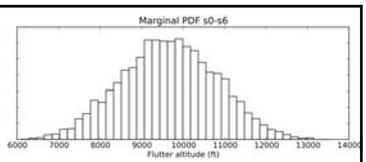

Bayesian assimilation of experimental data into simulation
(for Goland wing flutter)

Richard Dwight, Simao Marques

Overview

1. **Why not** uncertainty quantification?
2. **Why** uncertainty quantification?
3. **Probabilistic collocation** (PC) for UQ
4. **Data assimilation**

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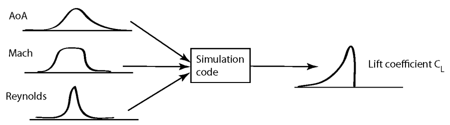
5. Application to **Goland wing**

Uncertainty quantification

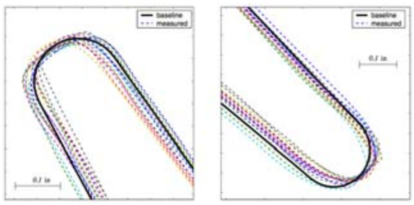


Goals:

- *Accurate* and *cheap* propagation of pdfs.
- No modification of the code required (*non-intrusive*).

Why not uncertainty quantification?

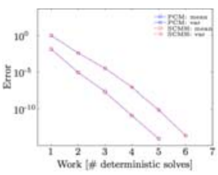
Best case scenario: measurements of *variability* exist.



from: Victor Garzon, *"Probabilistic Aerothermal Design of Compressor Airfoils"*, M.I.T. Doctoral Thesis, 2003.

Why not uncertainty quantification?

- Garzon measured **150** compressor blades.
- Monte-Carlo convergence rate........................ $\frac{1}{\sqrt{N}}$



from: Alex Loeven, *"Efficient uncertainty quantification in computational fluid dynamics"*, TU Delft PhD thesis, 2010.

Why uncertainty quantification?

1. Interval analysis is too conservative **(?)**
2. A tool for **data assimilation**.

Probabilistic collocation

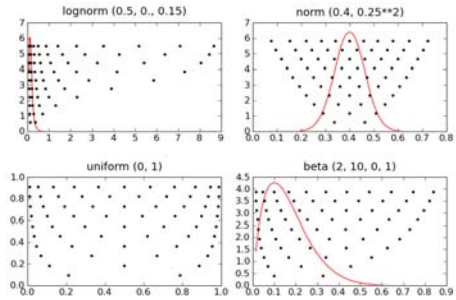
Central idea:

- Use a polynomial approximation to model response, that approximates accurately:

$$\mathbb{E}_f(u) = \int_{-\infty}^{\infty} u(\alpha) f(\alpha)\, d\alpha$$

$\Rightarrow$ Gauss rule weighted with $f(\cdot)$

Weighted Gauss rules = PC



Probabilistic collocation

- Approximate solution using Lagrange polynomials:

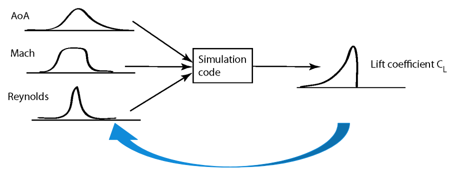
$$u(\alpha) \approx \sum_{i=1}^{N} u_i L_i(\alpha)$$

- **Ivo Babuška, Fabio Nobile, and Raúl Tempone**, *"A Stochastic Collocation Method for Elliptic Partial Differential Equations with Random Input Data"*, SIAM J. Numer. Anal. Volume 45, Issue 3, pp. 1005-1034 (2007).
- **G.J.A. Loeven**, *"Efficient Uncertainty Quantification in Computational Fluid Dynamics"*, PhD Thesis, TU Delft, 2010.

Data assimilation

- **Definition:**

  The incorporation of experimental data into numerical simulation, with the objective of obtaining a composite result that better reflects reality.



Data assimilation

Parameters				α
Experiment	$\Rightarrow$	Data	$\rightarrow$	d
Simulation	$\Rightarrow$	Model	$\rightarrow$	$m(\alpha)$

$$\boxed{m(\alpha) \text{ should approximate } d, \text{ for some } \alpha}$$

For example:

d = Measured C_L on an airfoil

$m(\alpha) = C_L(w, \alpha)$ where $R(w, \alpha) = 0$

R is a complete CFD solver.

