Use of Polynomial Chaos Expansions for Robust Design of Composite Wings for Flutter and Gusts

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Use of Composites in Aerospace

- Composites replacing metallic structures
- Main driver is weight savings
- Not using the full benefit of composites

Ironbridge. Metal structure derived from wood design

Boeing 787. A350. Composite structure derived using metal design philosophy

Aeroelastic Tailoring

- Make use of composite materials unidirectional properties to influence aeroelastic behaviour
  - Ply lay-up / thickness / ply percentages for each orientation
- Possible new configurations
  - Forward swept wings
  - Oblique wings
  - W shaped wings
- Virtually no application since 1980s – lots of studies

Dryden Flight Research Center EC87 0182-14 Photographed 1987
X-29

Objectives

- Investigate use of Polynomial Chaos Expansion to provide efficient probabilistic modelling of PDF variations in structural parameters for aeroelastic tailoring
- Several simple examples
- Illustrate approach for robust design using uncertain modelling

Background and Concept of PCE

- In simplified form random process can be written as

$$u(\theta) = \sum_{0}^{p} \beta_i \psi_i(\xi(\theta))$$

- For example 1-D chaos can be expressed

$$u = \beta_0 + \beta_1\xi + \beta_2(\xi^2 - 1) + \beta_3(\xi^3 - 3\xi) + \beta_4(\xi^4 - 6\xi^2 + 3) + \ldots$$

- β are unknown coefficients that must be found

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Polynomial Chaos Expansion

PCE with Latin Hypercube Sampling

Latin Hypercube Samples are taken ← **Input Parameters X(Samples)**

Input Parameters X(Samples) → **Response at each sample**; **Polynomial, ξ**

Response at each sample + **Polynomial, ξ** → **Coefficients β_i** → **Approximate Model: 2nd Order PCE, 3rd Order PCE** → **Residuals**

Parameter

(2-D 2nd Order)

$$u(\theta) = \sum_{0}^{p} \beta_i \psi_i(\xi(\theta))$$

$$u_{2nd} = \beta_0 + \beta_1\xi_1 + \beta_2\xi_2 + \beta_3(\xi_1^2 - 1) + \beta_4\xi_1\xi_2 + \beta_5(\xi_2^2 - 1)$$

$$u_{3rd} = \beta_0 + \beta_1\xi_1 + \beta_2\xi_2 + \beta_3(\xi_1^2 - 1) + \beta_4\xi_1\xi_2 + \beta_5(\xi_2^2 - 1) + \beta_6(\xi_1^3 - 3\xi_1) + \beta_7(\xi_1^2\xi_2 - \xi_2) + \beta_8(\xi_2^2\xi_1 - \xi_1) + \beta_9(\xi_2^3 - 3\xi_2)$$

Required parameter – e.g natural freq or flutter speed

(2-D 3rd Order)

Test cases → **Use regression to find coefficients** → **Efficient response emulation**

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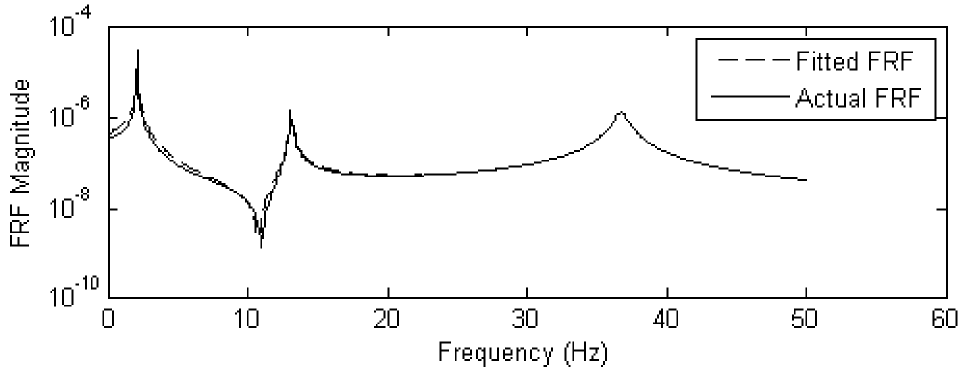
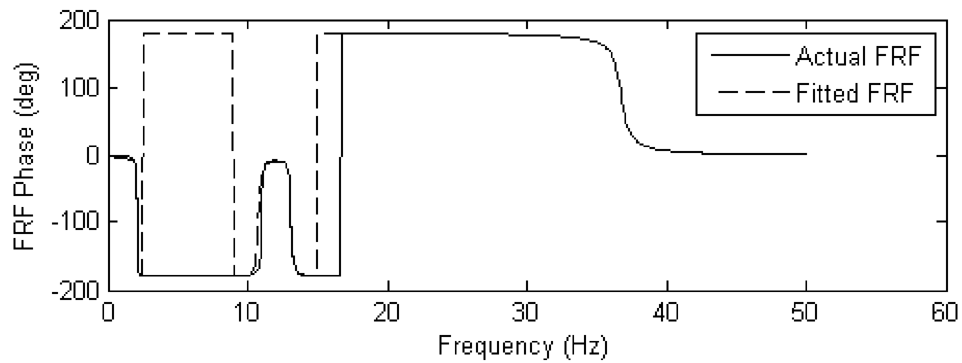
Example 1. Simple Beam

Variation of Beam FRF

- Simple beam FE model (Rayleigh damping)
- Freqs and damps from eigenvalue solution
- Curve-fit with standard FRF model



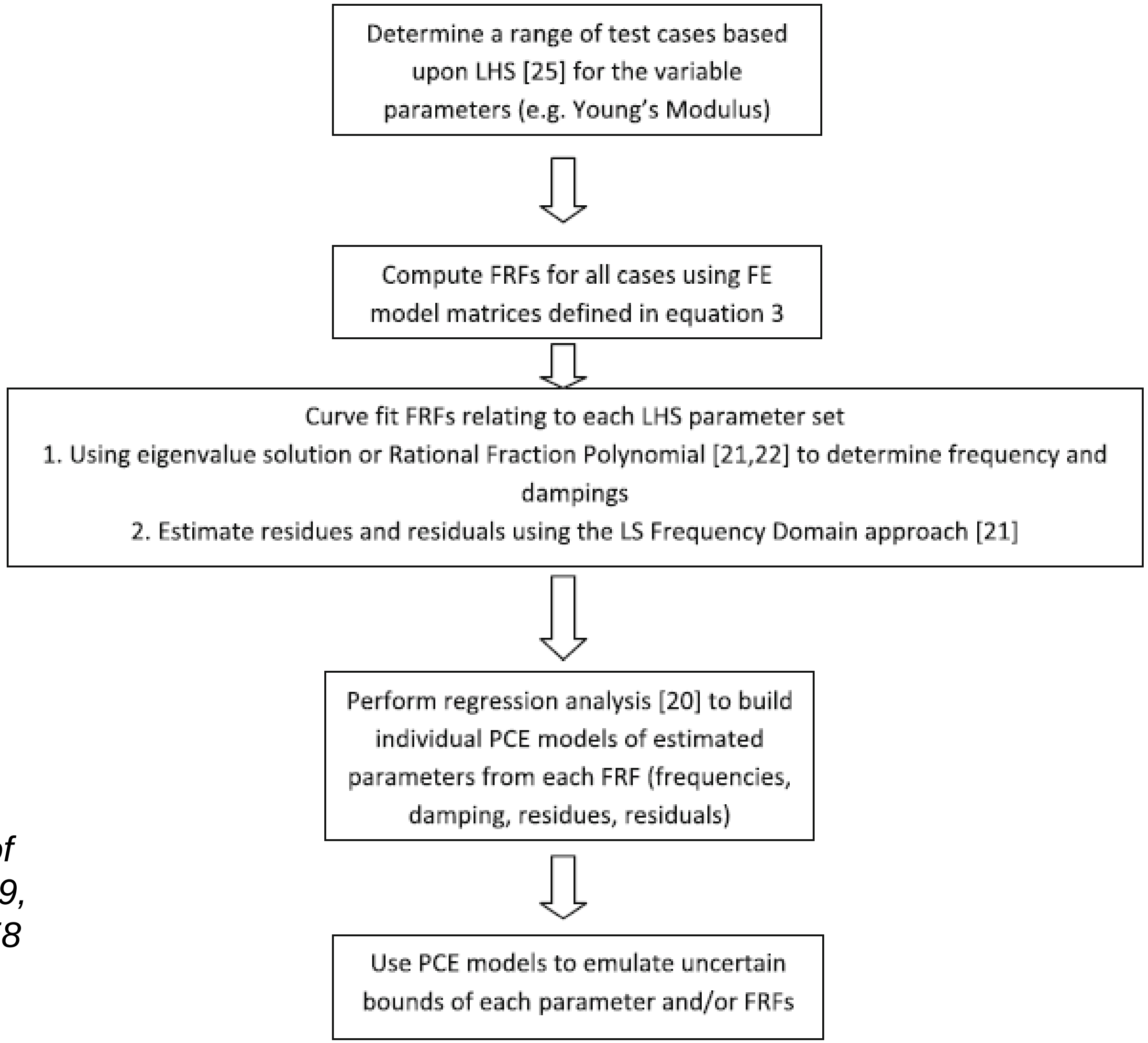
$$H_f(\omega) \cong \sum_{i=1}^{n} \frac{A_i}{\omega_i^2 - \omega^2 + 2j\zeta_i\omega_i\omega} + A_r + \frac{B_r}{\omega^2}$$



