How to Validate Stochastic Finite Element Models from Uncertain Experimental Modal Data

Yves Govers

Deutsches Zentrum für Luft- und Raumfahrt e.V.
in der Helmholtz-Gemeinschaft

Outline/ Motivation

- Validation of Finite Element Models on basis of modal data (eigenfrequencies and mode shapes) determined from Modal Survey Test or Ground Vibration Test (GVT)

- Use of gradient based Computational Model Updating Procedures

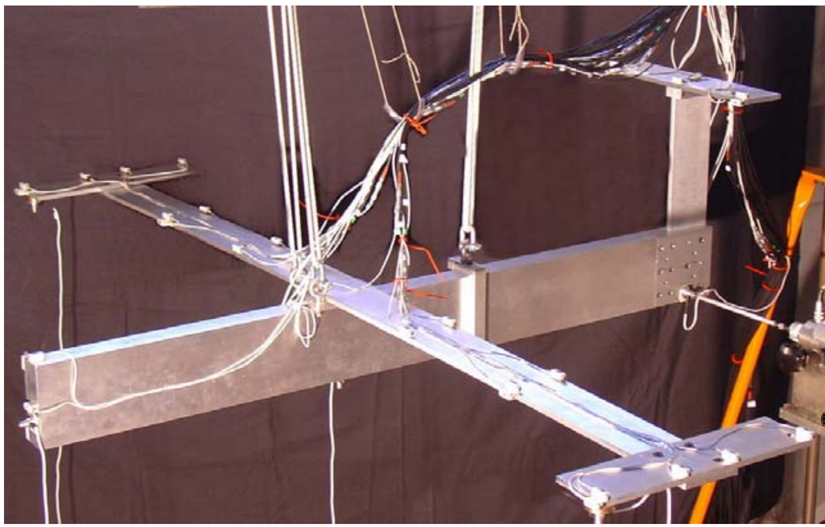
- State of the art: deterministic approach
(a single experimentally determined set of modal data is used to identify a deterministic Finite Element Model)

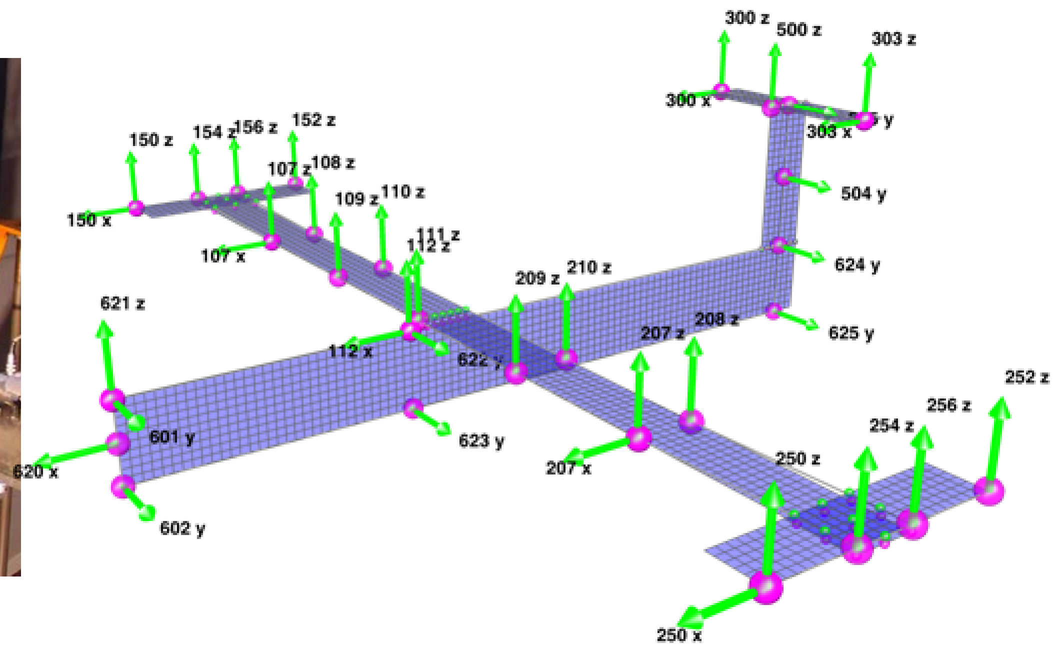
- Goal: probabilistic approach
(use multiple experimentally determined test data sets to identify a Finite Element Model with stochastic parameters)

Motivation

Modal Data Uncertainty

- DLR laboratory benchmark structure **AIR**craft **MOD**el (GARTEUR SM-AG19 replica)
- Made of aluminium with 6 beam like components connected by bolted joints