Bayes' theorem

$$\mathbb{P}(\alpha | d) \propto \mathbb{P}(d | \alpha) \cdot \mathbb{P}(\alpha)$$

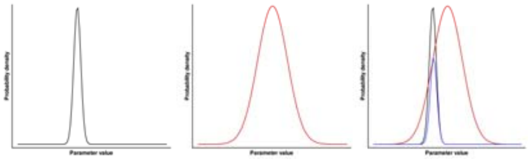
Bayesians believe:

1. Beliefs can be encoded in probability.
2. Knowledge can be separated into past belief (prior), and new observations.
3. Bayes Theorem relates prior belief to posterior (updated) belief.

Bayes' Theorem for Model-Data

Likelihood	**Prior**		**Posterior**
P(*d*|α)	P(α)	∝	P(α|*d*)

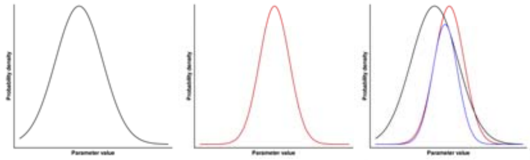
Informative data



Bayes' Theorem for Model-Data

Likelihood	**Prior**		**Posterior**
P(*d*|α)	P(α)	∝	P(α|*d*)

Non-informative data



Statistical model for likelihood

Need to specify: $\mathbb{P}(d|\alpha)$

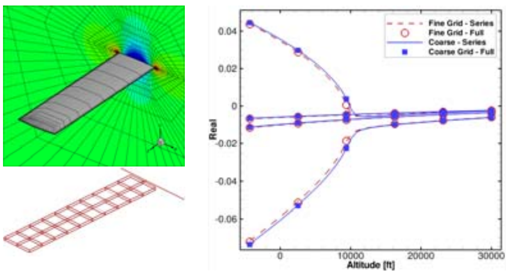
Assuming measurement noise is normal: $d = m(\alpha) + \epsilon$

$\epsilon \sim \mathcal{N}(0, \sigma_d^2)$

So: $\mathbb{P}(d|\alpha) = \rho_d(d - m(\alpha))$

$$\rho_d(x) \propto \exp\left[\frac{-x^2}{\sigma_d^2}\right]$$

Goland wing flutter – with Schur code



Liverpool's Schur flutter analysis code
Presentation of Simao Marques – *"Eigenvalue stability formulation..."*

Goland flutter uncertainty

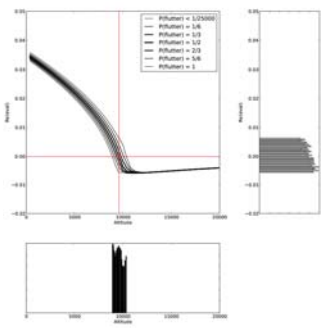
- **Seven (7)** uncertain structural parameters:
  - Thicknesses of the upper and lower skins
  - Thicknesses of the leading- and trailing-edge spars
  - Areas of the leading-, center- and trailing-edge spar caps
- **Uniformly** distributed **+-10%** of deterministic values.

PC(1): 2^7 = 128 samples
PC(2): 3^7 = 2187 samples...

Goland flutter uncertainty

1 parameter

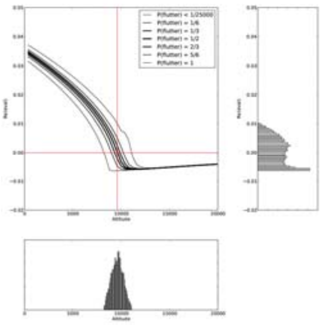
PC(2)



Goland flutter uncertainty

2 parameters

PC(2)

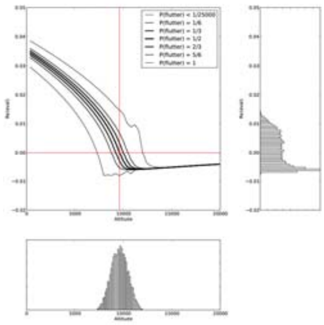


TUDelft Richard Dwight 19

Goland flutter uncertainty

3 parameters

PC(2)



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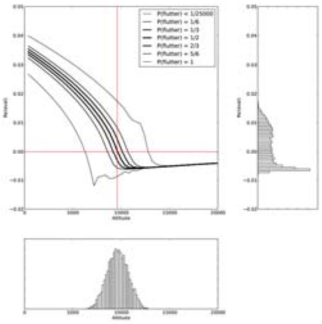
Goland flutter uncertainty