PCE Modelling

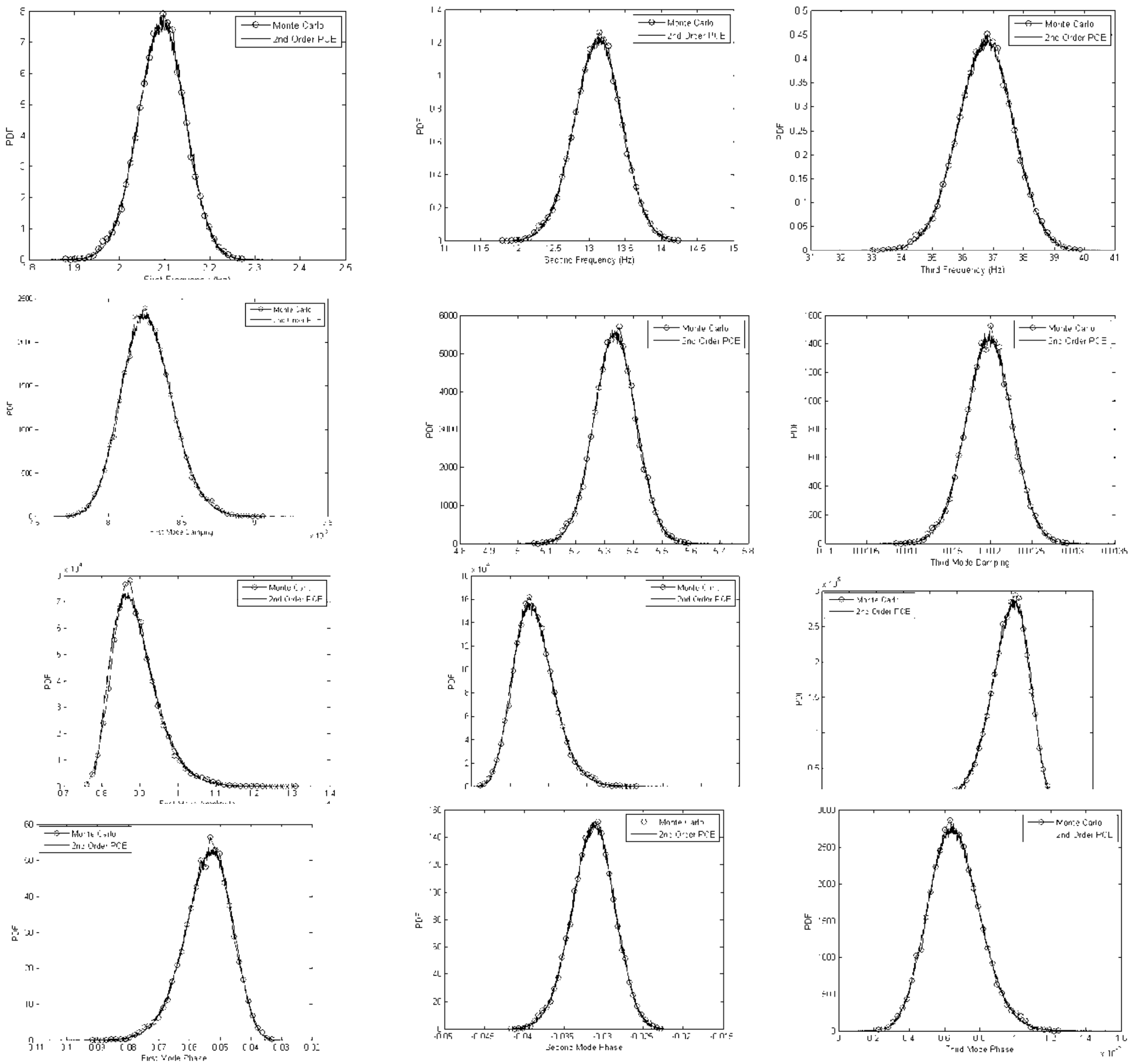
- Uncertainty
  - Young's modulus
  - Young's modulus and cross-section area
- Perform Latin Hypercube defined tests
- Fit PCE model to FRF fit for each mode
  - Frequency
  - Damping
  - Residues
  - Residuals