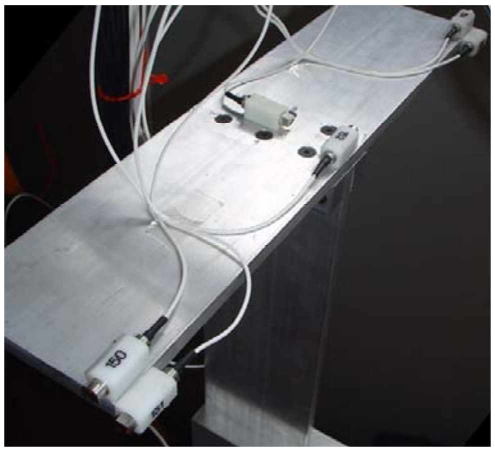




Motivation

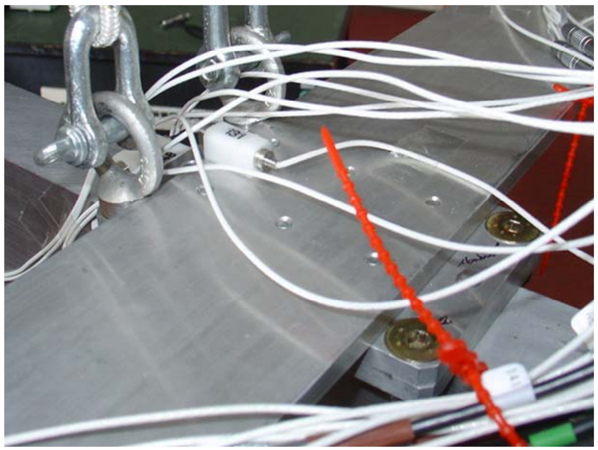
A Source of Modal Data Uncertainty

- the uncertainty of joint stiffness parameters are generally unknown

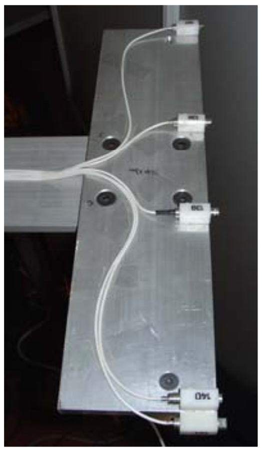


VTP vs. HTP

VTP vs. fuselage



wing vs. fuselage



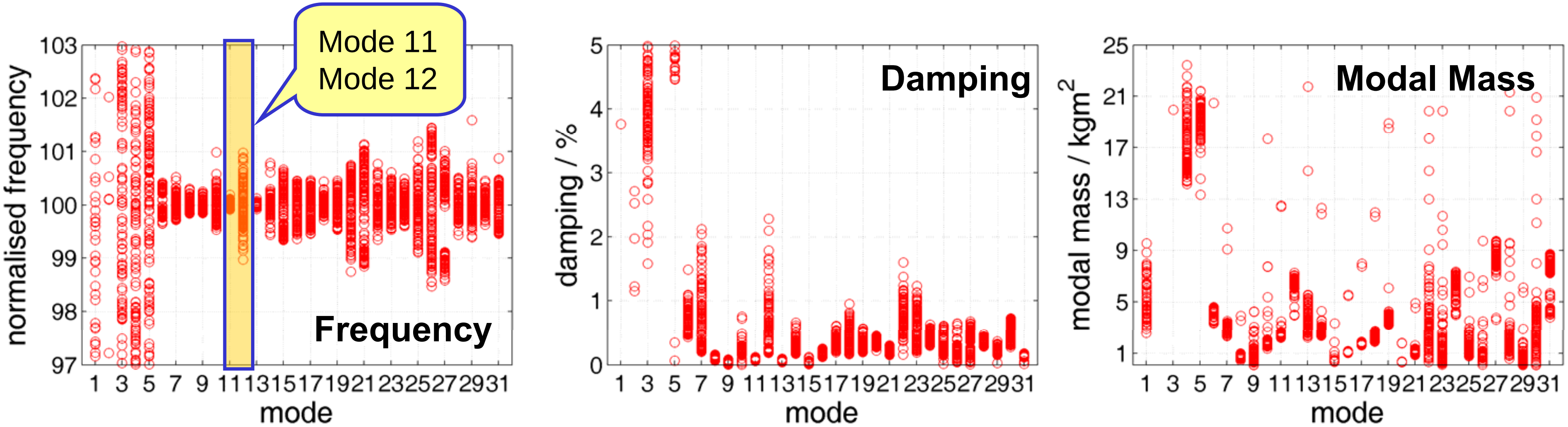
winglet

Repeated Modal Survey Test on AIRMOD

Motivation

Modal Data Uncertainty – Results of AIRMOD Test Campaign

- 130 times assembled, disassembled, reassembled and re-tested with random excitation and subsequent automated modal parameter estimation

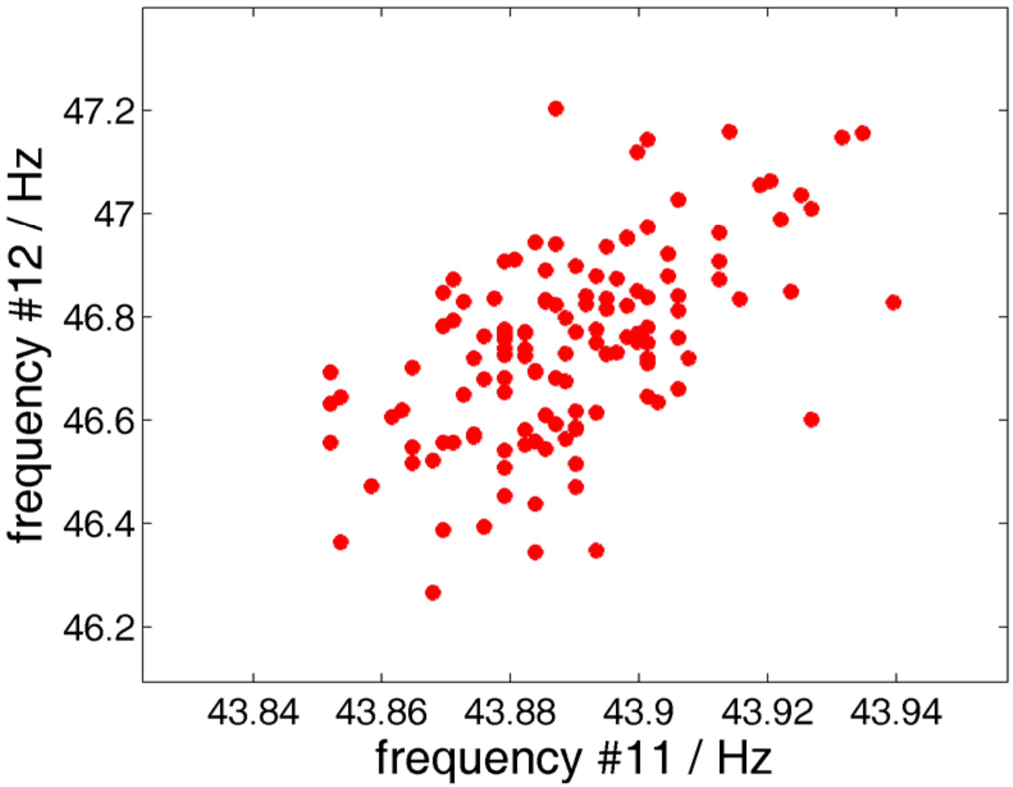


- Significant variation on modal parameters!

Introduction

Frequency Clouds

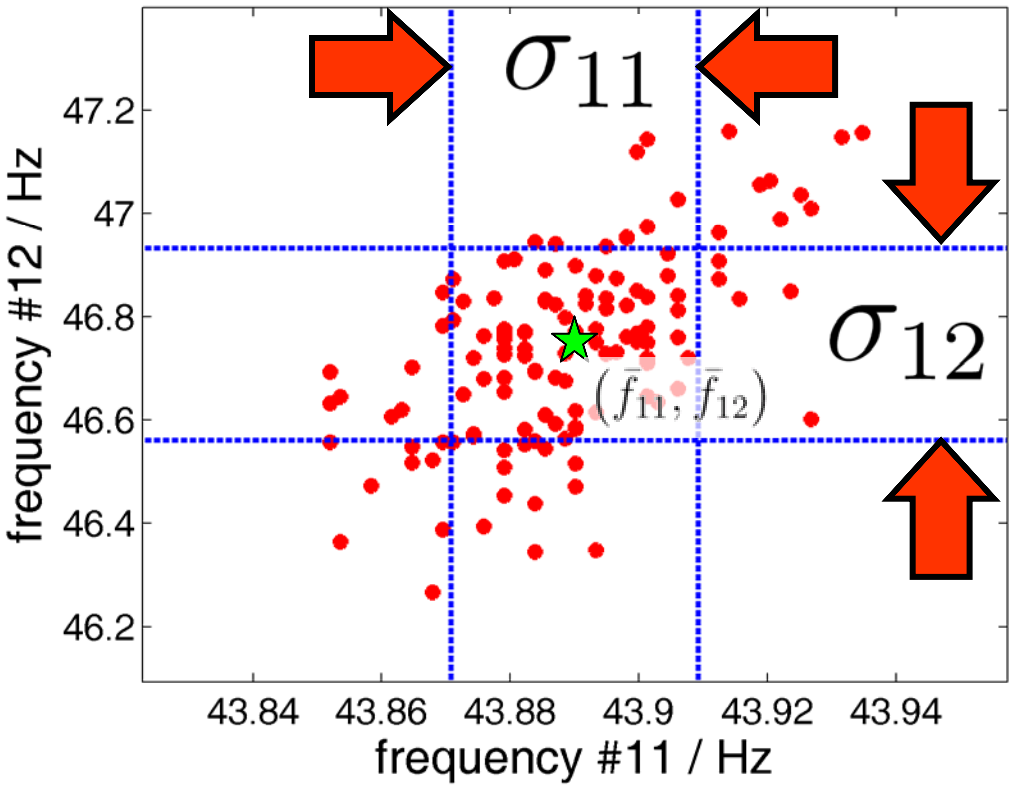
- a number of n tests has been performed
- → n frequency pairs $(f_{11,1},f_{12,1})$, $(f_{11,2},f_{12,2})$,… $(f_{11,n},f_{12,n})$ can be plotted
- a scatter diagram makes the correlation between two frequencies visible



