LABORATORY

# Outcomes of the ECERTA project

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Hamed Khodaparast, Marco Prandina, John Mottershead

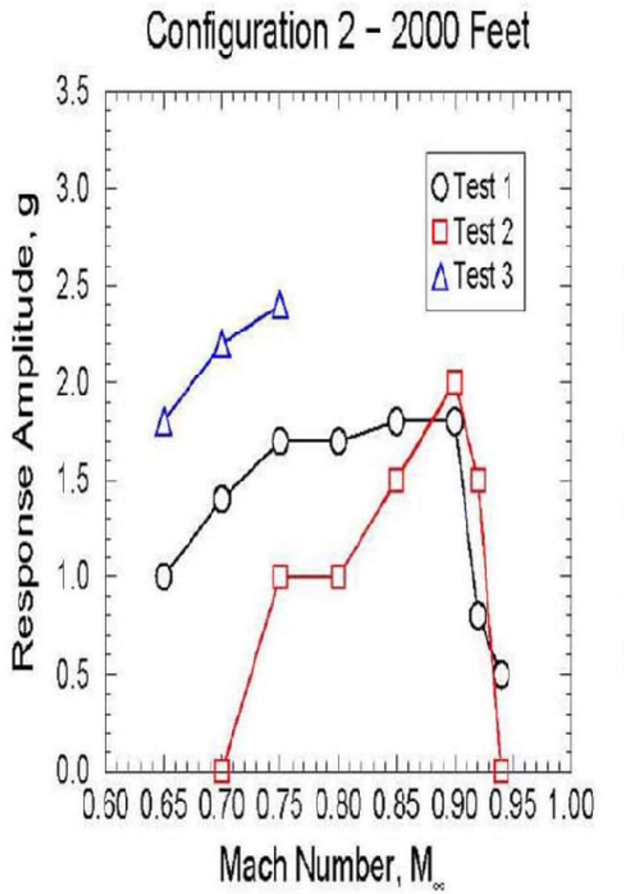
LABORATORY

www.cfd4aircraft.com

Marie Curie Excellence Team

Enabling Certification by Analysis

MARIE CURIE ACTIONS




Thomas, SDM, 2006

# Simulation Requirements

- Physics Based Simulation
  - Nonlinearity
- Envelope Search for Unanticipated Events
  - Computational cost
- Sensitivity and Variability
  - Probabilistic/Possibilistic
- Integration of Available Measurements
  - Updating/Identification

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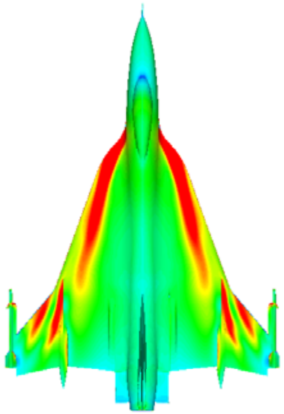
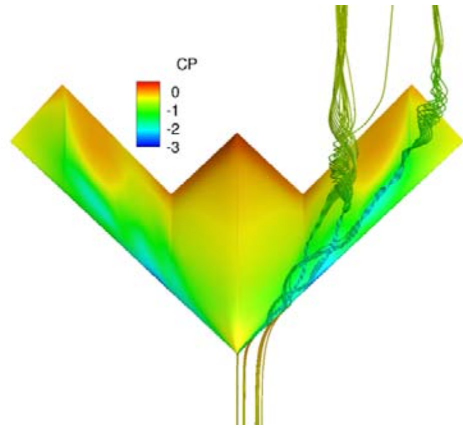
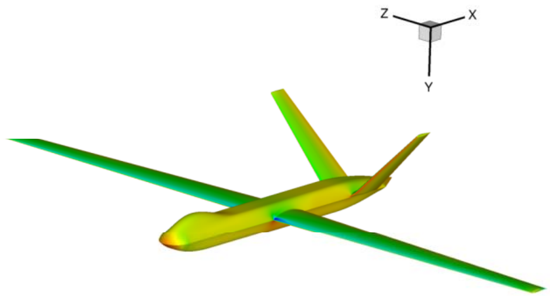
→ **Aeroelasticity and uncertainty is the workshop topic**