FRF-PCE Modelling



A. Manan, J.E. Cooper *Journal of Sound and Vibration*, *Volume 329, Issue 16*, *2010*, *Pages 3348-3358*

Fitted and Monte-Carlo PDFs



Frequency

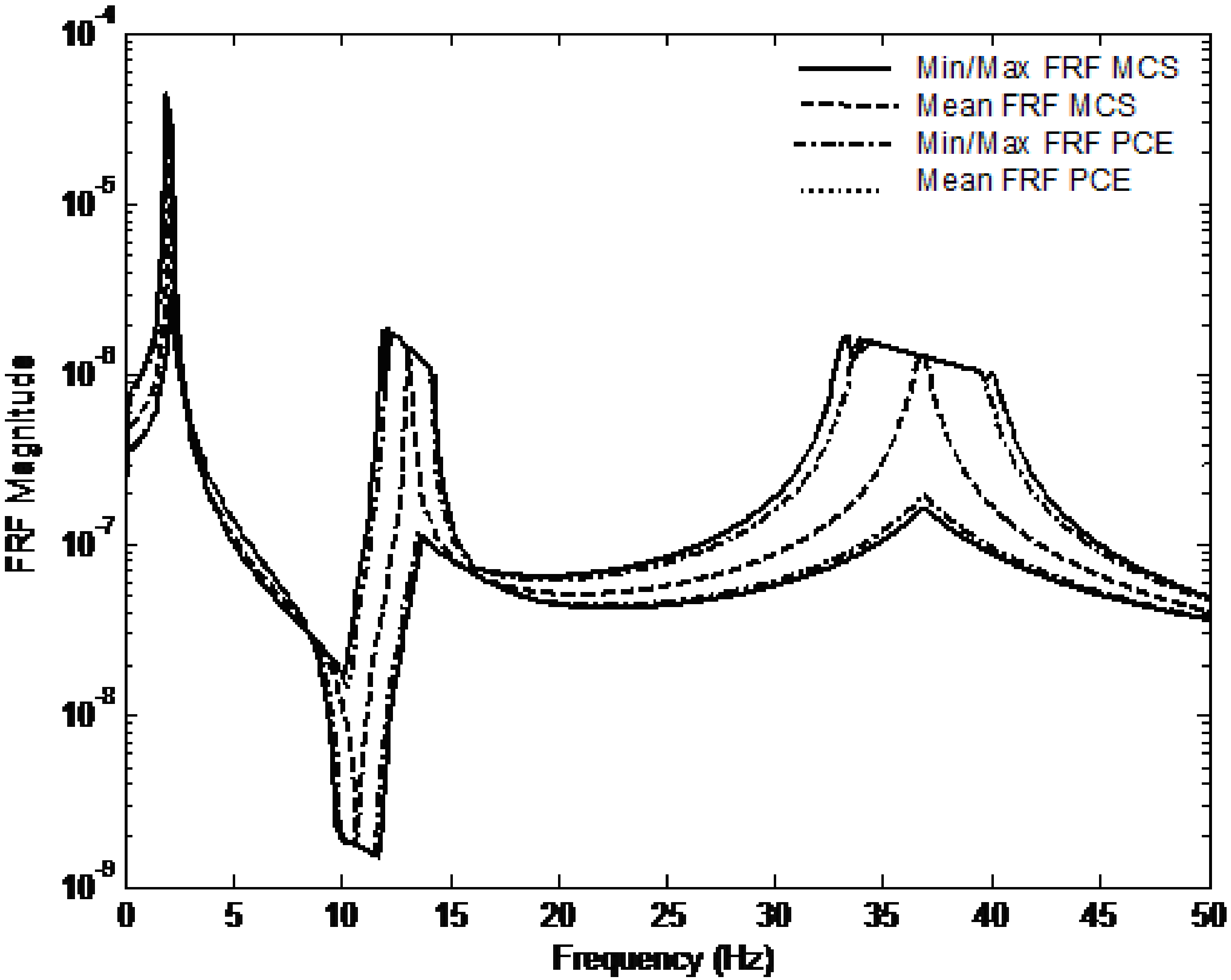
Damping

Residue amplitude

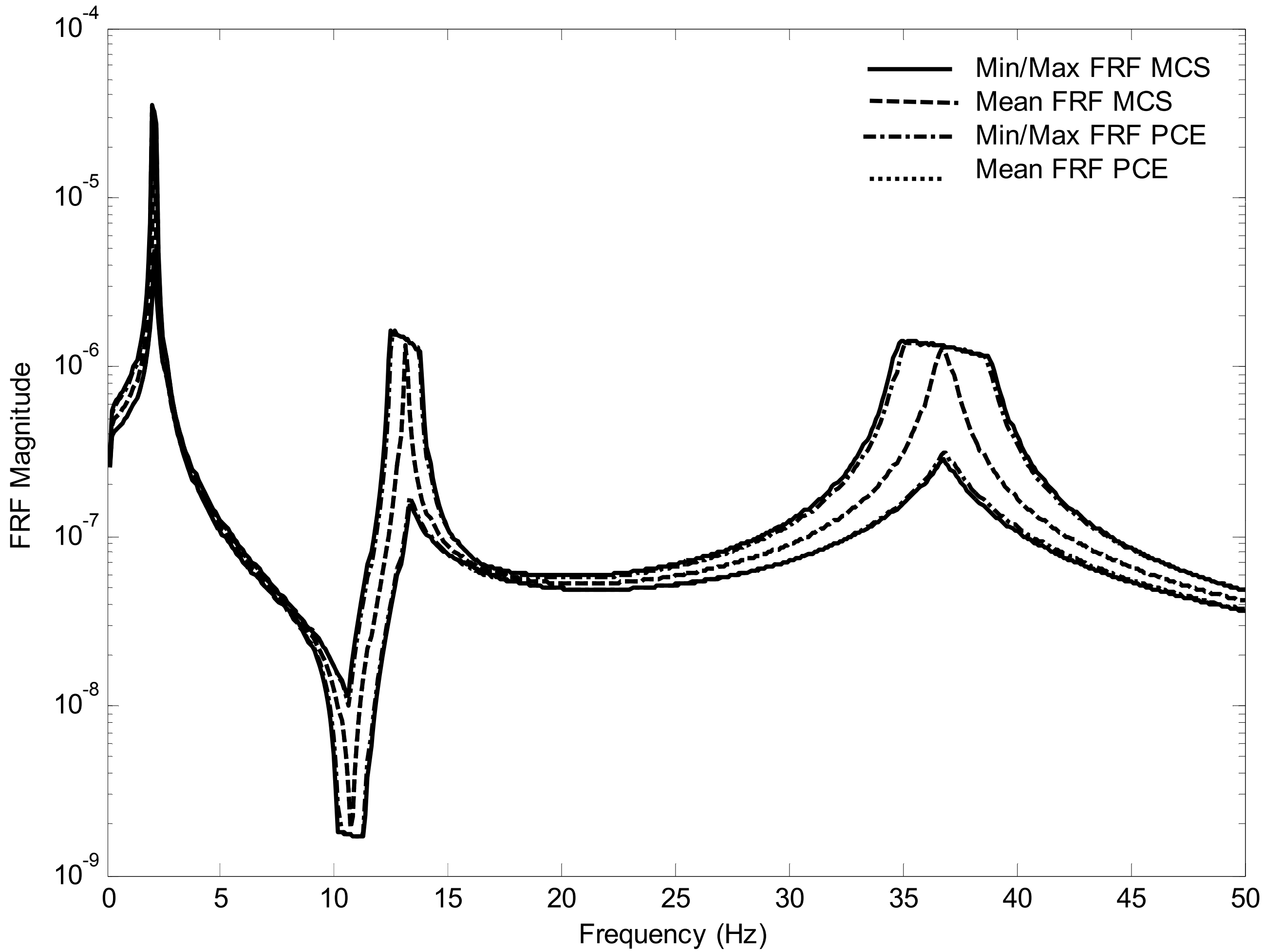
Residue Phase

Mode 1 | Mode 2 | Mode 3

99% FRF Confidence Bounds - Young's Modulus Variation

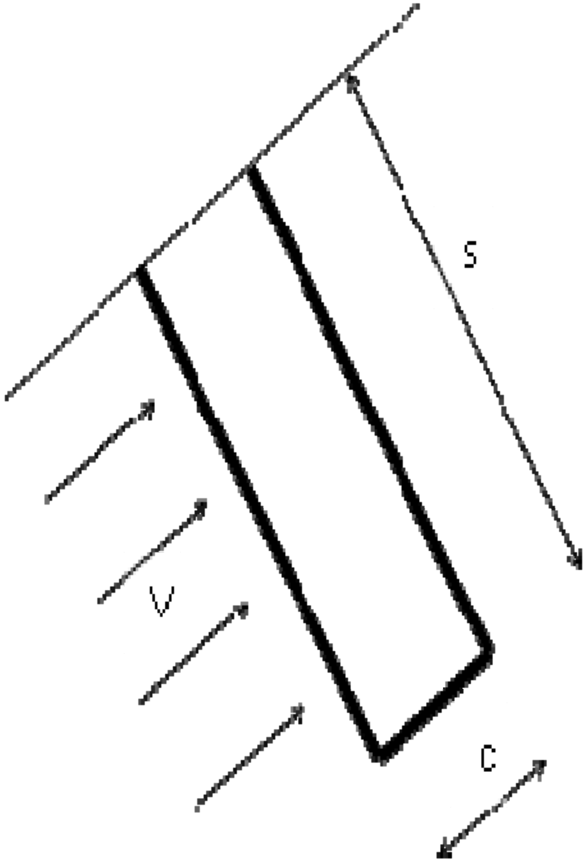


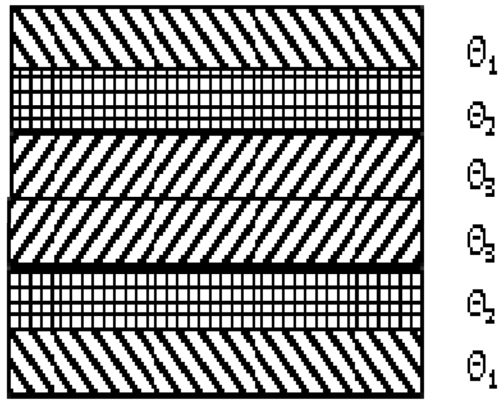
99% FRF Confidence Bounds - Young's Modulus + CSA Variation



Example 2.
Simple Rectangular Composite Wing - Flutter Speed

Wing Model





Composite Layers

Aeroelastic Modelling

- Structure
  - Assumed modes model
- Aerodynamics
  - Modified strip theory including unsteady terms
- Combine using Lagrange

$$\mathbf{A}\ddot{\underline{\mathbf{q}}} + (\rho V \mathbf{B} + \mathbf{D})\dot{\underline{\mathbf{q}}} + (\rho V^2 \mathbf{C} + \mathbf{E})\underline{\mathbf{q}} = \underline{\mathbf{0}}$$

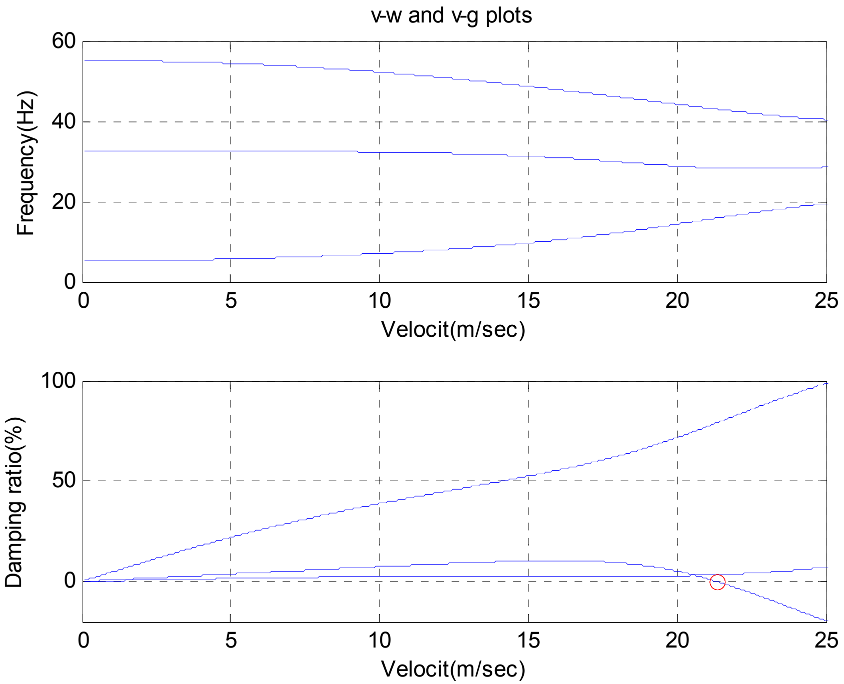
⇓ Mass Matrix

⇓ Overall Damping Matrix

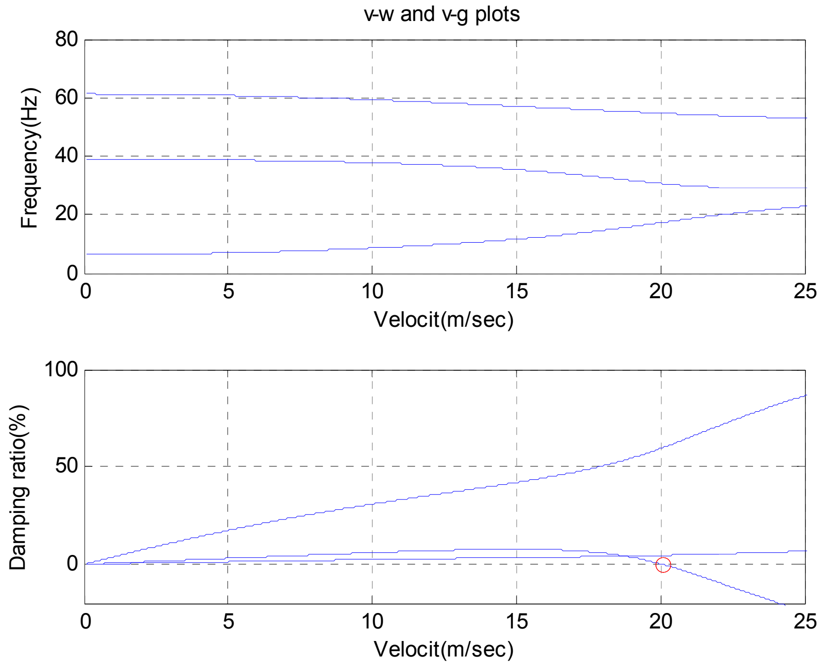
⇓ Overall Stiffness Matrix

Frequency and Damping Trends

- Aeroelastic Modelling of composite wing



V-g and V-ω plot for [-45,-45,0]s laminate (*xf =0.5c*)