Introduction

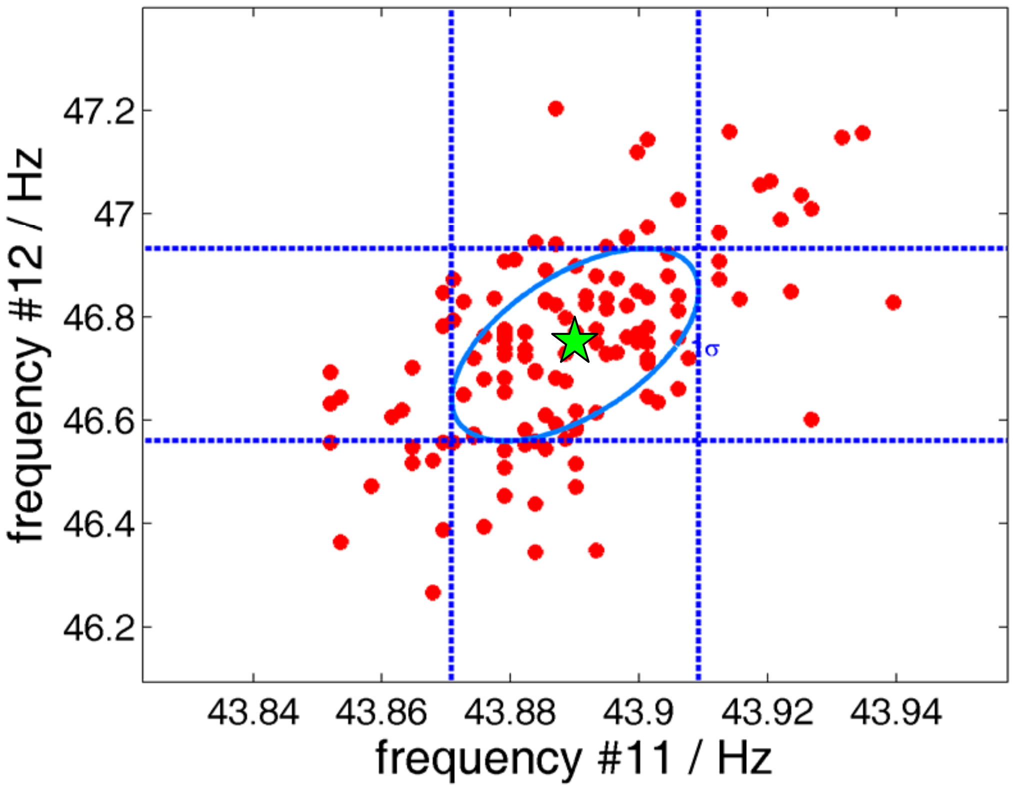
Frequency Clouds

- The mean values $(\bar{f}_{11}, \bar{f}_{12})$ and the standard deviation σ_{11} and σ_{12}
- In case of correlated frequency pairs the direction of the frequency cloud is important



Introduction
Frequency Clouds

- The so called covariance ellipse is a contour line of equal probability
- Here: 1 x σ



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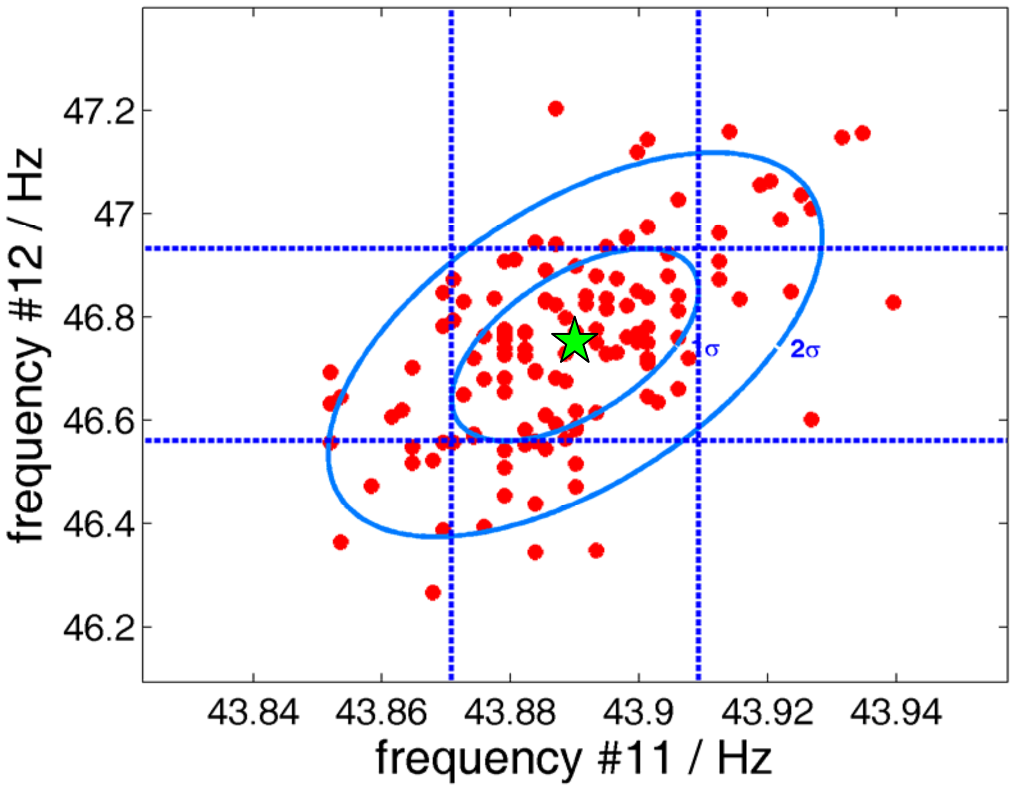

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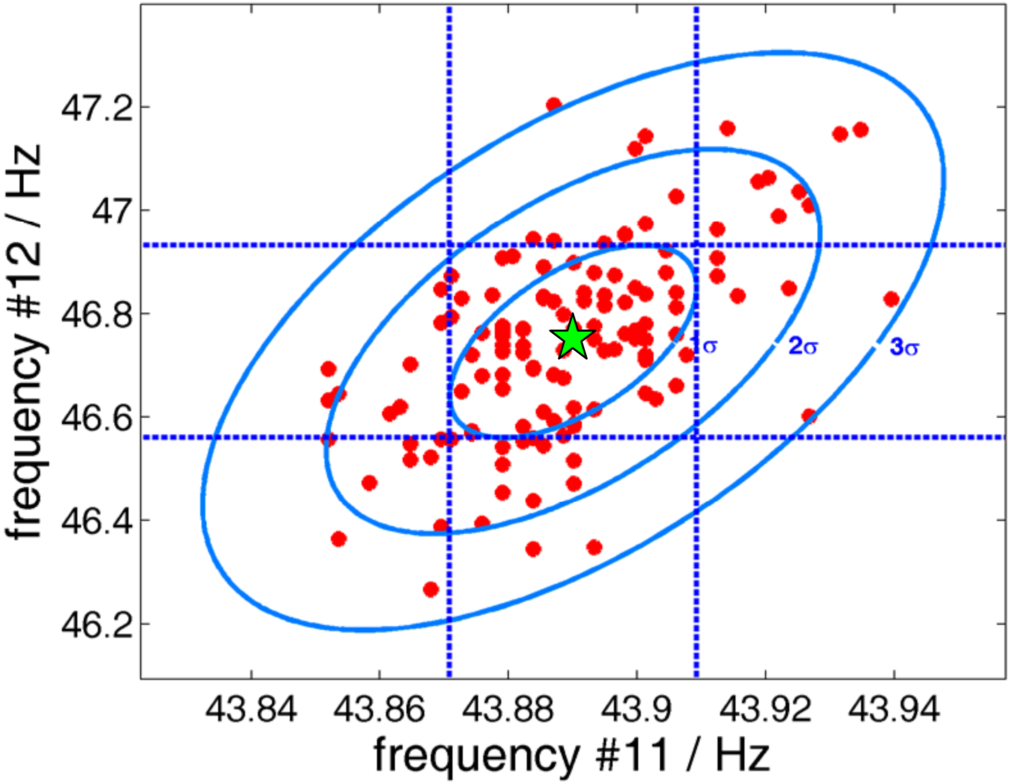
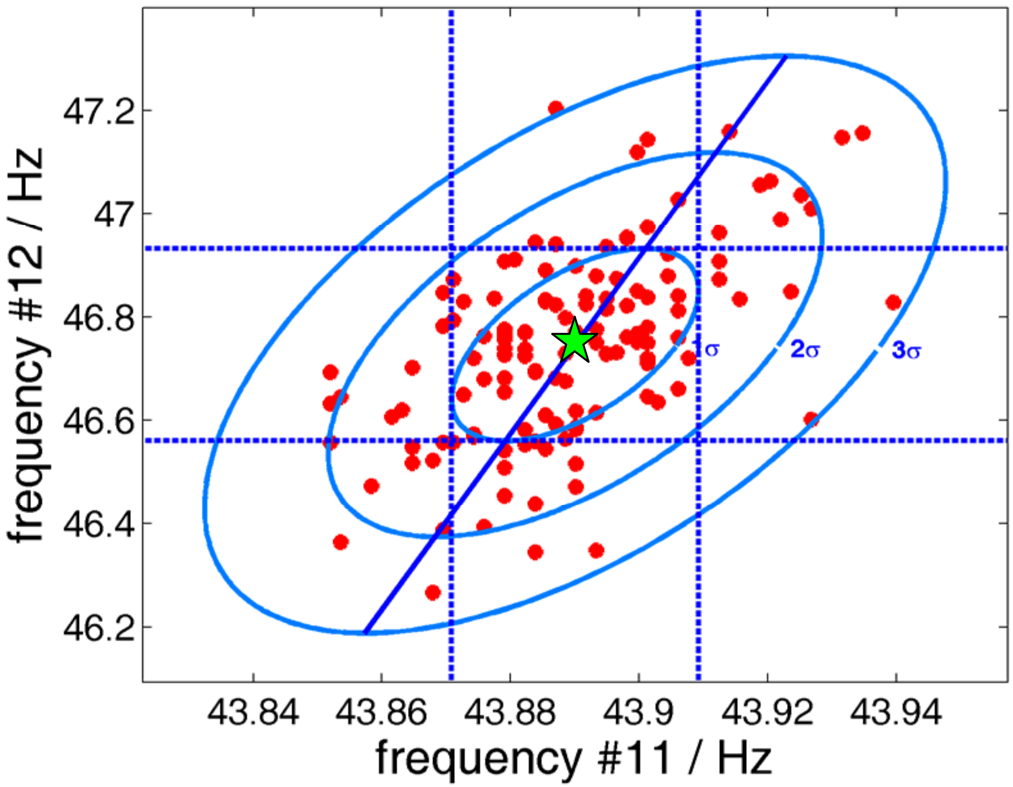