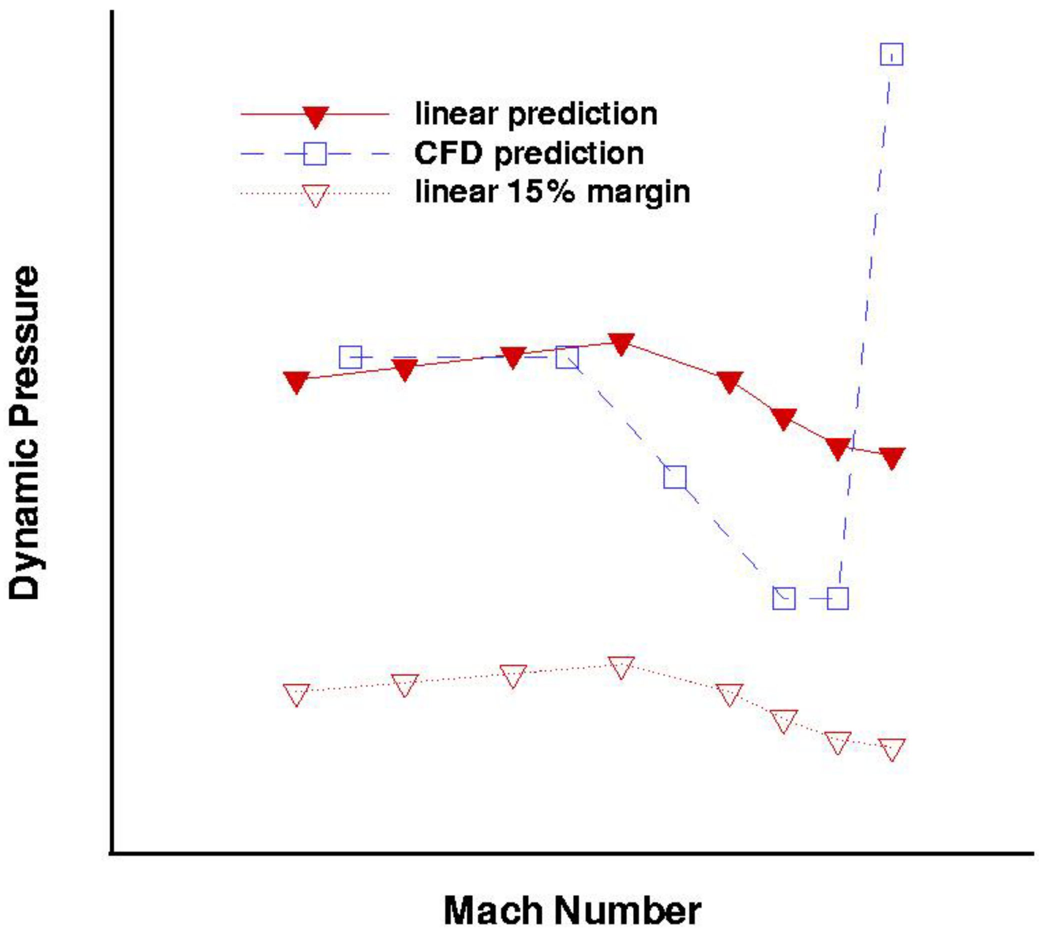


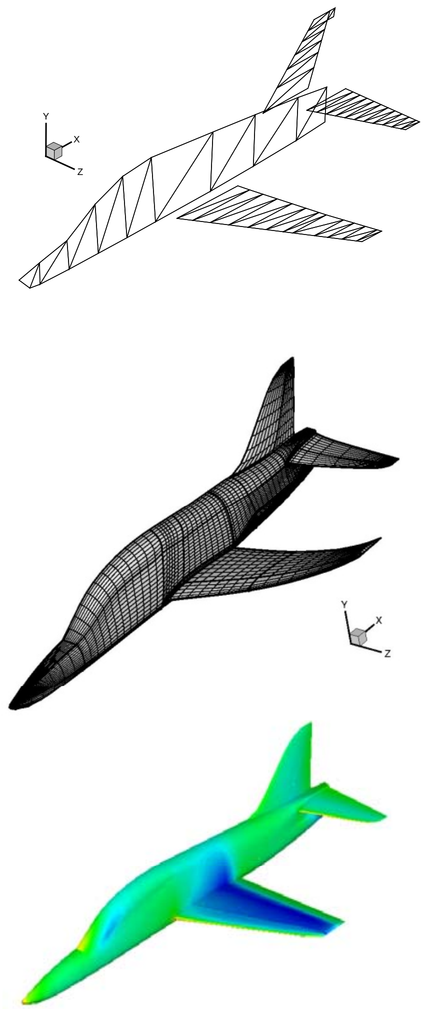



LABORATORY

2008 - 25th Anniversary



Woodgate, M. Badcock, K.J. Rampurawala, A.M. Richards, B.E. Nardini D. and Henshaw M. Aeroelastic calculations for the Hawk aircraft using the Euler equations, **Journal of Aircraft, 42(4),** 2005, 1005-1012.

Denley, C.J., Eccles, T.A., Cross, A.G.T., Practical Unsteady CFD Application to Aircraft Flutter and Limit Cycle Oscillation, **RTO-AVT-152**, Loen, May, 2008

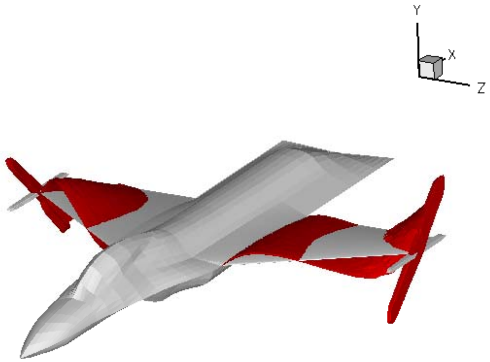
Mares C, Mottershead JE and Friswell MI (2006) Stochastic model updating: Part 1- theory and simulated example. **Mechanical Systems and Signal Processing** vol 20 issue 7 pp 1674-1695

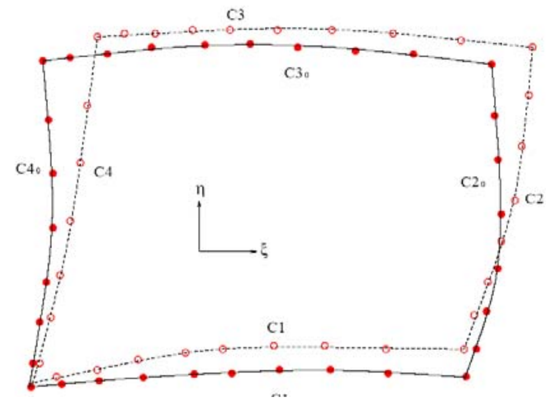
$$\frac{d\mathbf{w}}{dt} = \mathbf{R}(\mathbf{w}, \mu)$$

$$\mathbf{w} = \begin{bmatrix} \mathbf{w}_f \\ \mathbf{w}_s \end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix} \mathbf{R}_f \\ \mathbf{R}_s \end{bmatrix}$$

# Eigenvalue Problem

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \begin{bmatrix} p_f \\ p_s \end{bmatrix} = \lambda \begin{bmatrix} p_f \\ p_s \end{bmatrix}$$



# SCHUR METHOD

- Stability studied from an eigenvalue problem:

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \begin{bmatrix} p_f \\ p_s \end{bmatrix} = \lambda \begin{bmatrix} p_f \\ p_s \end{bmatrix}$$

- Schur Complement formulation:

$$E = S(\lambda) p_s - \lambda p_s = 0$$

$$S(\lambda) = A_{ss} - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}$$

Badcock, K.J. and Woodgate, M.A., Prediction of Bifurcation Onset of Large Order Aeroelastic Models, **AIAA Journal,** 48(6), 2010, 1037-1046

$$E = S(\lambda) p_s - \lambda p_s = 0$$
$$S(\lambda) = A_{ss} - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}$$

- Solved by Newton's Method

$$\frac{\partial \mathbf{E}}{\partial \mathbf{u}} \Delta \mathbf{u} = -\mathbf{E}$$

Approximate Jacobian to drive convergence

Exact or Approximate Residual

$$\mathbf{u} = [\mathbf{p}_s, \lambda]^T$$

$$E = S(\lambda) p_s - \lambda p_s = 0$$

$$S(\lambda) = A_{ss} - A_{sf} (A_{ff} - \lambda I)^{-1} A_{fs}$$

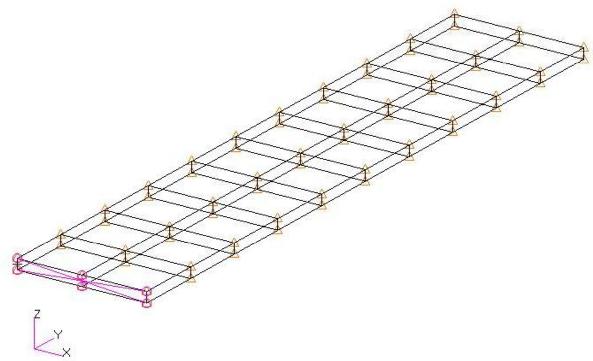
- Solved by Newton's Method

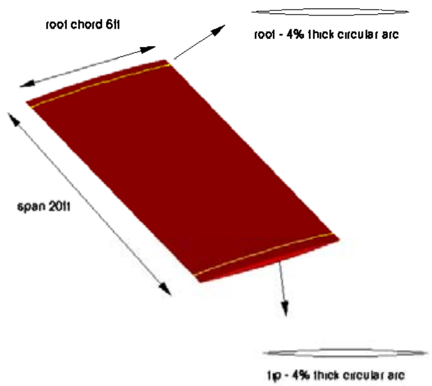
$$\frac{\partial \mathbf{E}}{\partial \mathbf{u}} \Delta \mathbf{u} = -\mathbf{E}$$