4 parameters

PC(2)



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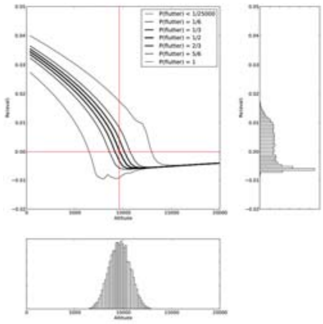
Goland flutter uncertainty

5 parameters

PC(2)



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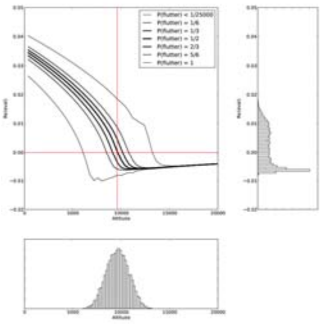
Goland flutter uncertainty

6 parameters

PC(2)



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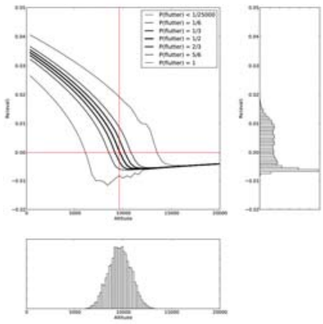
Goland flutter uncertainty

7 parameters

PC(2)



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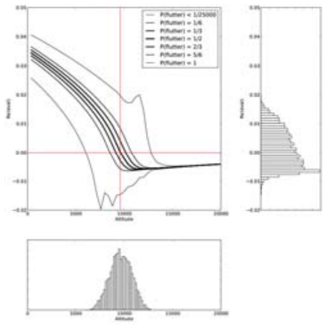
Goland flutter uncertainty

7 parameters

PC(1)



Variable #1

#2

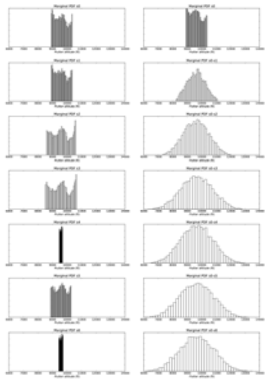
#3

#4

#5

#6

#7



Variables #1

#1-2

#1-3

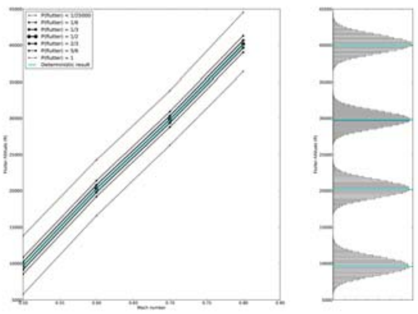
#1-4

#1-5

#1-6

#1-7

Goland flutter uncertainty



Goland data assimilation

- **How can we reduce this uncertainty?**
  - If we have measurement data...
  - (Actually we don't have measurement data for the Goland wing – so generate some...)

$$\mathbb{P}(\alpha | d) \propto \mathbb{P}(d | \alpha) \cdot \mathbb{P}(\alpha)$$