Introduction

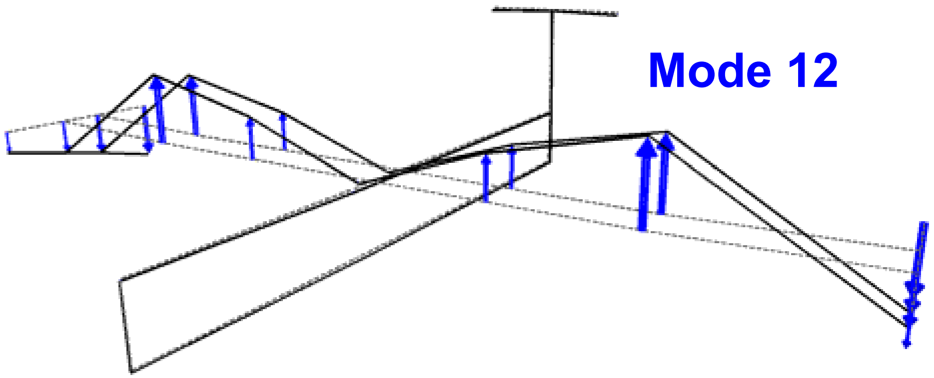
Frequency Clouds

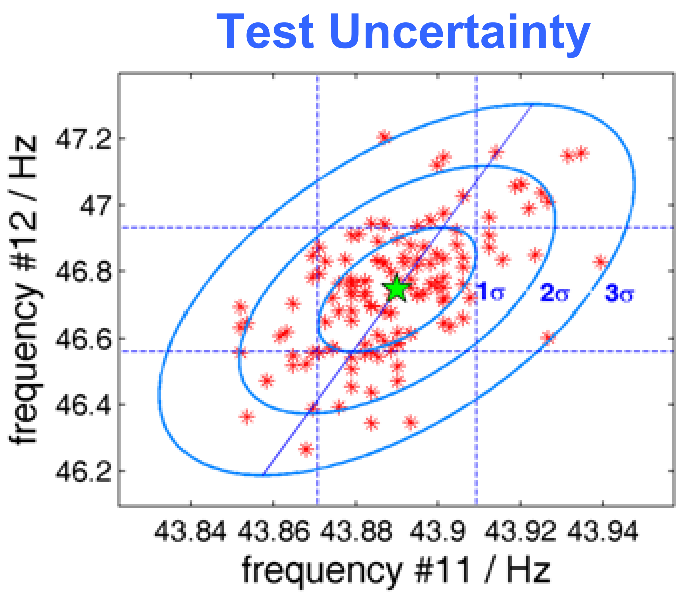
- the orientation of the ellipse can be visualised by the principal axes
- It shows if the two frequencies are positively or negatively correlated



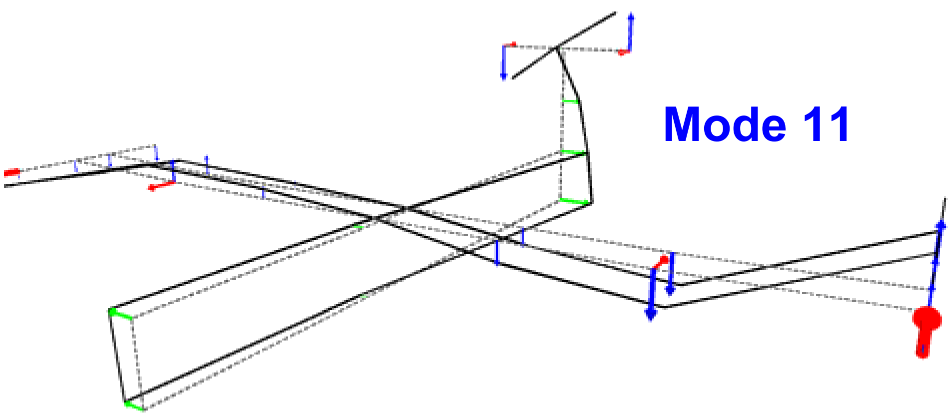
Introduction

Frequency Clouds - Test Data



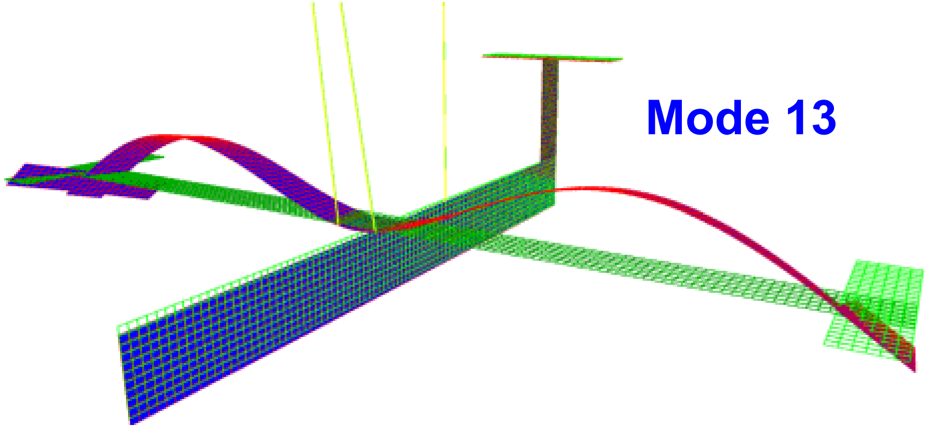


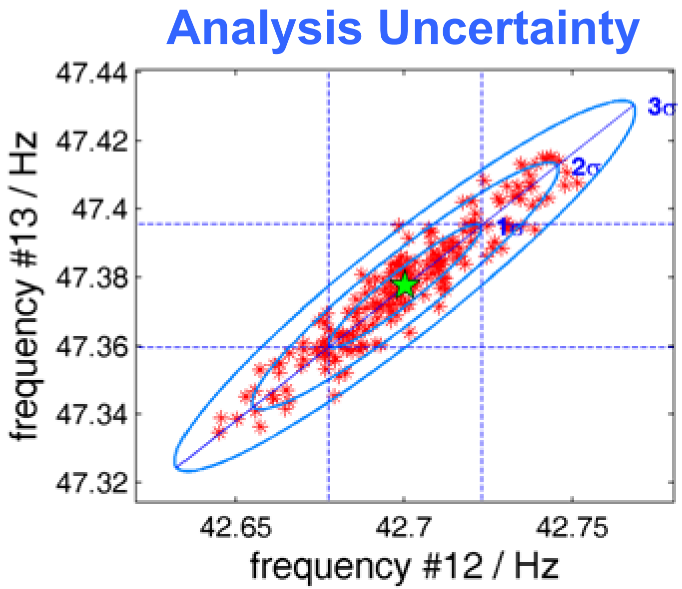
- Uncertain experimental modal data by **multiple** tests on nominal identical structures
- Uncertainty and correlation of modal data becomes visible if two frequencies are plotted against each other