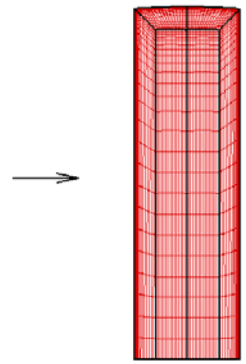
| Approximate Jacobian to drive convergence | Exact or Approximate Residual |
|---|---|

$$(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1} + \ldots$$

# GOLAND WING

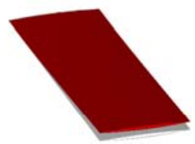




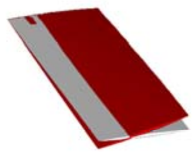



236k points

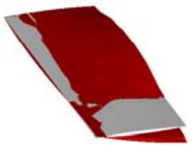
$1x10^6$ DoF



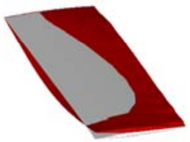
1.72 Hz



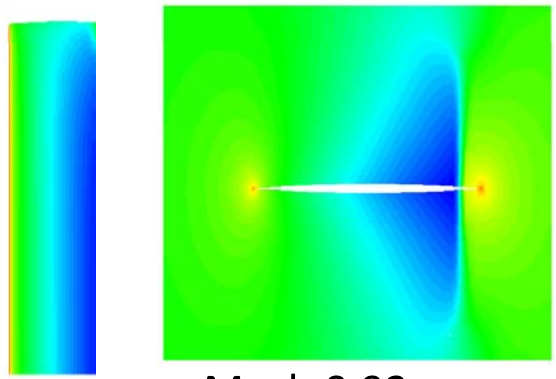
3.05 Hz



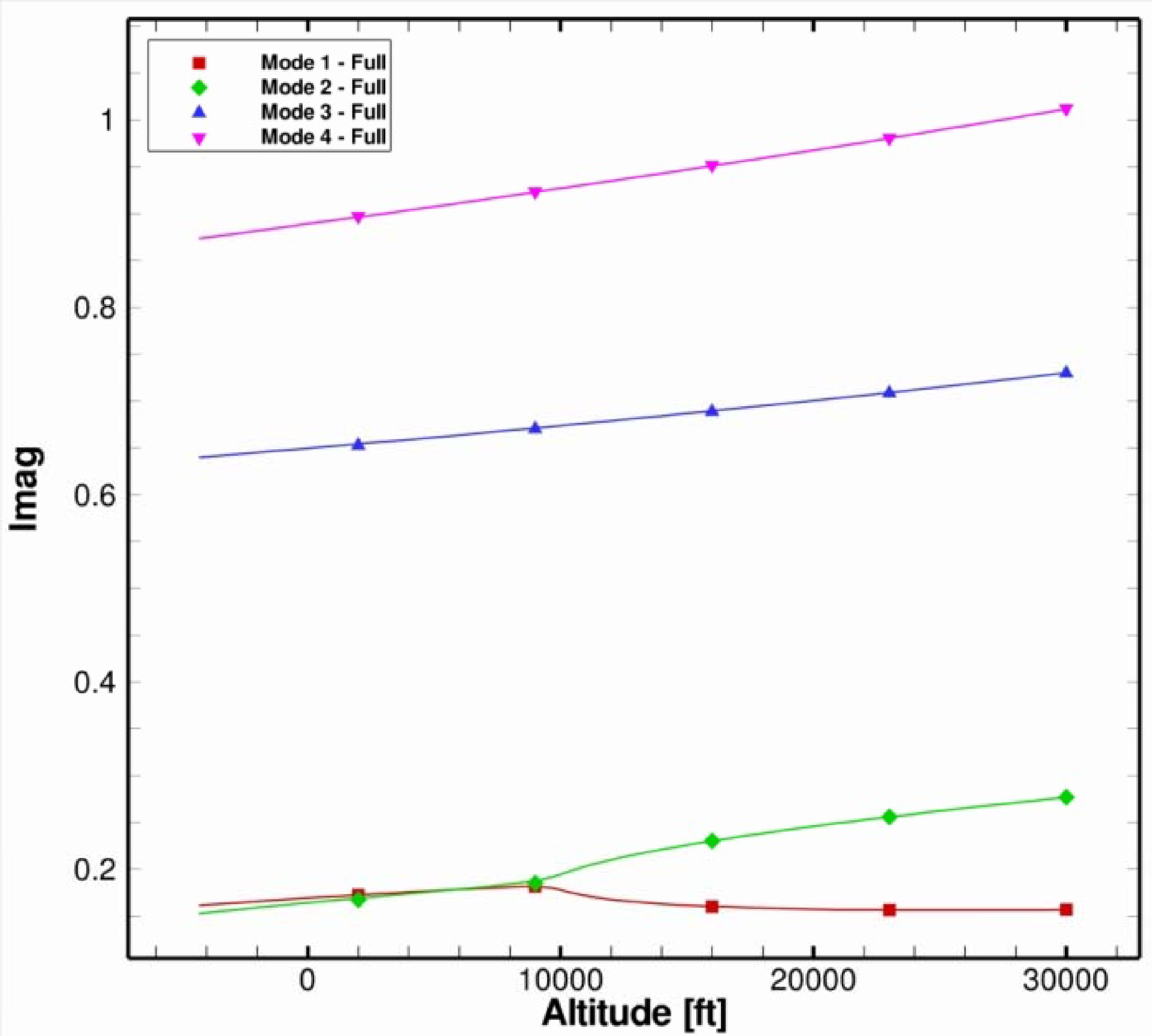
9.18 Hz