V-g and V-ωplot for [-30,-40,0]s laminate (*xf =0.5c*)

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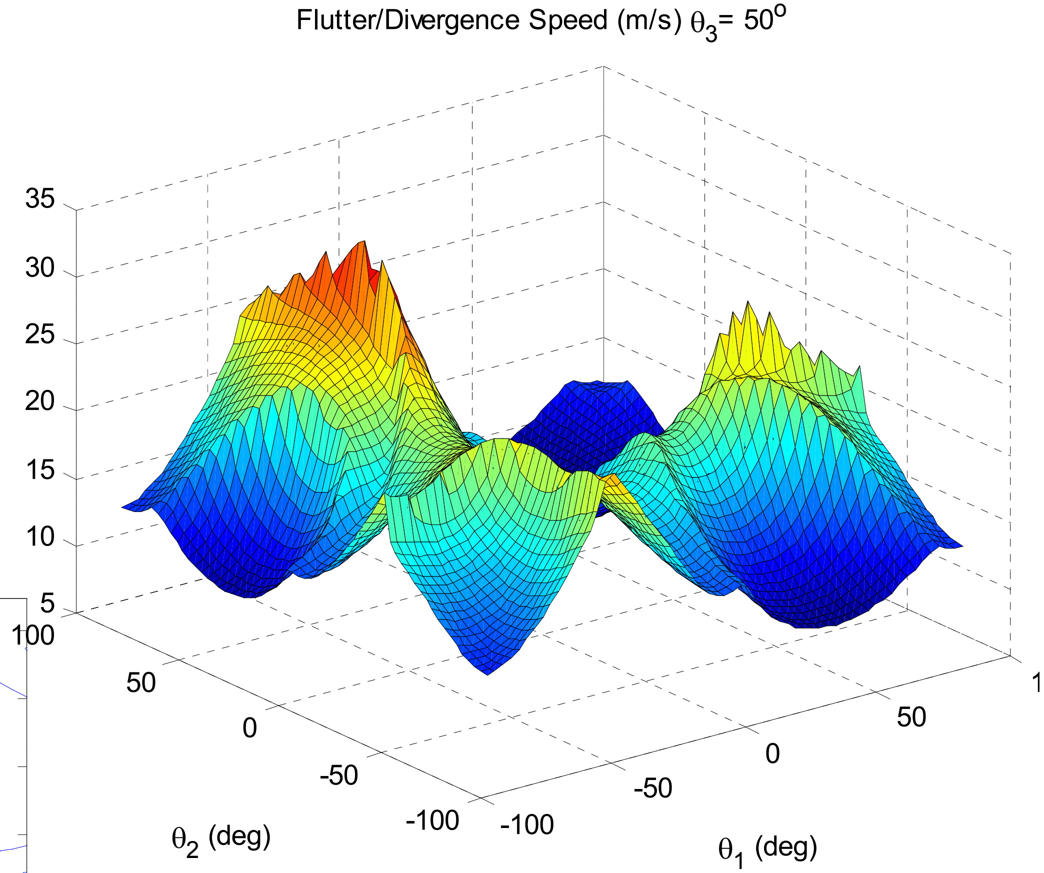
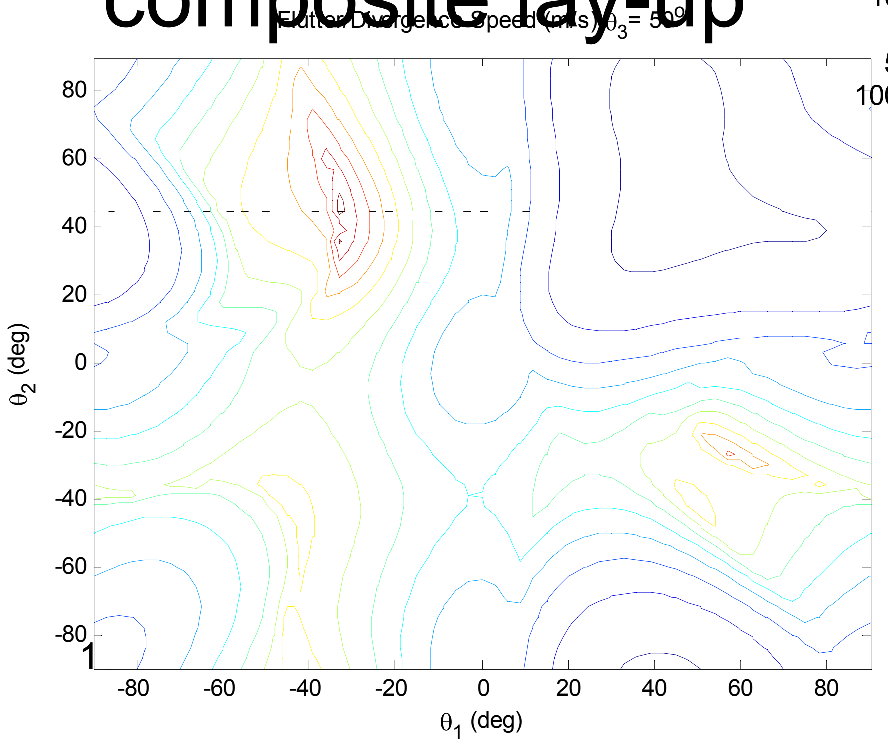
Deterministic Optimization Strategy

- Objective
  - Maximize Speed at which flutter and divergence occur
- Variables
  - Fibre Angle Orientation
- Studied Example
  - Composite Wing is selected
  - Aspect Ratio=4
  - Number of Layers =6
  - Fibre Angle → $(\theta_1, \theta_2, \theta_3)_s$

Composite Wing Orientation Example

- Composite rectangular wing (3 types of layer)
- Determine best composite lay-up





Manan A, Vio GA, Harmin YF & Cooper JE "Optimization of Aeroelastic Composite Structures using Evolutionary Algorithms" Engineering Optimisation.v42 n2 2010 pp 171 – 184.

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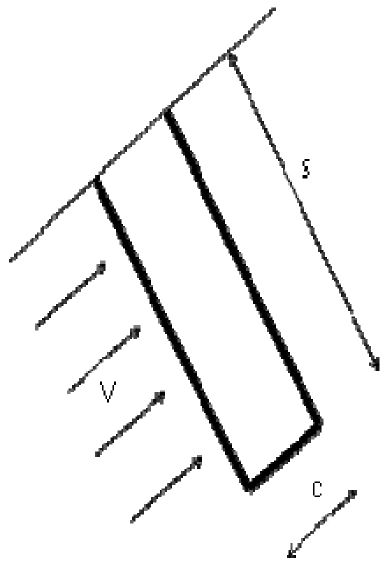
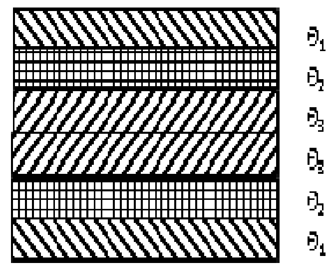
Uncertainty of Flutter Speed

- Same wing as previous optimisation study
- 1D
  - Longitudinal Young's Modulus
- 2D
  - θ_1 and θ_2
- 3D
  - Longitudinal Modulus
  - Shear modulus
  - Thickness

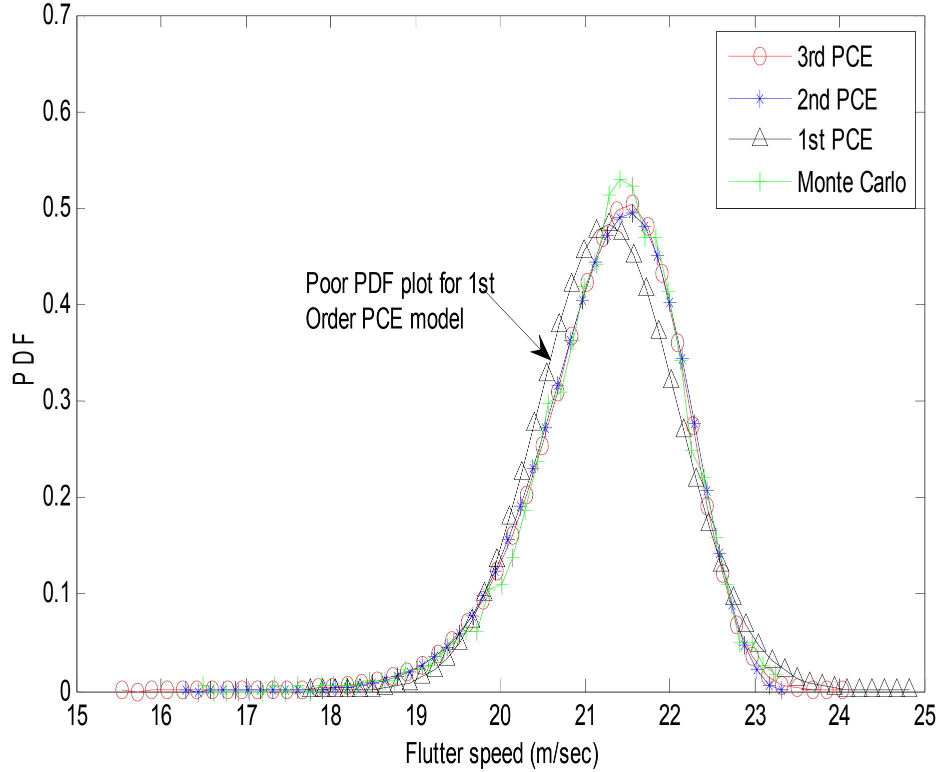




Composite Layers

Probabilistic Aeroelastic Model

- 1-D Polynomial Chaos
  - Longitudinal Young's modulus(E_1),
  - coef of variation = 0.2
  - 10 samples to get β terms
  - PDF then generated from PCE
- Compared with Monte Carlo =1288 simulations



