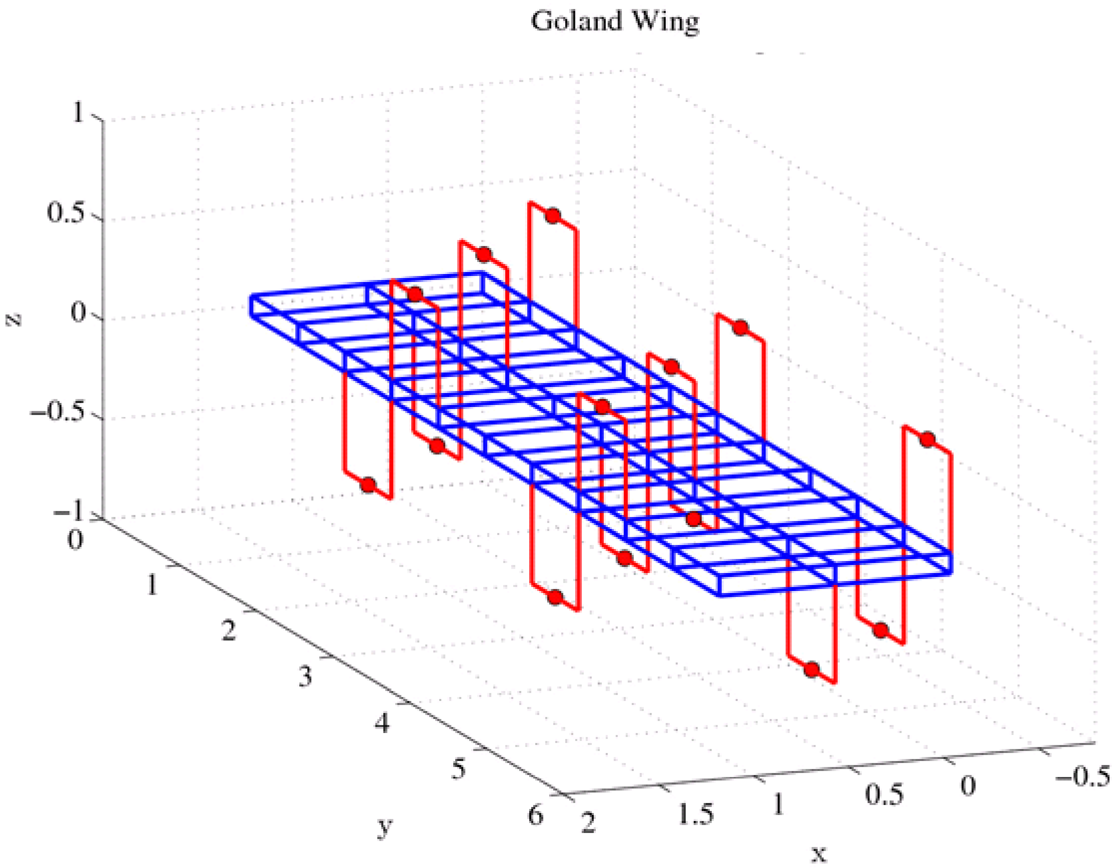








Propagation of structural damping to Aeroelastic analysis

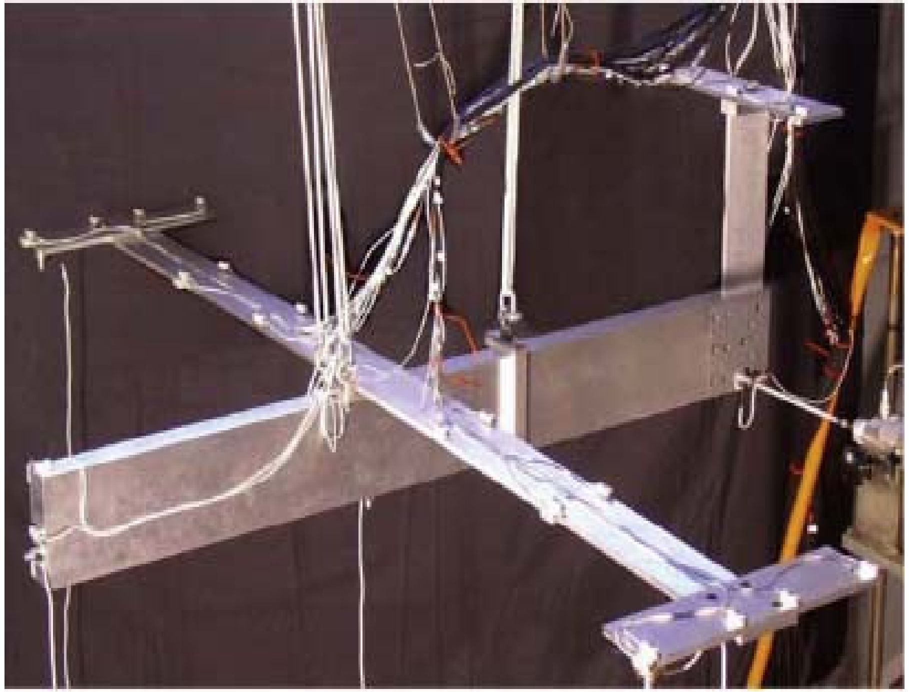


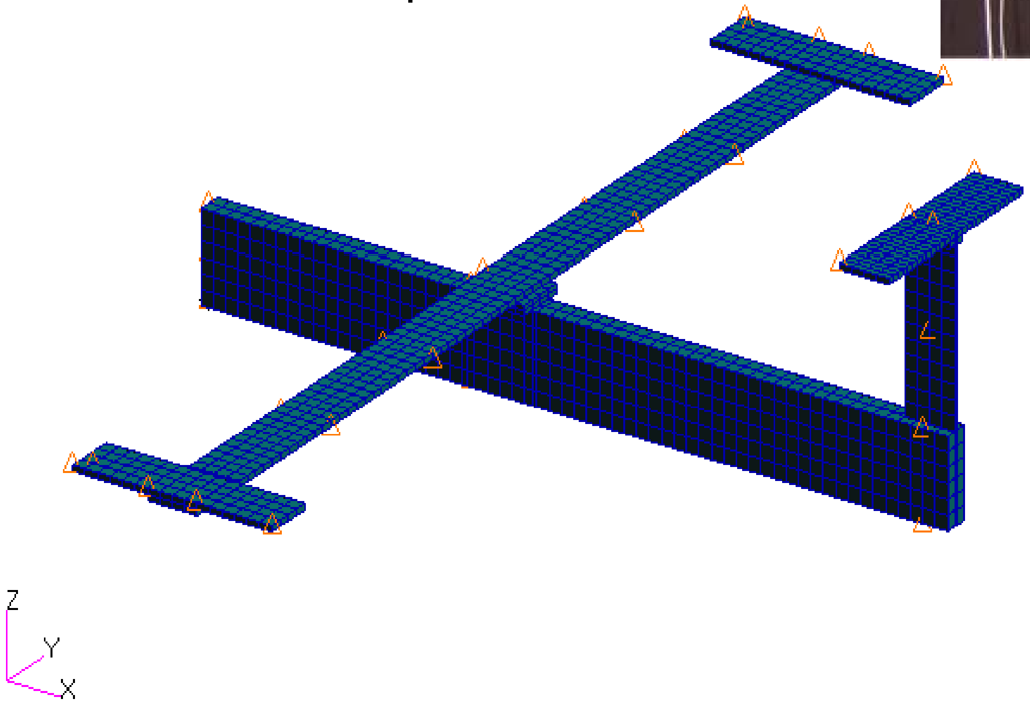
Dr Marco Prandina

Current and future work

GARTEUR SM-AG19

Application of forward propagation methods in LCO problem





Conclusions

- Different forward propagation methods have applied to the problem of flutter analysis in aeroelastic system.
- Interval analysis (with Response-Surface optimisation) was found to be efficient and produces enough information about uncertain aeroelastic system responses.
- The flutter boundary becomes increasingly sensitive to structural variability, as the stability threshold is approached.
- Efficent perturbation methods have been developed for identification of the ranges of input varitaion from measured output variations.
- Interval model updating has developed and it is shown that the method is capable of producing quite accurate results even in the presence of small number of measured data.
- The stochastic updating methods are verified experimentally.

Acknowledgement

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Thanks for your attention !