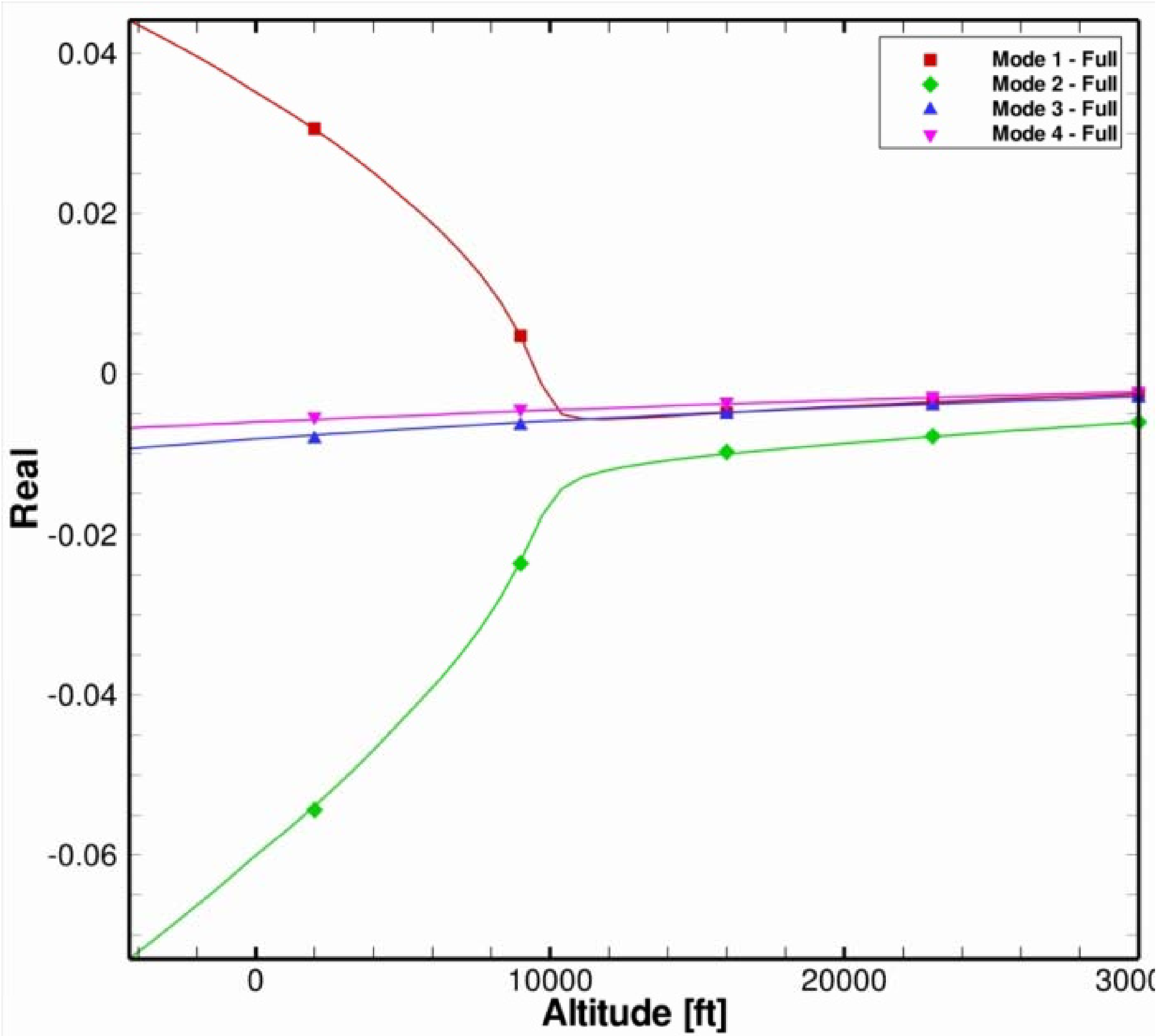
11.10 Hz



Mach 0.92

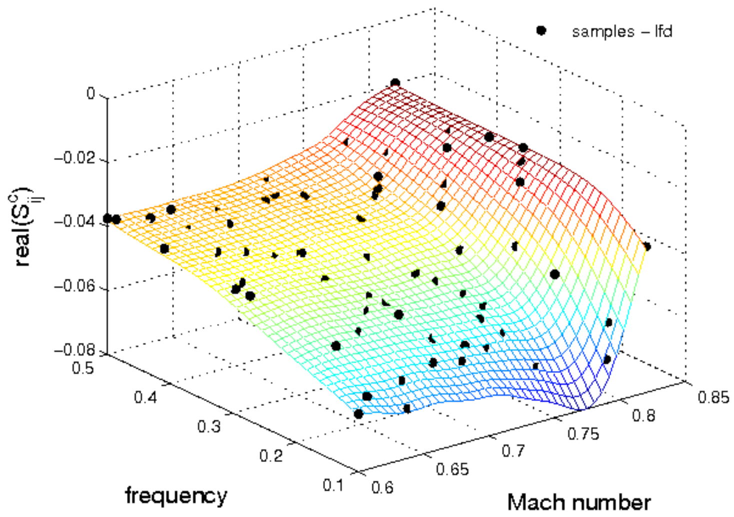


Mode 1 - Full
Mode 2 - Full
Mode 3 - Full
Mode 4 - Full
Imag
1
0.8
0.6
0.4
0.2
0
10000
20000
30000
Altitude [ft]



Mode 1 - Full
Mode 2 - Full
Mode 3 - Full
Mode 4 - Full
0.04
0.02
0
-0.02
-0.04
-0.06
Real
0
10000
20000
Altitude [ft]

# Approximation of S(λ)

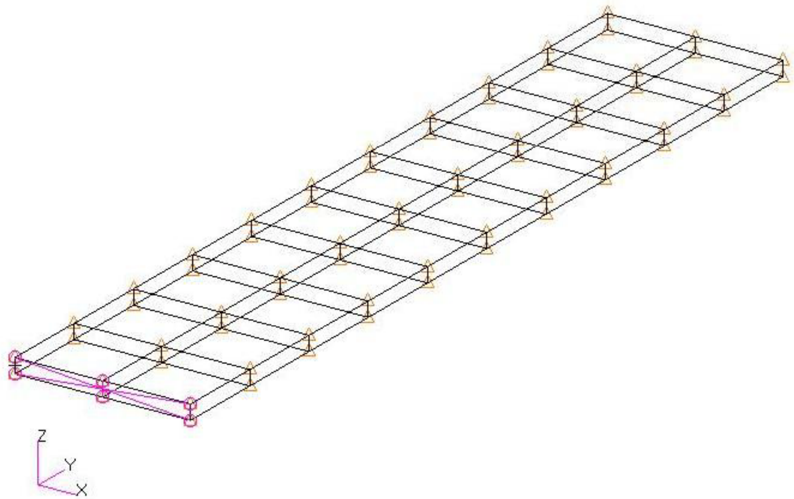


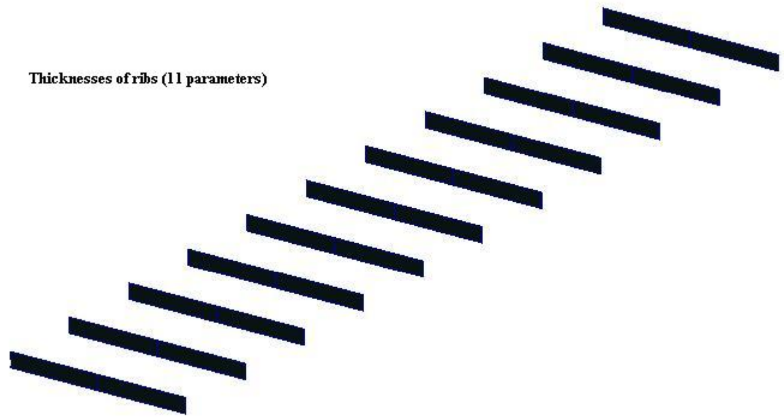

- Kriging Approximation
  - Sampling Methods
  - Update with higher order information