Deterministic Flutter Speed m/s	2^{nd} Order PCE μ m/s	3rd Order PCE μ m/s	Monte Carlo μ m/s	2^{nd} Order PCE σ	3^{rd} Order PCE σ	Monte Carlo σ
21.352	21.2697	21.2700	21.2876	0.8306	0.8441	0.8522

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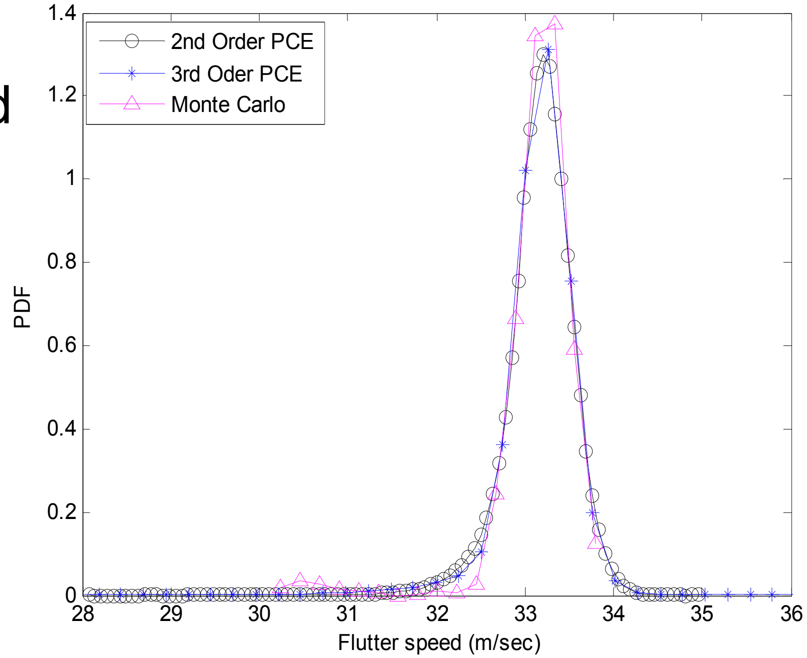
Probabilistic Aeroelastic Model

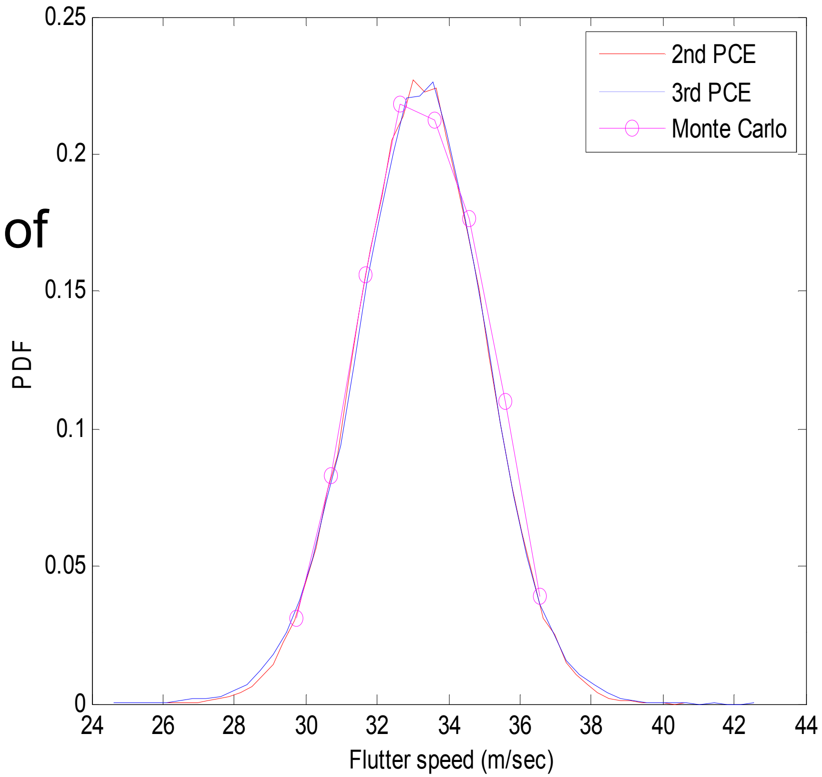
- 2-D Polynomial Chaos
  - Coefficient of variation of θ_1 and θ_2 is 0.01(~ 2 degree variation)
  - $(-33.75, 28.125, 90)_s$ lay-up
  - 30 samples are taken.
  - MCS = 2500 simulations



Deterministic Flutter Speed m/s	2nd Order PCE μ m/s	3rd Order PCE μ m/s	Monte Carlo μ m/s	2nd Order PCE σ	3rd Order PCE σ	Monte Carlo σ
33.298	33.1708	33.1420	33.1448	0.3746	0.4701	0.4787

Probabilistic Aeroelastic Model

- 3-D Polynomial Chaos
  - Longitudinal Young's modulus, in-plane shear modulus and total thickness of the laminate are treated as random variables
  - For moduli cov=0.1 and for thickness cov=0.02
  - 60 sample solutions
  - MCS = 900

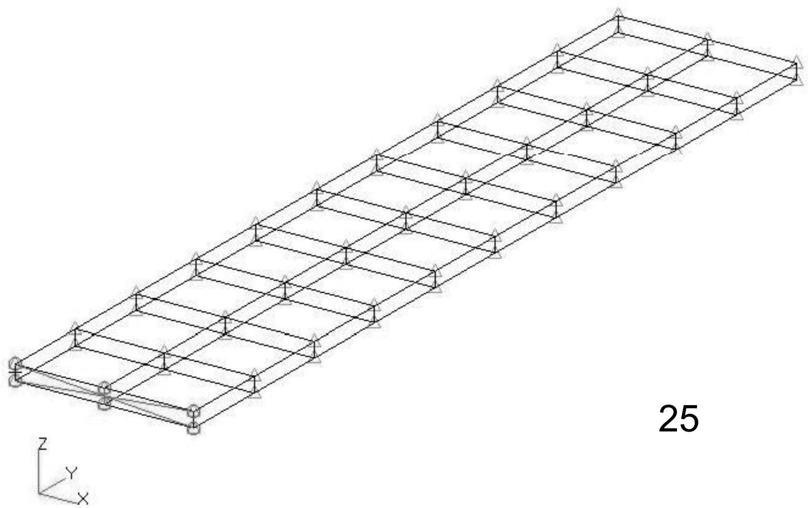


Deterministic Flutter Speed m/s	2nd Order PCE μ m/s	3rd Order PCE μ m/s	Monte Carlo μ m/s	2nd Order PCE σ	3rd Order PCE σ	Monte Carlo σ
33.298	33.2260	33.2277	33.2355	1.7268	1.7487	1.6476