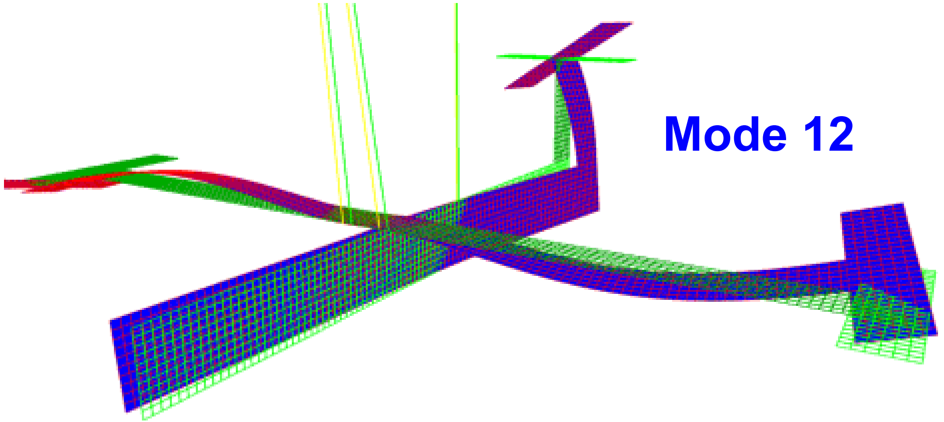
Introduction

Frequency Clouds - Analysis Data



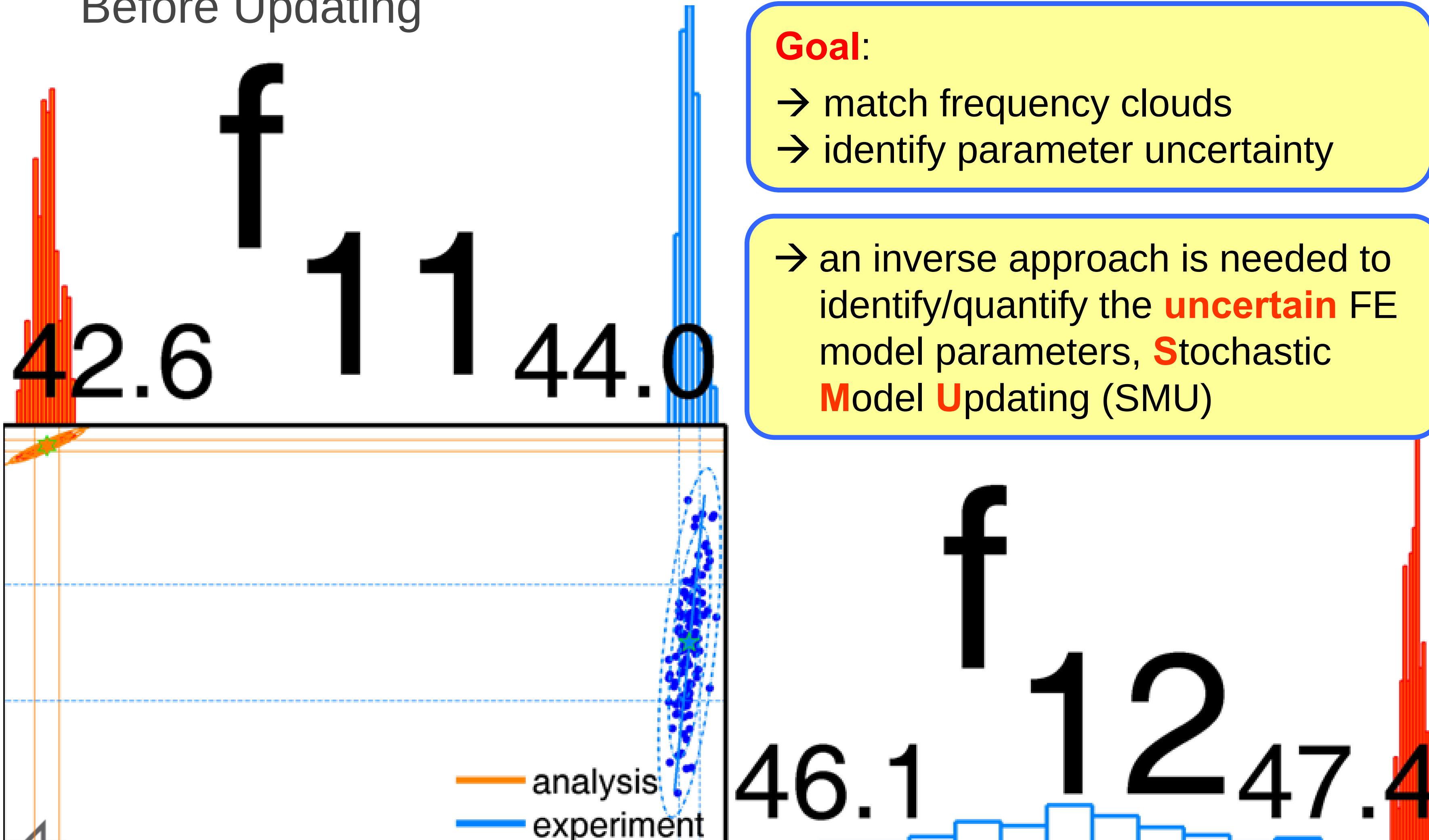


- Uncertain analysis modal data by **randomising** a number of **design parameters**
- here: **Monte Carlo Simulation** is utilised in combination with **Latin Hypercube Sampling**



Introduction

Before Updating



Goal:

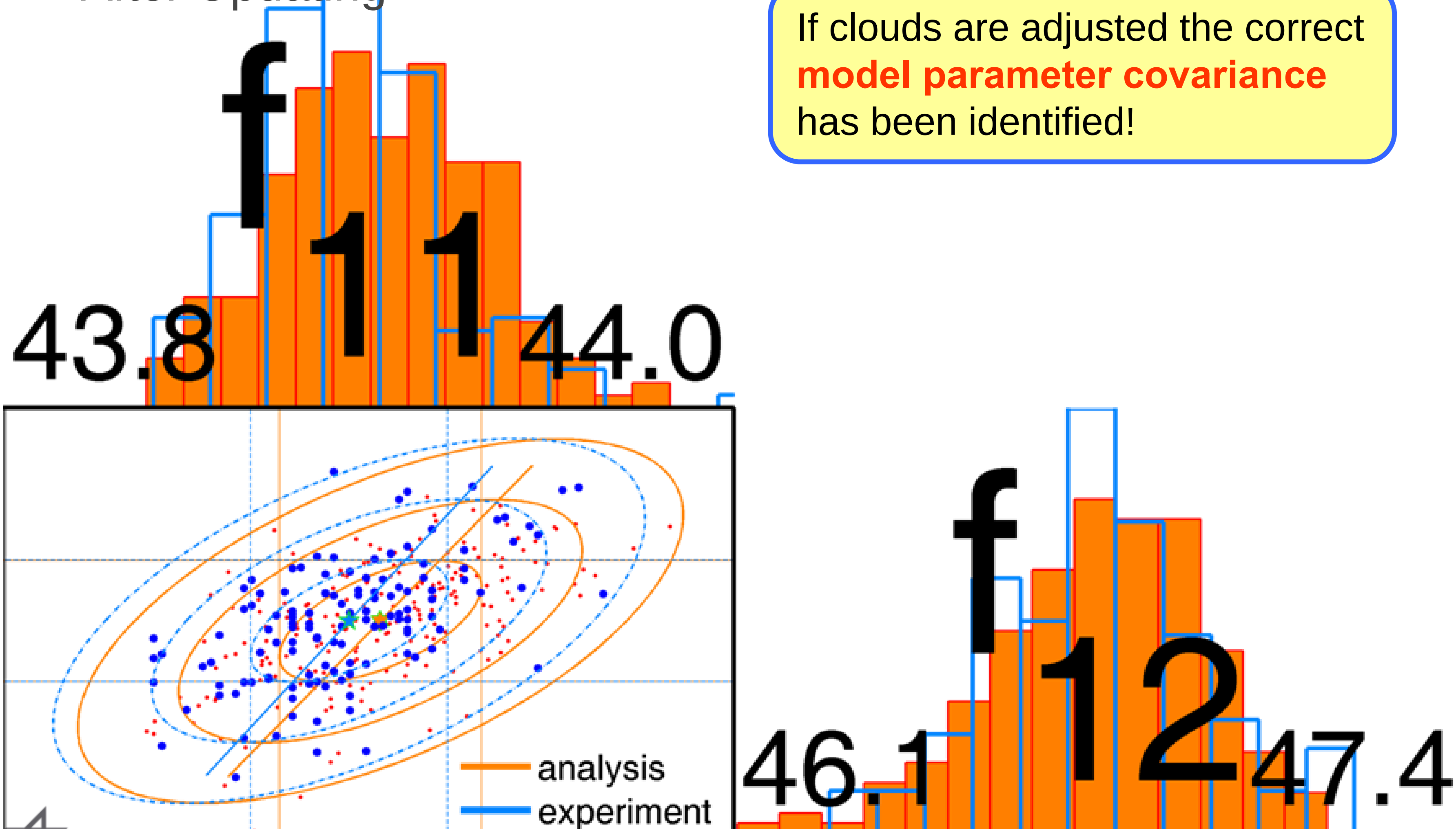
→ match frequency clouds
→ identify parameter uncertainty

→ an inverse approach is needed to identify/quantify the **uncertain** FE model parameters, **S**tochastic **M**odel **U**pdating (SMU)

Introduction

After Updating

If clouds are adjusted the correct **model parameter covariance** has been identified!



Stochastic Model Updating

Mean Parameter Adjustment

- the difference between mean analytical and measured values can be assembled in a weighted residual vector

$$\{\bar{\varepsilon}_w\} = [W_v](\{\bar{v}_m\} - \{\bar{v}_a(p)\})$$

- the vector of analytical values $\{v_a\}$ can be described by a linearized Taylor series where $[G]_i$ represents the sensitivity matrix

$$\{\bar{v}_a(p)\}_{i+1} = \{\bar{v}_a\}_i + [G]_i\{\Delta\bar{p}\}_i$$

- by minimizing following objective function

$$J = \{\bar{\varepsilon}\}^T[W_\varepsilon]\{\bar{\varepsilon}\} + \{\Delta\bar{p}\}^T[W_{p,\varepsilon}]\{\Delta\bar{p}\} \rightarrow \min$$

a regularization term $[W_p]_i$ is used in case of ill-conditioning to improve convergence

Stochastic Model Updating

Mean Parameter Adjustment

- the parameter changes are derived using the pseudoinverse of $[G]_i$

$$\{\Delta\bar{p}\}_i = [T_\varepsilon]_i\{\bar{r}\}_i$$ with $$\{\bar{r}\}_i = \{\bar{v}_m\} - \{\bar{v}_a\}_i$$

- where [T] is the transformation matrix

$$[T_\varepsilon]_i = ([G]_i^T[W_\varepsilon][G]_i + [W_{p,\varepsilon}]_i)^{-1}[G]_i^T[W_\varepsilon]$$

Stochastic Model Updating

Covariance Matrix Adjustment

- the difference of the covariance matrix of the measured samples and the corresponding analytical covariance matrix can be summarized in a residual matrix

$$[S_\Delta]_i = [S_{v_m}] - [S_{v_a(p)}]_i$$

- the analytical covariance matrix can derived from the Taylor series expansion of the analytical vector under the assumption of $\{v_a\}$ and $\{\Delta p\}$ to be uncorrelated at iteration step i

$$\begin{aligned}[S_{v_a(p)}]_{i+1} &= \left[\mathbf{Cov}\left(\{v_a\}_i + [G]_i\{\Delta p\}_i, \{v_a\}_i + [G]_i\{\Delta p\}_i\right)\right] \\ &= [S_{v_a}]_i + [G]_i[S_{\Delta p}]_i[G]_i^T\end{aligned}$$

Stochastic Model Updating

Covariance Matrix Adjustment

- by minimizing following objective function with the Frobenius Norm of the residual matrix $[S_\Delta]$

$$J_S = \frac{1}{2} \left\| [W_S][S_\Delta][W_S]^T \right\|_F^2 \rightarrow \min$$

- the parameter covariance matrix changes (increments) are derived from

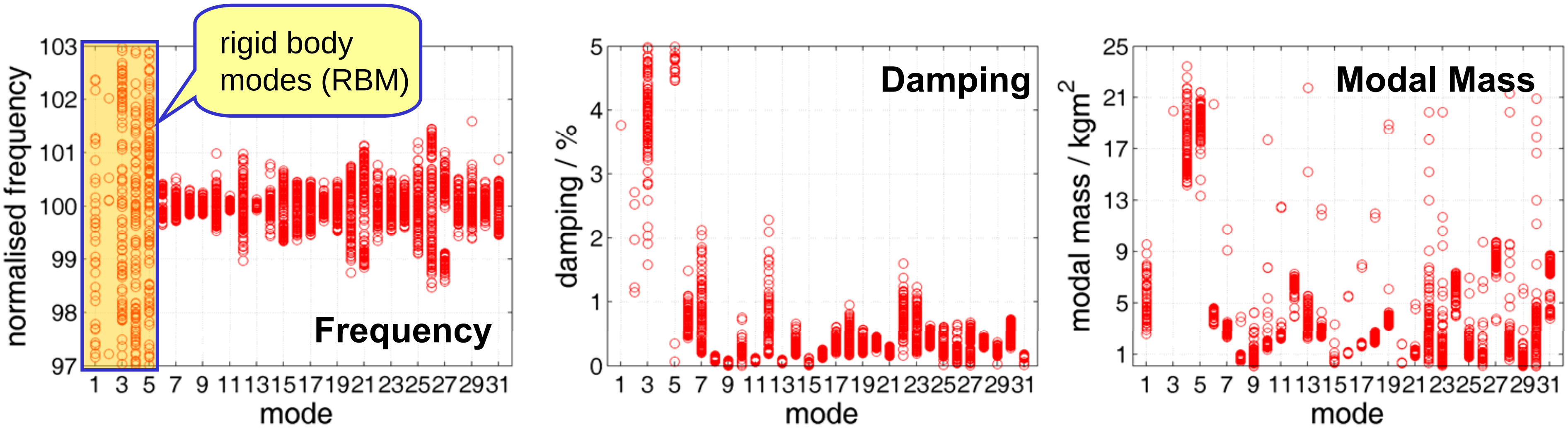
$$[S_{\Delta p}]_i = [T_\Sigma]_i \; [R]_i \; [T_\Sigma]_i^T \qquad \text{with} \quad [R]_i = [S_{v_m}] - [S_{v_a}]_i$$