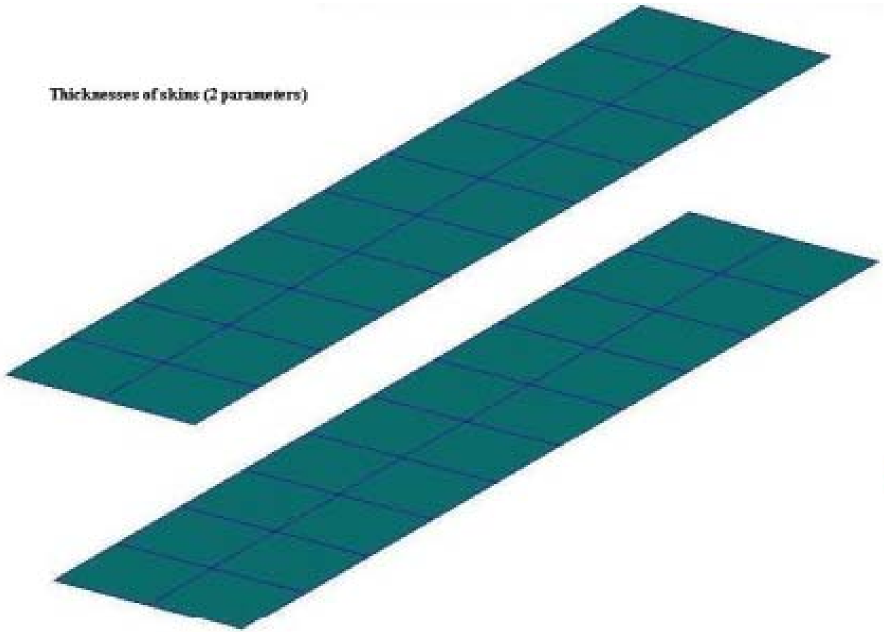
$$A_{sf}\left(A_{ff}-\lambda I\right)^{-1}A_{fs}$$

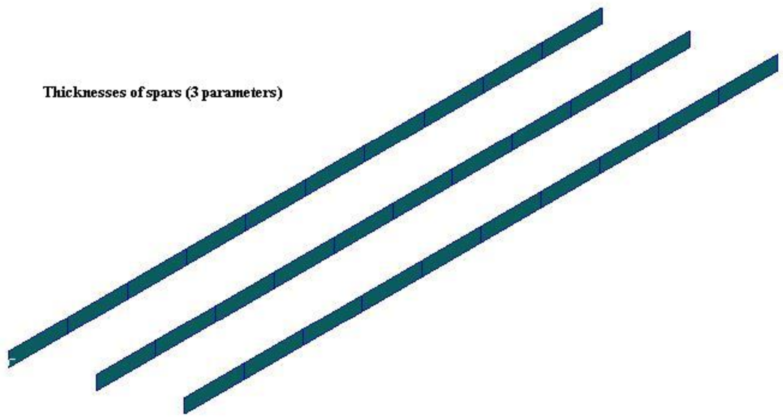
Cost of evaluating one sample ≈ 1 steady state calculation


Timme, S. and Badcock, K.J., Searching for Transonic Aeroelastic Instability Using an Aerodynamic Model Hierarchy, **AIAA-2010-3048.**


Z
Y
X


Thicknesses of ribs (11 parameters)


Thicknesses of skins (2 parameters)


Thicknesses of spars (3 parameters)

# Structural Model Variability

- Vector of structural parameters θ
- θ is uncertain in real engineering structure
  - Lack of knowledge
  - Variability
- **Probabilistic**
  - θ is defined by a PDF
  - What are the eigenvalue PDF's?
- **Possibilistic**
  - θ is defined by an interval
  - What is the worst case eigenvalue?

Haddad Khodaparast H, Mottershead JE and Badcock KJ (2010) Propagation of structural uncertainty to linear aeroelastic stability**. Computers and Structures** vol 88 issue 3-4 pp 223-236

$$E = S(\lambda) p_s - \lambda p_s = 0$$

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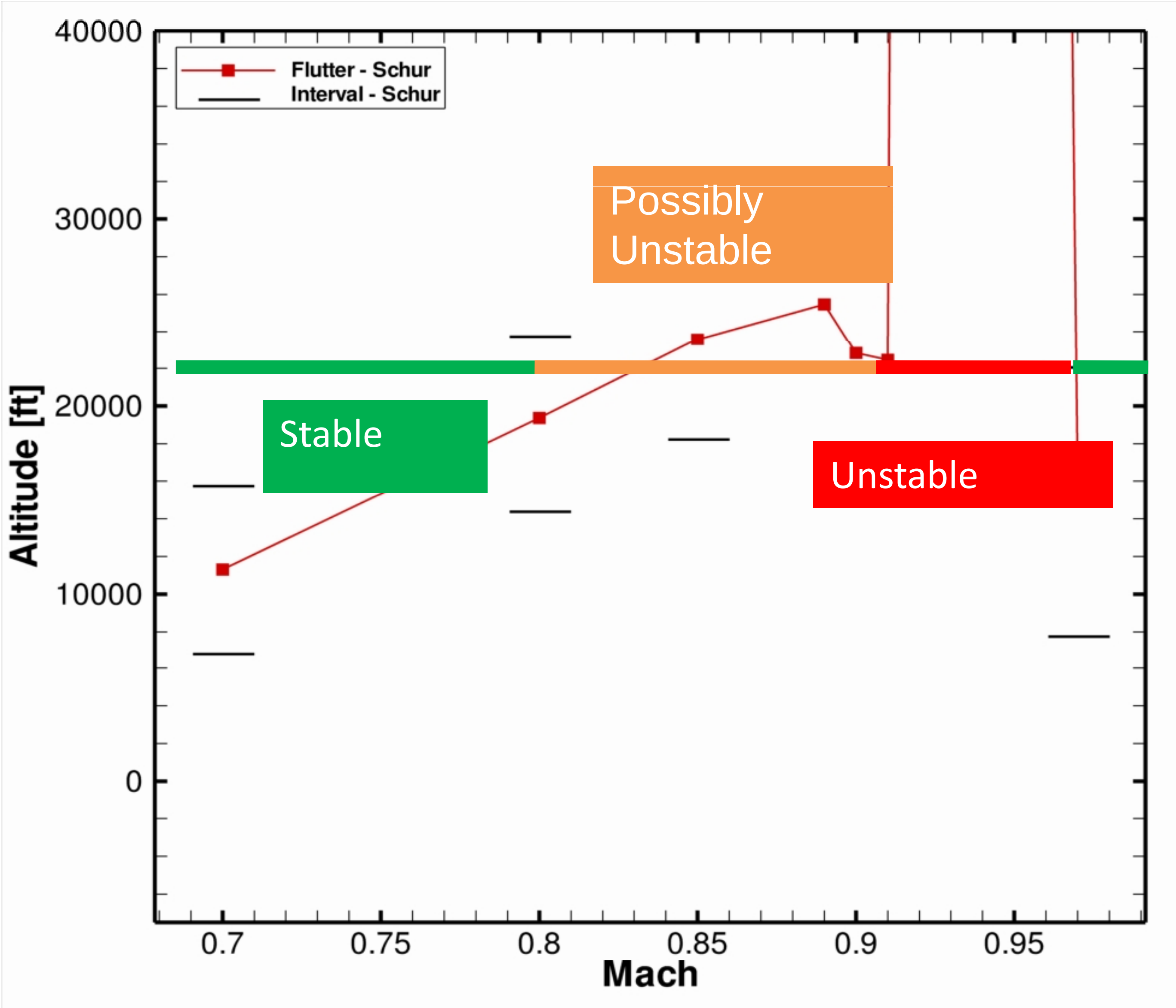
- Solved by Newton's Method