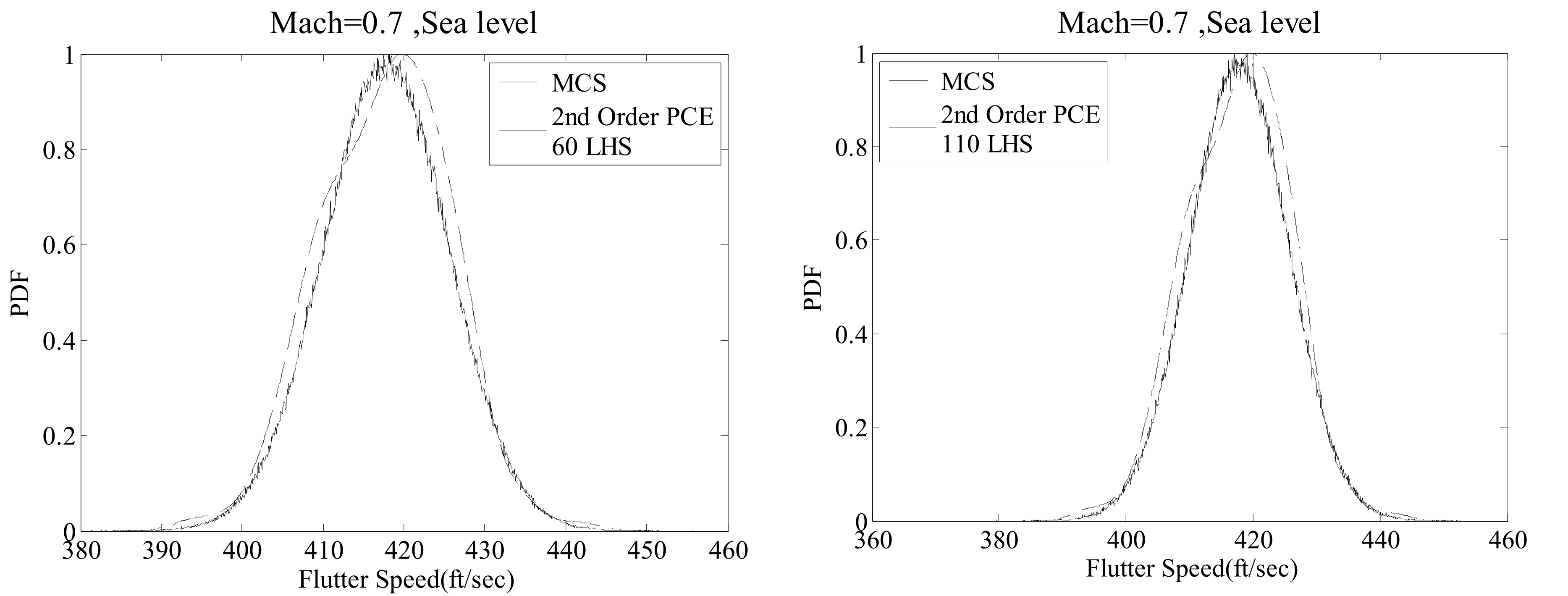
Example 3.
Goland Wing - Flutter Speed

Goland Wing Example

- **Uncertain variables**
- Upper Wing skin=[0.017825-0.013175] ,Mean= 0.0155
- Lower Wing Skin=[0.017825-0.013175] ,Mean= 0.0155
- Leading Edge Spar =[0.00069-0.00051] ,Mean =0.0006
- Trailing Edge Spar =[0.00069-0.00051] ,Mean =0.0006
- Leading Edge Spar Caps=[0.04784-0.03536], Mean =0.0416
- Trailing Edge Spar Caps=[0.17204-0.12716], Mean =0.1496
- Centre Spar Cap=[0.04784-0.03536], Mean =0.0416



Goland Wing 7D PCE Model



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Robust Aeroelastic Design using PCE Models

Robust Design

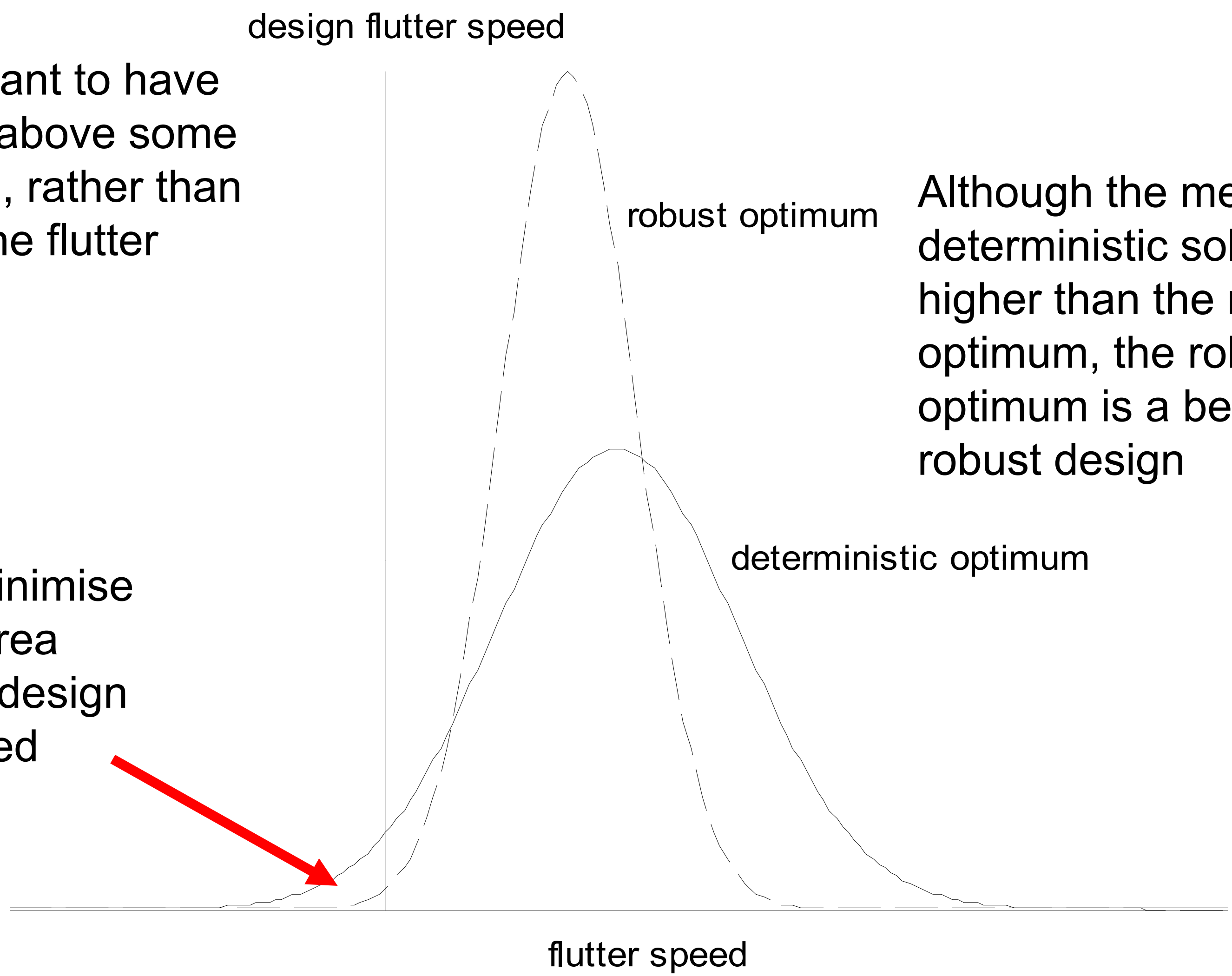
- Deterministic approach
  - Maximise some function
- Robust approach
  - Consider position of PDF compared to some design objective

Manan.A & Cooper J.E. "Design of Composite Wings Including Uncertanties – A Probabilistic Approach" J.Aircraft. v46n2 2009 pp601-607

Robust Design for Flutter

In practice, want to have flutter speed above some design speed, rather than maximising the flutter speed

Want to minimise the PDF area below the design flutter speed



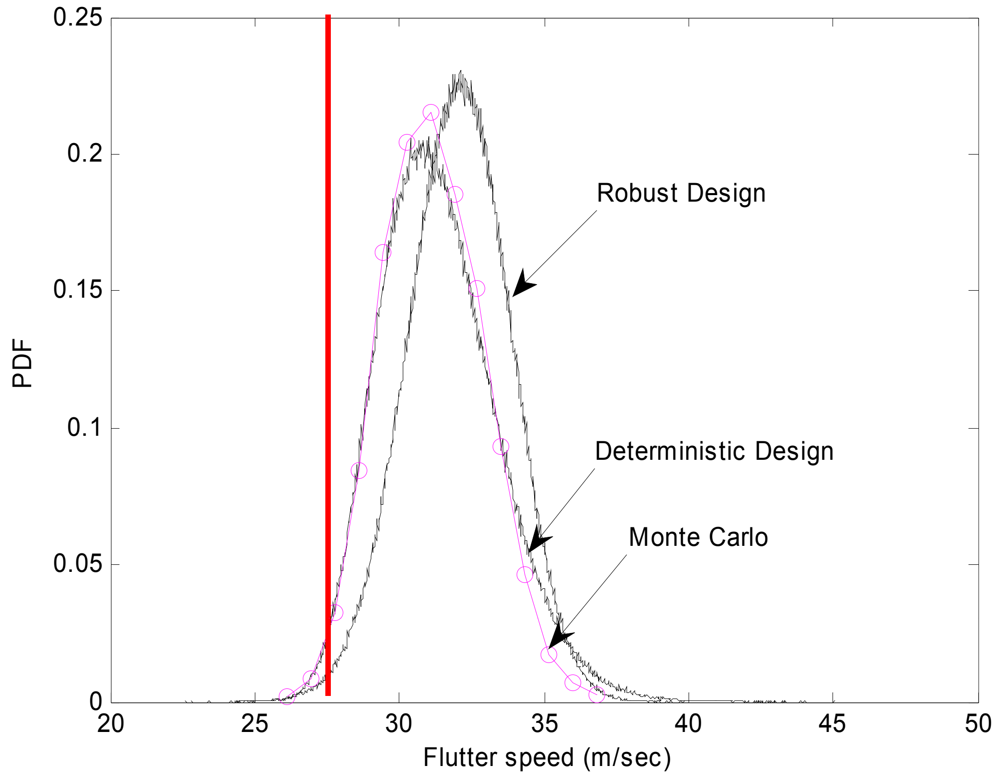
Although the mean of the deterministic solution is higher than the robust optimum, the robust optimum is a better robust design

Robust Optimisation using PSO

- **Same wing but E_1,G_{12} and total thickness are random variables**
  - 8 particles in swarm are selected
  - For each particle a PDF is generated from which area below Design Flutter Speed is calculated
  - This means in each loop 8 PDFs are assessed
  - 0.00 probability value is flutter free and 1.00 is total failure.