- and the transformation matrix $[T_\Sigma]$

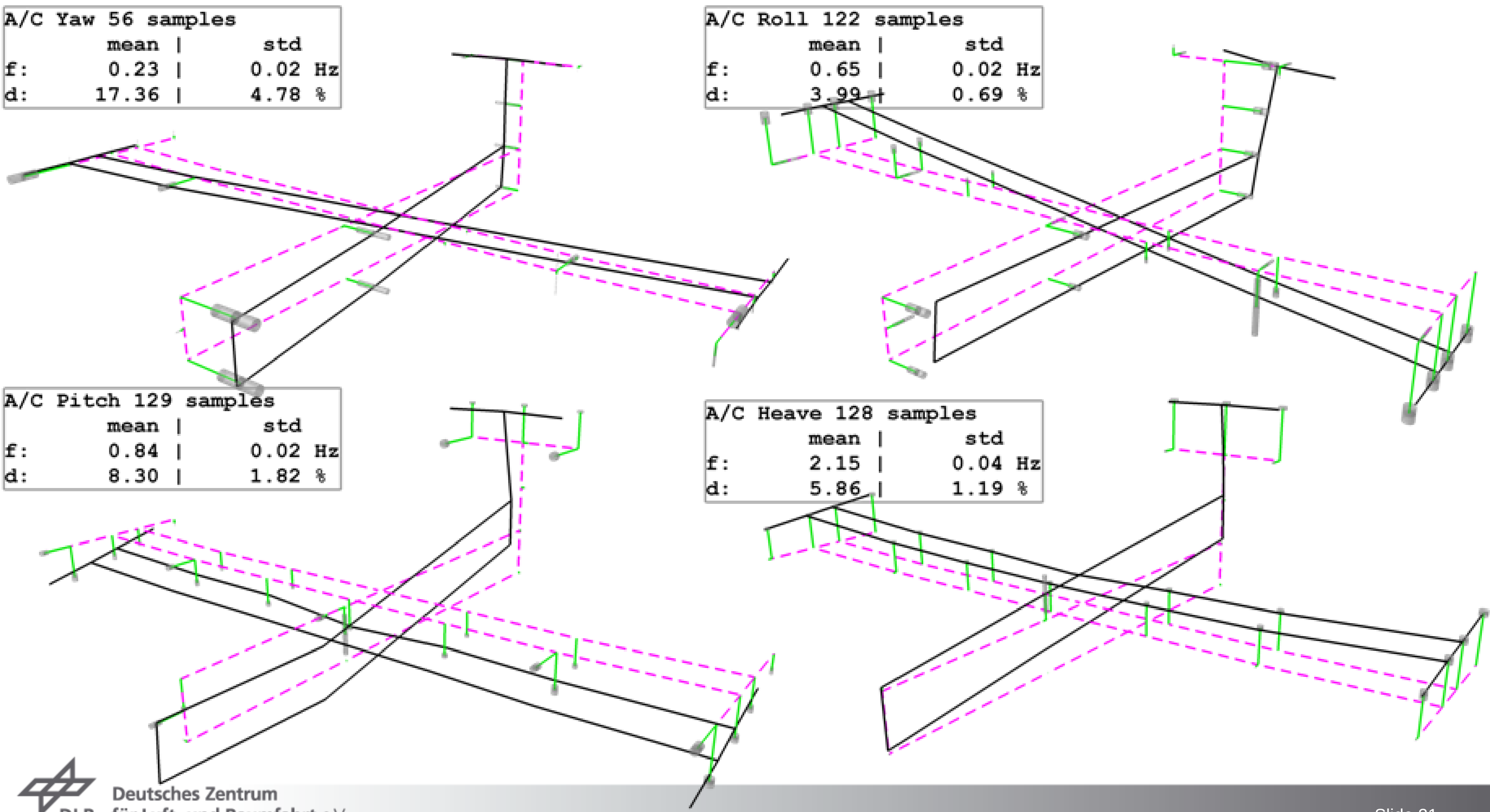
$$[T_\Sigma]_i = \left([G]_i^T [W_\Sigma][G]_i + [W_{p,\Sigma}]_i \right)^{-1} [G]_i^T [W_\Sigma]$$