$$\frac{\partial \mathbf{E}}{\partial \mathbf{u}} \Delta \mathbf{u} = -\mathbf{E}$$

Nominal state Jacobian to drive convergence

Exact or Approximate Residual for realisation

Marques, S., Badcock, K.J., Khodaparast, H.H. and Mottershead, J.E., Transonic Aeroelastic Stability Predictions Under the Influence of Structural Variability, **Journal of Aircraft**, 47(4), 2010, 1229-1239

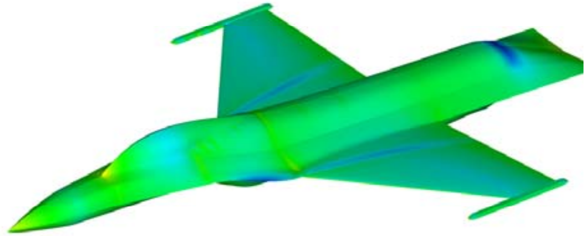


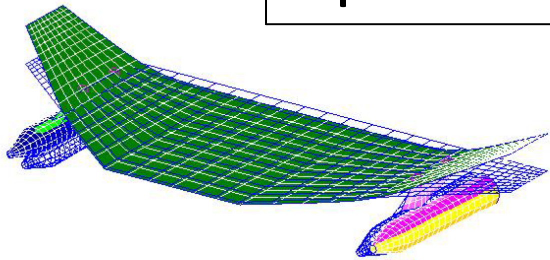
Flutter - Schur
Interval - Schur
Possibly Unstable
Stable
Unstable
Altitude [ft]
Mach
40000
30000
20000
10000
0
0.7
0.75
0.8
0.85
0.9
0.95

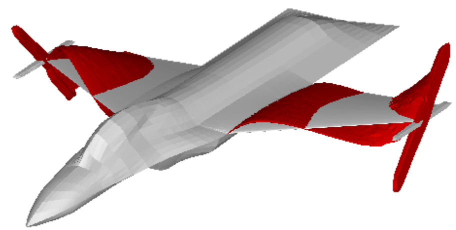
# Overview

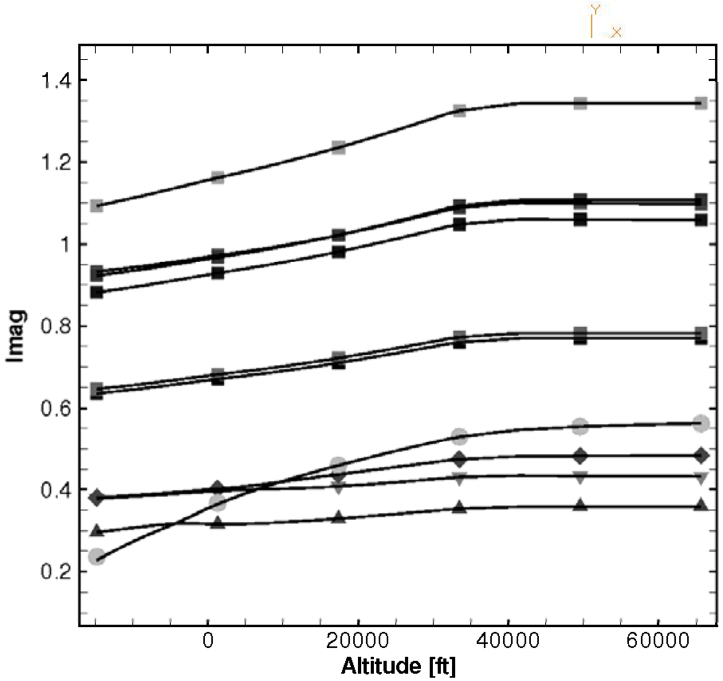
- **Reformulation of eigenvalue problem**
  - Reduces computational task to approximating a function → exploit Kriging and sampling (Timme)
- **Exploit Interval methods**
  - Propagate structural parameter uncertainties to flutter solution (Khodaparast)
- **Approximate Newton solver for Eigenvalue problem**
  - Allows non-deterministic calculations for CFD based flutter predictions (Marques)
- **Too Difficult to model structural damping**
  - New identification methods from tests (Prandina)