Robustness of Composite Wing

Deterministic Optimisation Flutter Speed	32.90 m/sec
Mean of Monte Carlo Simulations Applied to Deterministic Optimum	31.15 m/sec
Mean of PCE Applied to Deterministic Optimum	31.17 m/sec
Mean of PCE Applied to Robust Optimum	32.10 m/sec
Deterministic Robust Optimisation Flutter Speed	32.24 m/sec



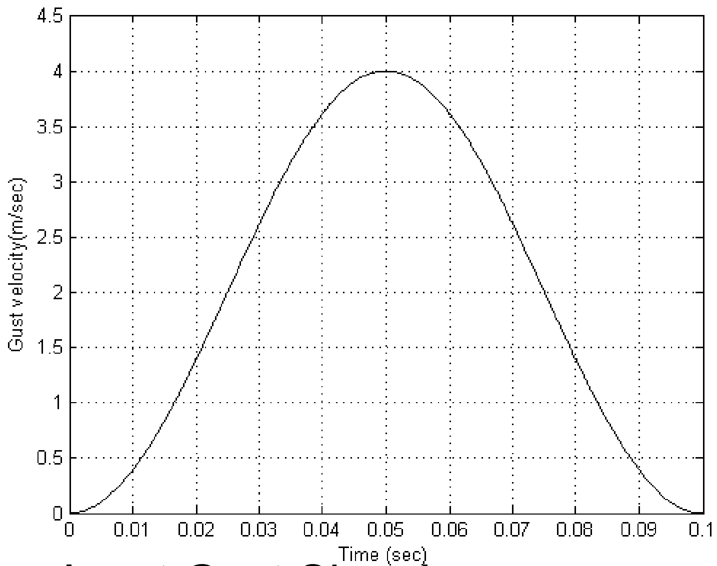
Note skewed behaviour of deterministic design

Design Flutter Speed	Deterministic Optimum	Robust Optimum
28 m/sec	0.0292	0.0133
32 m/sec	0.6533	0.4845

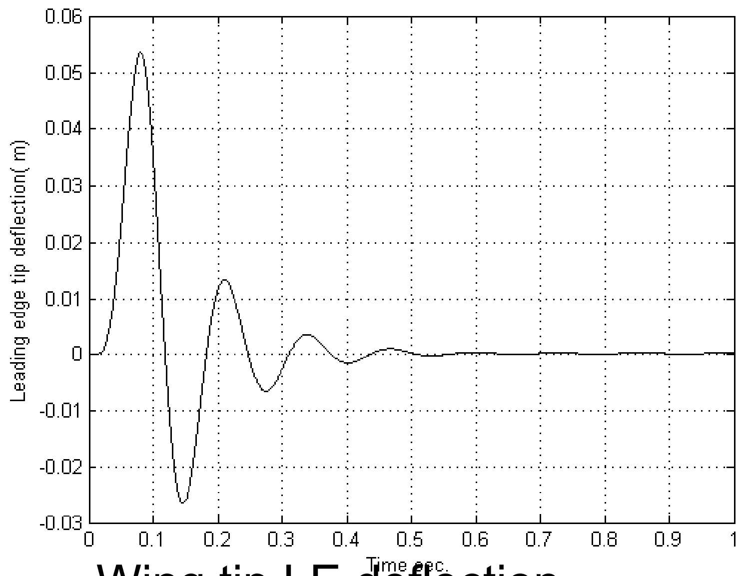
Table for Probability of Failure

1-Cosine Gust Modelling and Design

- "1-cosine" gust excitation applied to composite wing
- 20 Particles, 100 runs conducted to minimise root bending moment
- Optimum layup [14.858,14.858,-74.543]s.
- Passive design using wash-out at tip



Input Gust Signal

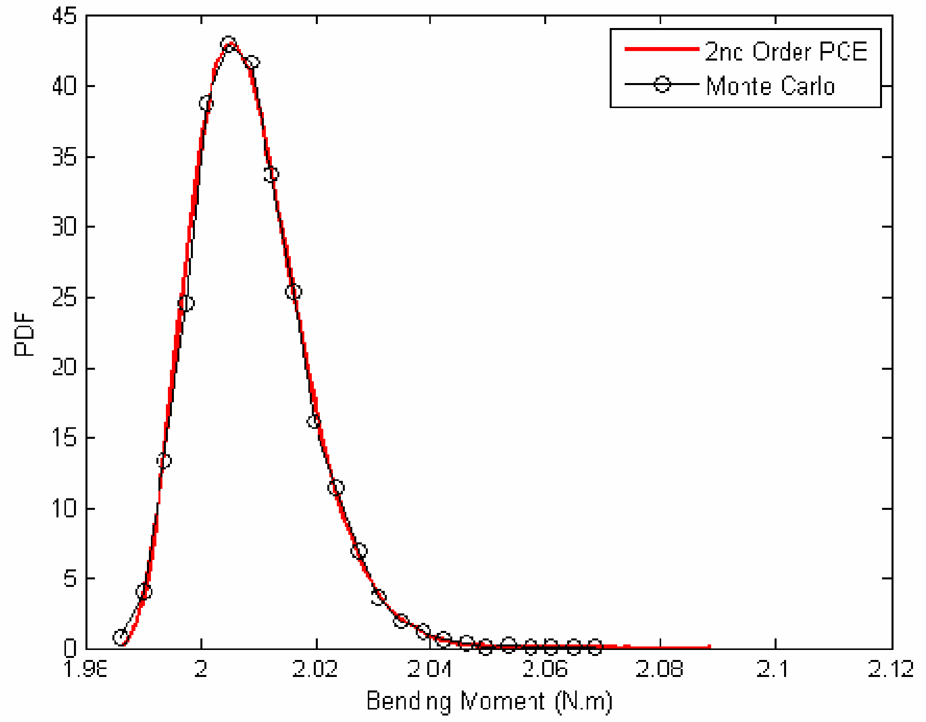


Wing tip LE deflection

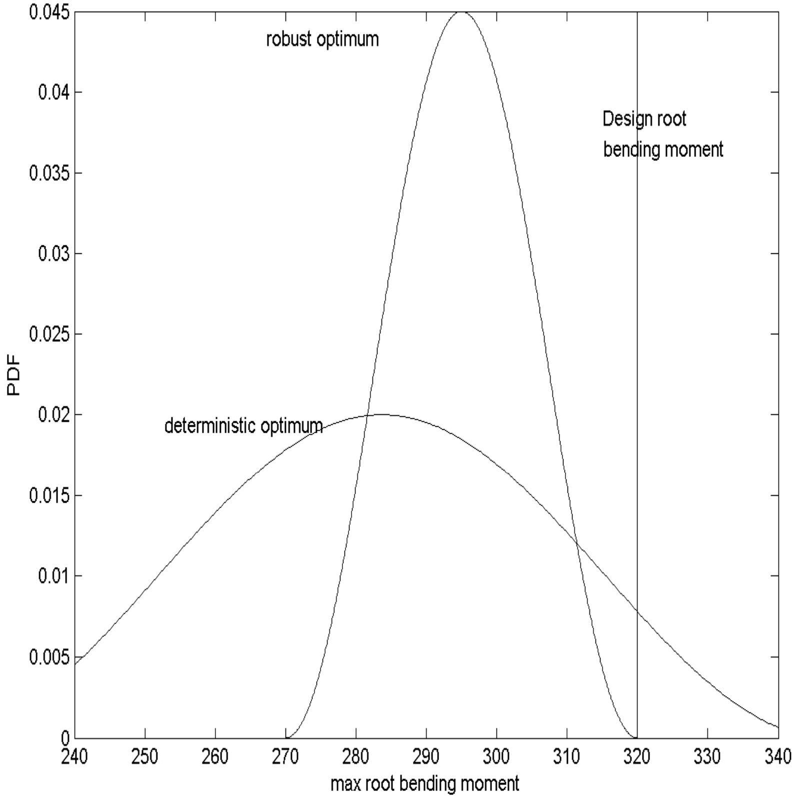
1-D Chaos Model for Gust Design

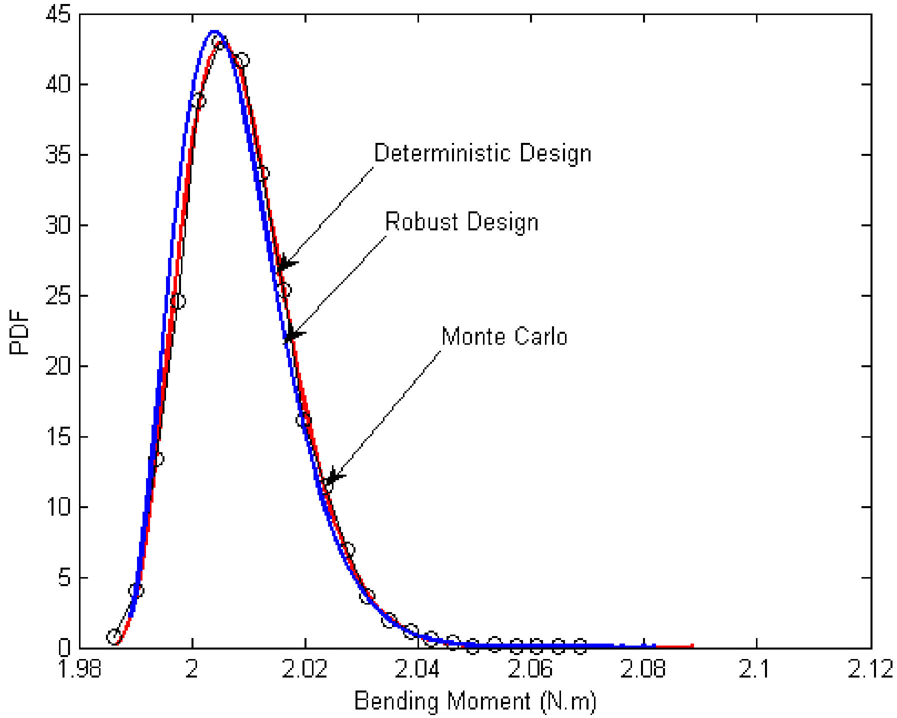
•The longitudinal Young's modulus with coefficient of variation of 0.2 was taken. A 2nd Order PCE model was derived and PDF plot was generated.