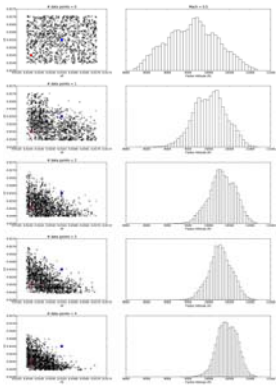
No experimental data

Data at Mach 0.5

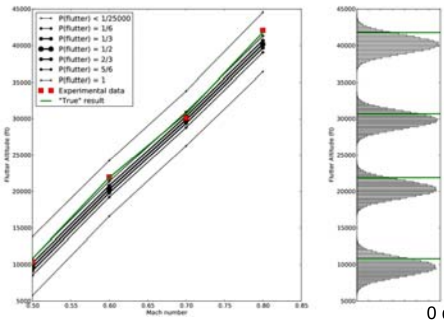
Mach 0.5, 0.6

Mach 0.5 - 0.7

Mach 0.5 - 0.8

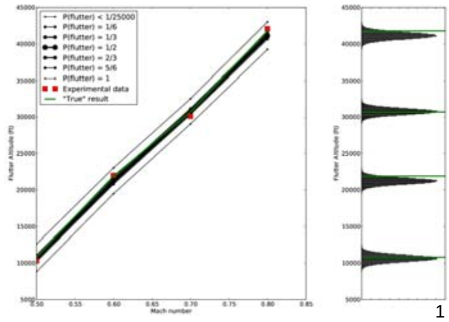


Goland flutter – Data assimilation



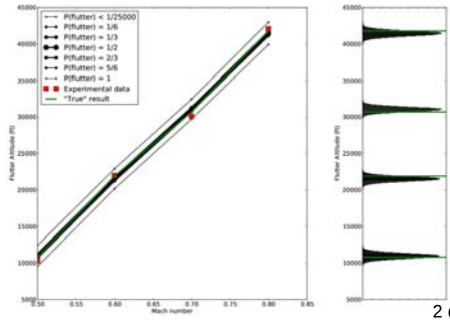
0 data points

Goland flutter – Data assimilation



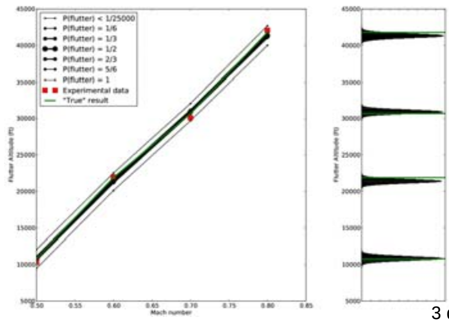
1 data point

Goland flutter – Data assimilation



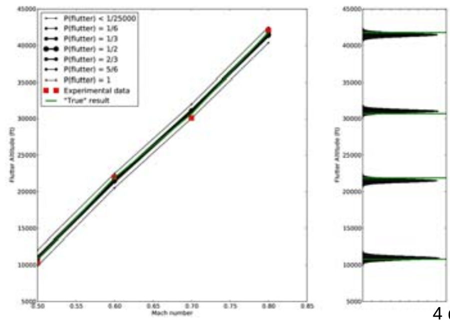
2 data points

Goland flutter – Data assimilation



3 data points

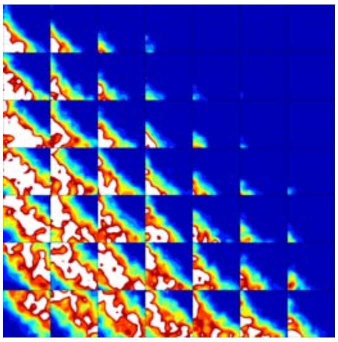
Goland flutter – Data assimilation



4 data points

Goland flutter – Parameter space

Increasing stiffness

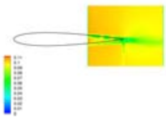


Conclusions

- Reasonable structural uncertainty leads to massive flutter uncertainty.
- Interval analysis may be too conservative. Need for UQ with good tail prediction.
- Even tiny amounts of data can be used to calibrate models, reducing uncertainty dramatically.
- Looking for nice applications / data.

Further work: To combine PIV and CFD

Experiment

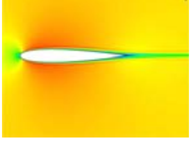


- Measurement noise
- Systematic error
- Parametric uncertainty

- Limited data

- Cheap topology changes

Simulation



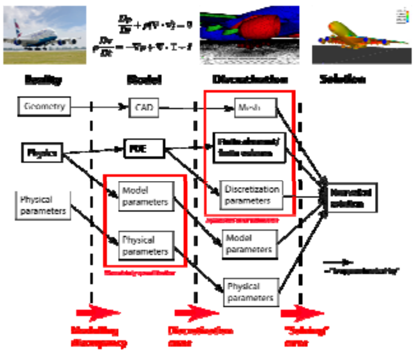
- Discretization error
- Model discrepancy
- Parametric uncertainty

- Complete data

- Cheap shape changes

Thank you for your attention

Errors in Simulation



UQ Literature

Starting points:

G.J.A. Loeven, "**Efficient Uncertainty Quantification in Computational Fluid Dynamics**", 2010 (http://repository.tudelft.nl/, search Loeven)

J.A.S. Witteveen, "**Efficient and Robust Uncertainty Quantification for CFD and Fluid-Structure Interaction**", 2009 (http://repository.tudelft.nl/, search Witteveen)

Habib N. Najm, **"Uncertainty quantification and Polynomial Chaos techniques in Computational Fluid Dynamics"**, Annual Review Fluid Mechanics 41 (2009) 35-52. http://dx.doi.org/10.1146/annurev.fluid.010908.165248