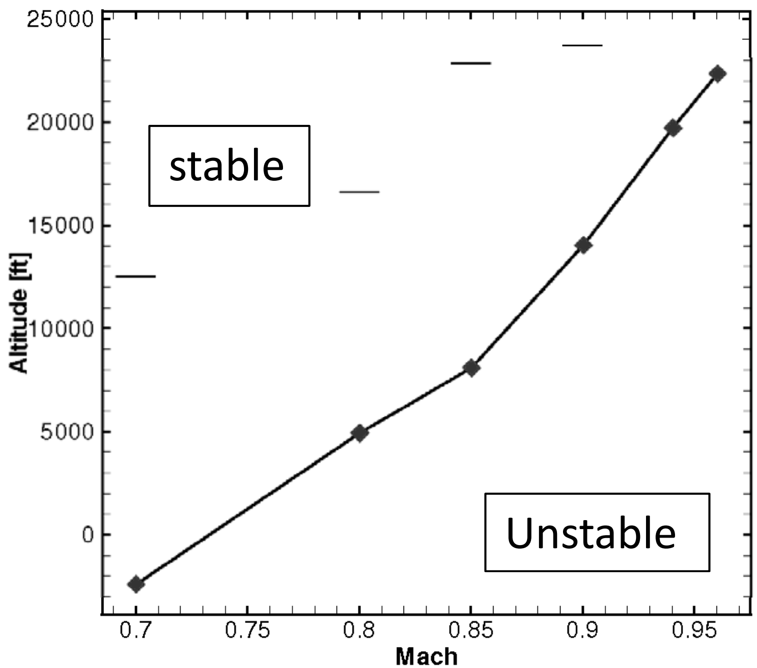
# Open Source Fighter



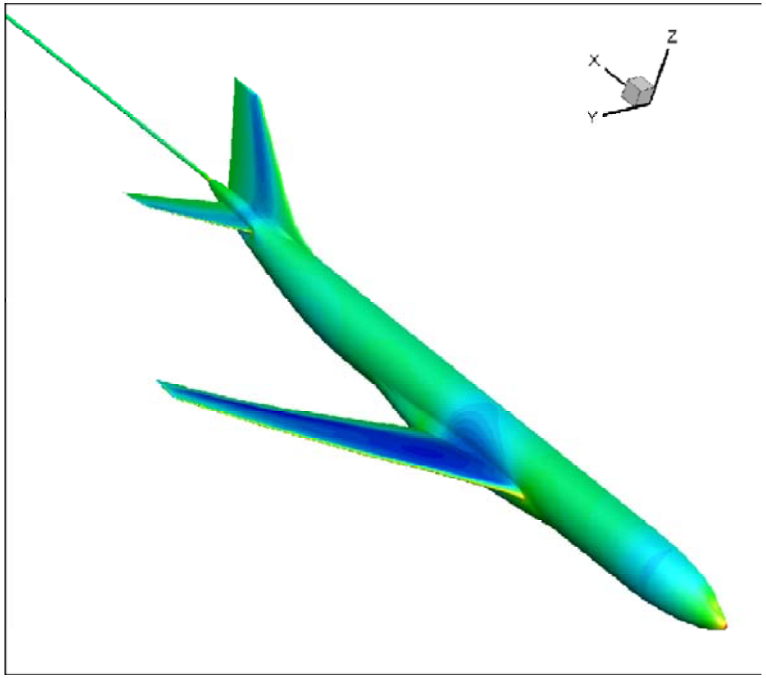


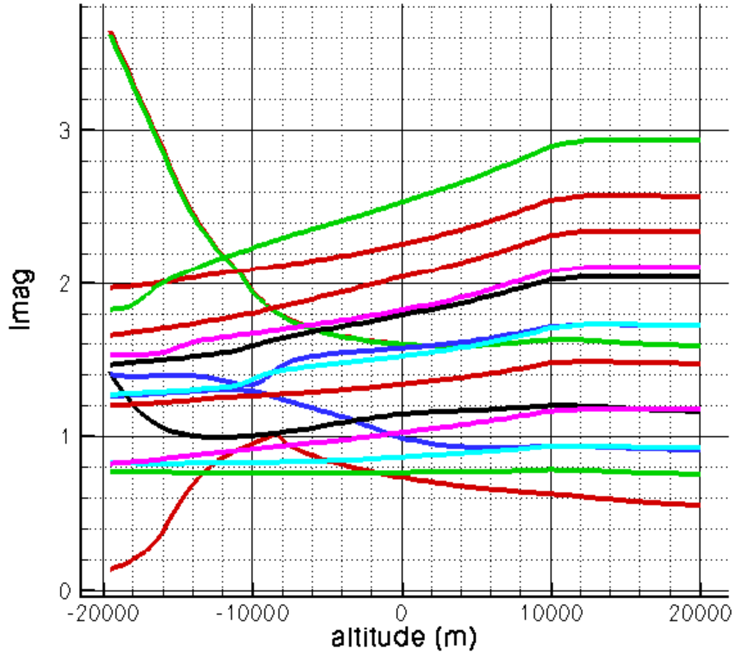


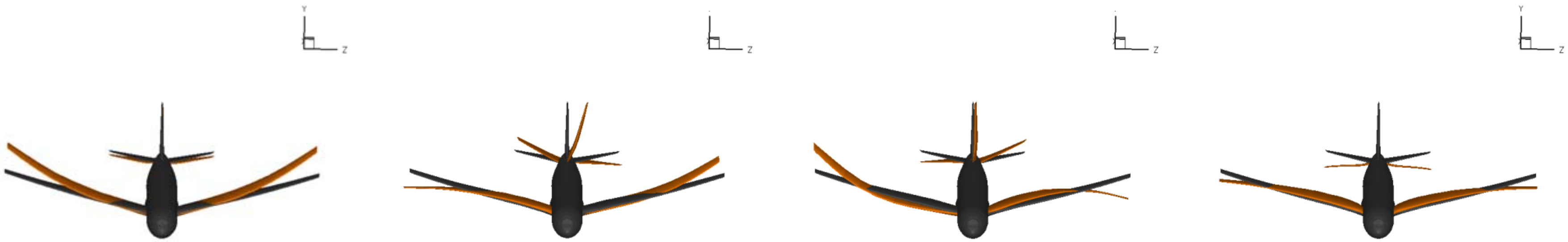



Marques, S., Badcock, K.J., Khodaparast, H.H. and Mottershead, J.E., Transonic Aeroelastic Stability Predictions Under the Influence of Structural Variability, **Journal of Aircraft**, 47(4), 2010, 1229-1239





Airbus model

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Costs of computing flutter boundaries low

**10-30x cost of a single steady state calculation**

# Current Work

- Influence of parameter variation on LCO
- Inverse problem for using flight data
- Consolidation of codes/test cases/publications

$$AP = iwP \qquad A^T Q = -iwQ$$

$$A\overline{P} = -iw\overline{P} \qquad A^T \overline{Q} = -iw\overline{Q}$$

Change coordinates

$$z = < P, \overline{w} >$$

$$y = \overline{w} - < P, \overline{w} > Q - < \overline{P}, \overline{w} > \overline{Q}$$