•10,000 Monte Carlo simulations are conducted and excellent agreement was observed.



Robust Gust Design





20 particles with 100 runs

Multi-Objective Design

Designs for gust alleviation and improved flutter speeds oppose each

Deterministic

$$\Omega_{det} = \min(w_f * \frac{V_d}{V_{max}} + w_g * \frac{R_{min}}{R_d})$$

Probabilistic

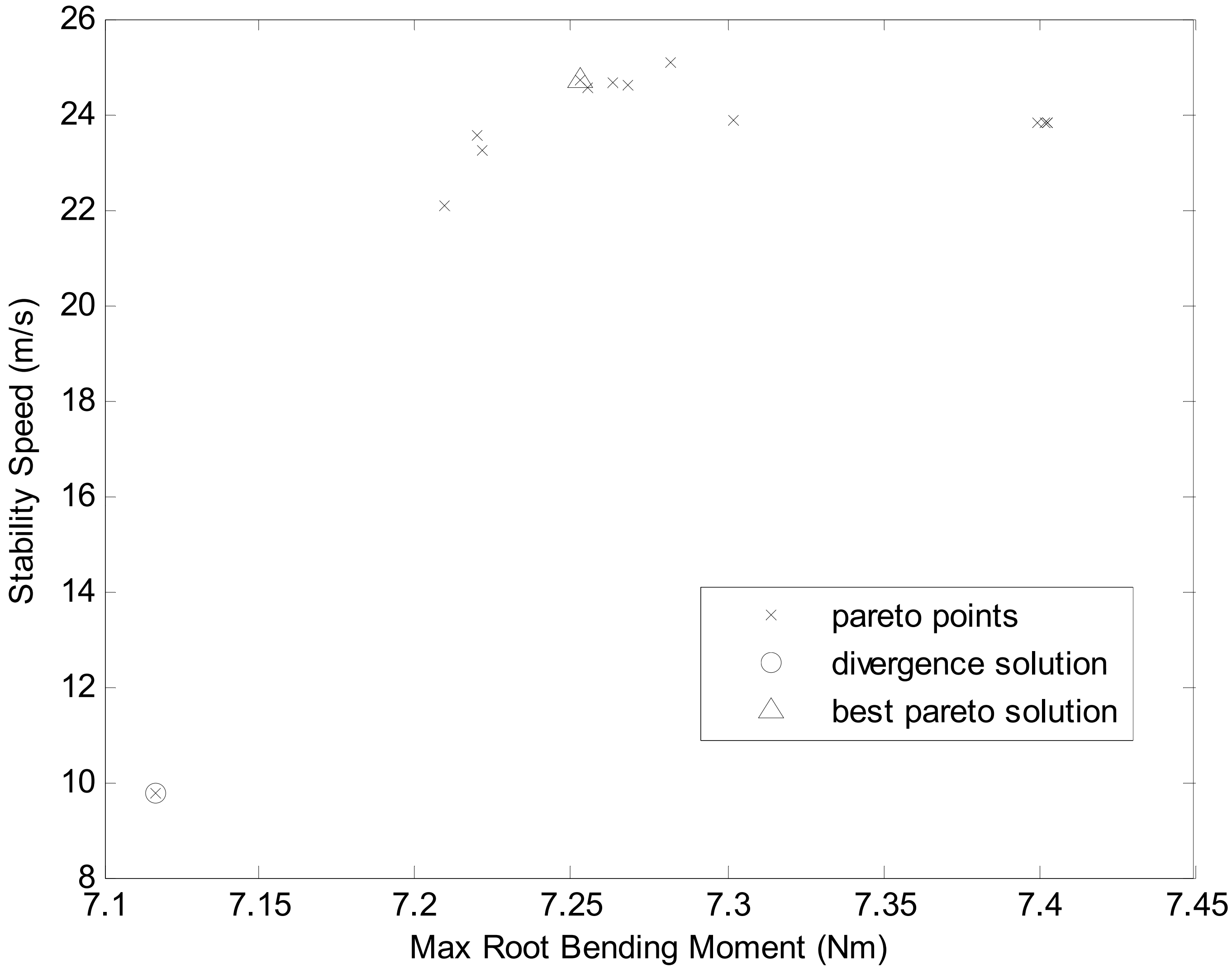
$$\Omega_{robust} = \min(w_f * \alpha + w_g * \beta)$$

α: Probability of flutter failure

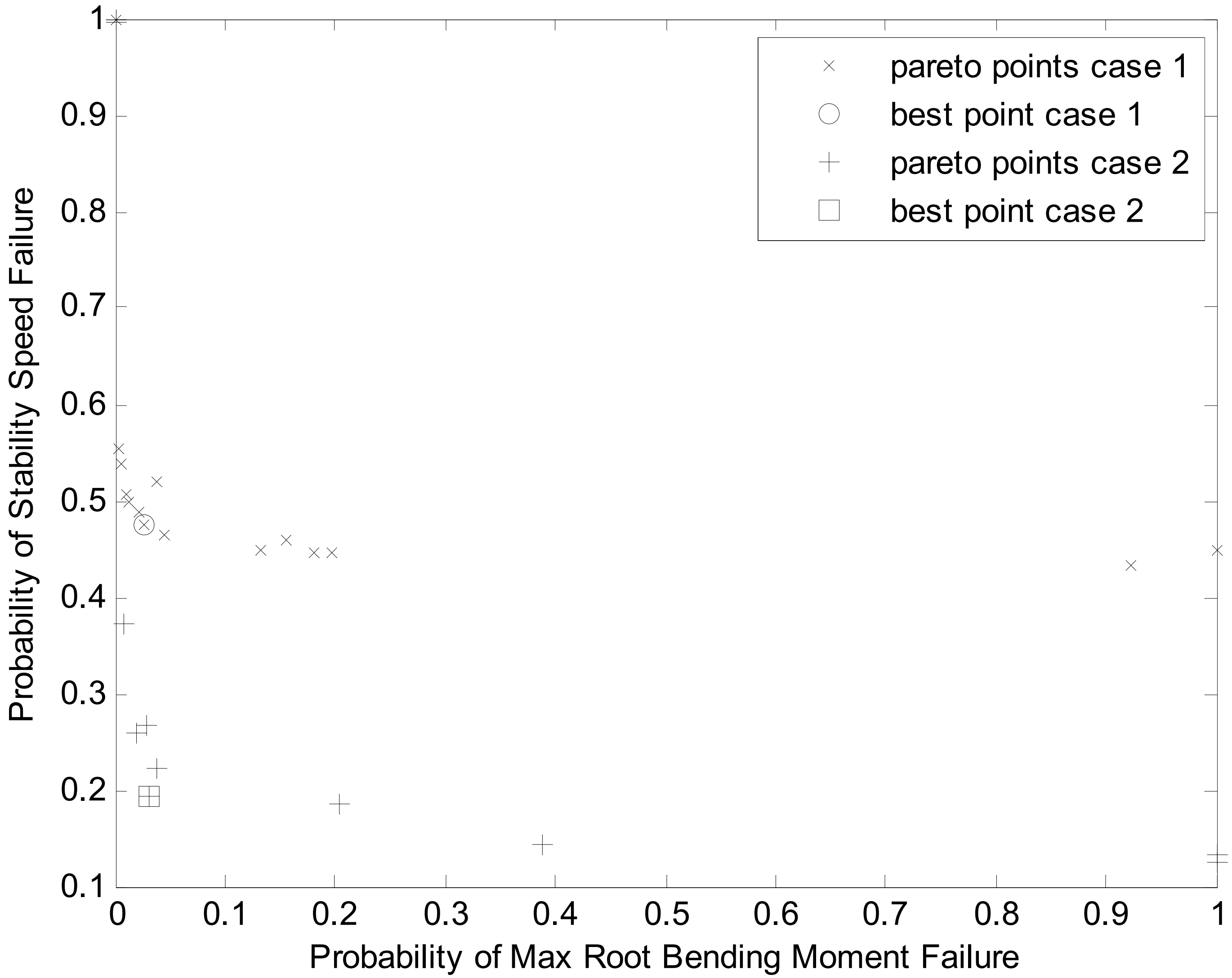
β: Probability of gust failure

A Manan & J E Cooper, " Multi-Objective Aeroelastic Tailoring including Uncertainty" to appear in the Aeronautical Journal

Deterministic Pareto Front



Robust Pareto Front



Conclusions

- Several successful applications of Polynomial Chaos have been shown for.
  - FRF calculations
  - flutter and gust response
- Application to robust design for flutter and gusts
- Development of robust Pareto Frontiers for multi-objective problems

- Further work required on
  - application to multi-parameter systems
  - More realistic structures