Test Case – AIRMOD

Rigid Body Modes



Test Case – AIRMOD
Rigid Body Modes

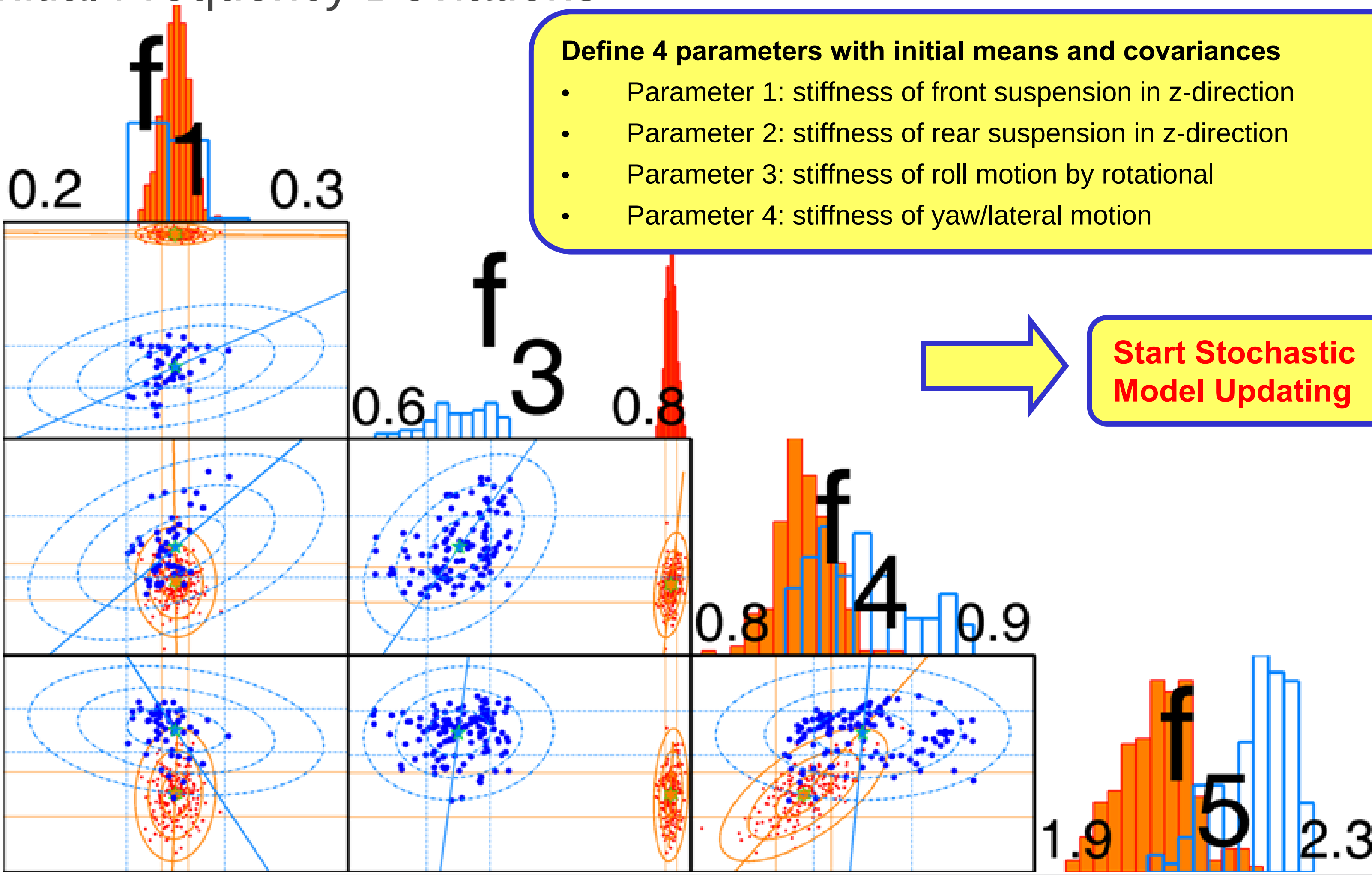


Test Case – AIRMOD rigid body modes

Initial Frequency Deviations

Define 4 parameters with initial means and covariances

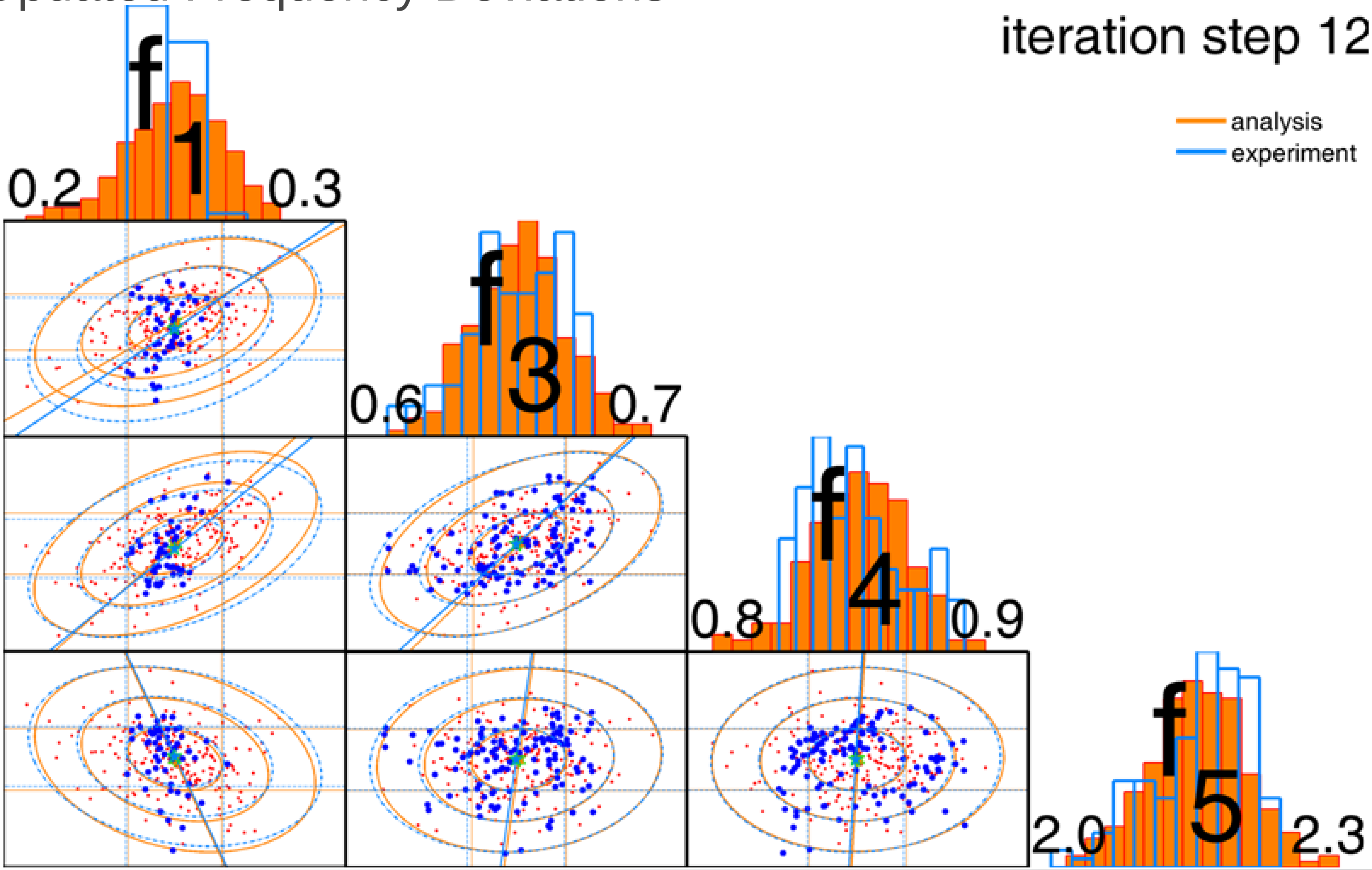
- Parameter 1: stiffness of front suspension in z-direction
- Parameter 2: stiffness of rear suspension in z-direction
- Parameter 3: stiffness of roll motion by rotational
- Parameter 4: stiffness of yaw/lateral motion



Start Stochastic Model Updating

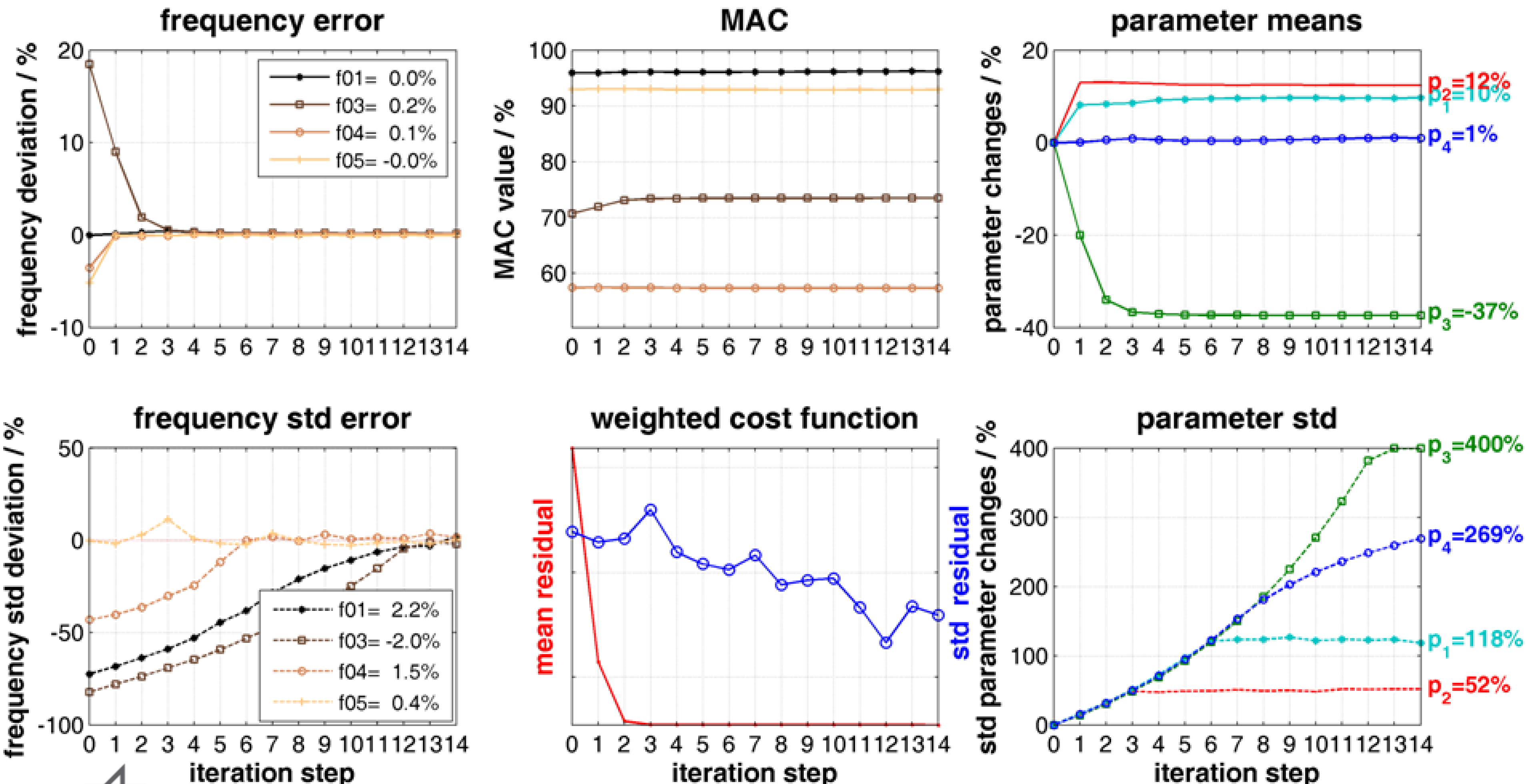
Test Case – AIRMOD rigid body modes

Updated Frequency Deviations



Test Case – AIRMOD rigid body modes

Convergence



Conclusions and Outlook

- Conventional model updating procedure has been extended by an equation adjusting the model parameter covariances
- Developed algorithm was applied to the rigid body modes of an aircraft like laboratory structure AIRMOD
- Test case shows a good convergence
- Frequency clouds match well: adjusted parameters represent the uncertainty of the measurement data
- In a second step the elastic modes will be updated

Thank you for your attention!

Yves Govers, German Aerospace Center (DLR)