Taylor Series Expansion of R

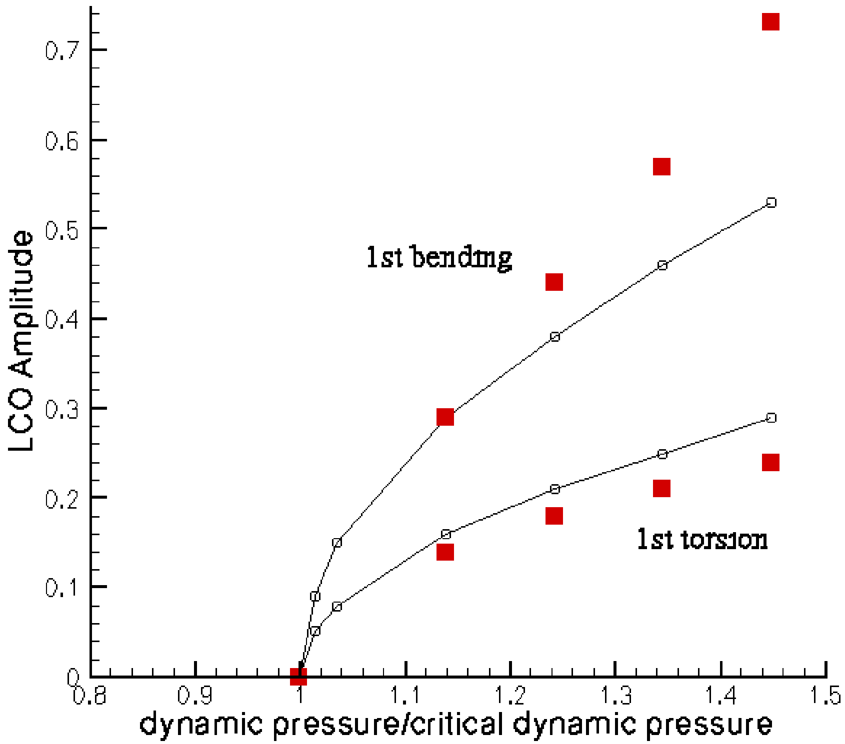
Expand system

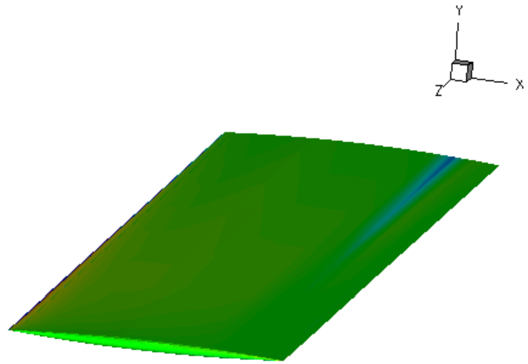
$$\frac{d\overline{w}}{dt} = A\overline{w} + \frac{\partial R}{\partial \mu}\overline{\mu} + \frac{\partial A}{\partial \mu}\overline{\mu w} + .....$$

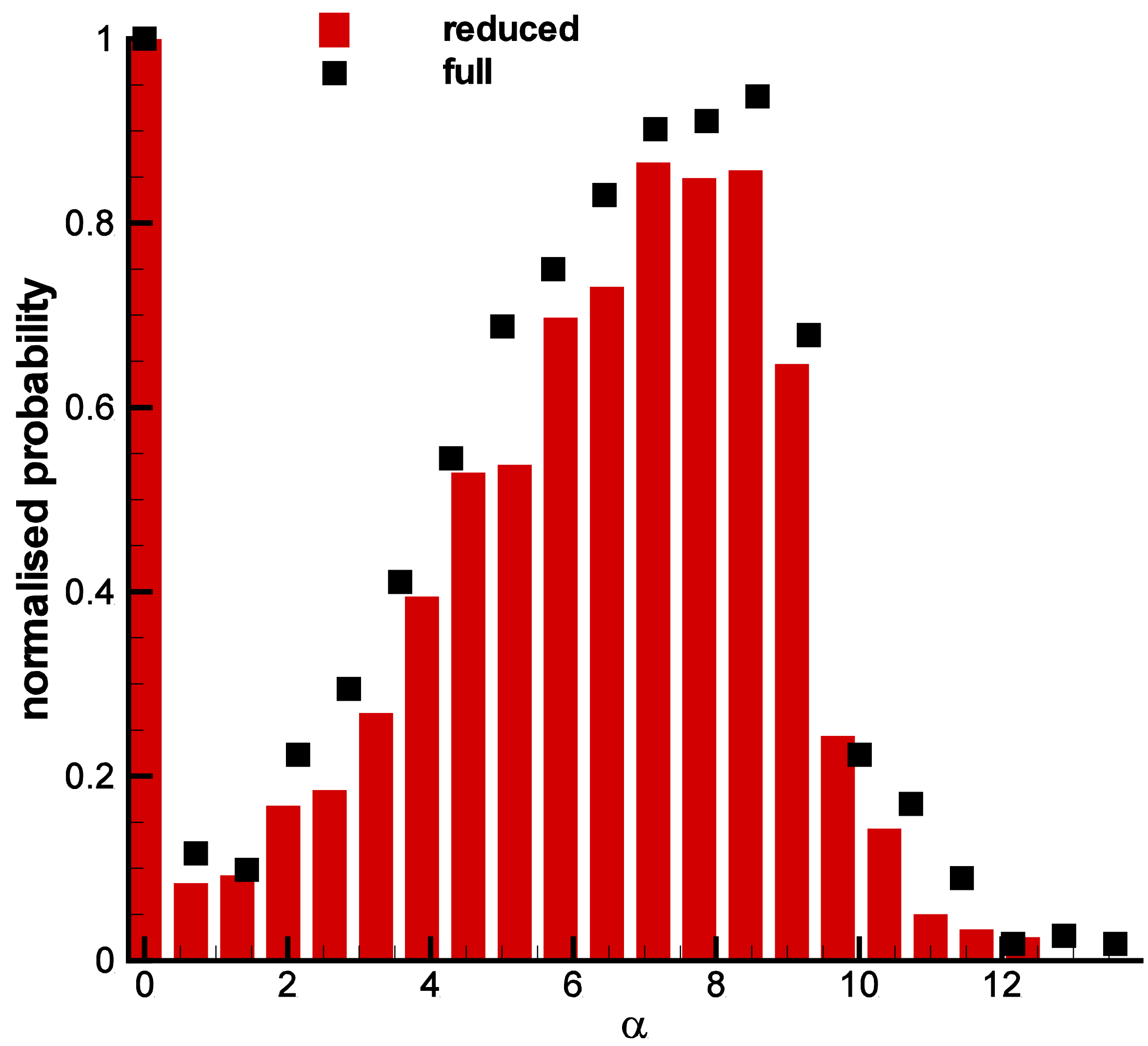
Inner Product with P

$$\dot{z} = i\omega z + < P, R_\mu \overline{\mu} > + < P, A_\mu \overline{\mu w} >$$

Badcock, K.J. and Woodgate, M.A., On the Fast Prediction of Transonic Aeroelastic Stability and Limit Cycles, **AIAA Journal** 45(6), 2007**.**

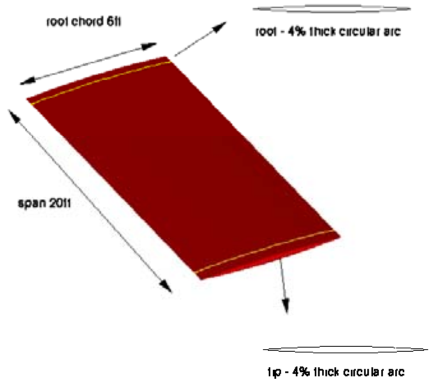

LCO Amplitude
dynamic pressure/critical dynamic pressure
1st bending
1st torsion
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
1.5

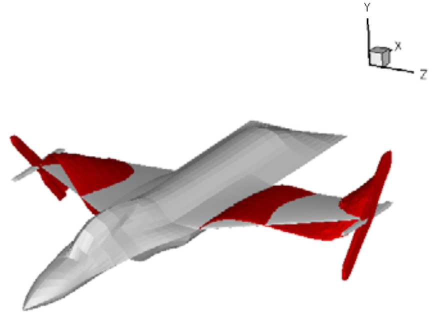

Y
X
Z

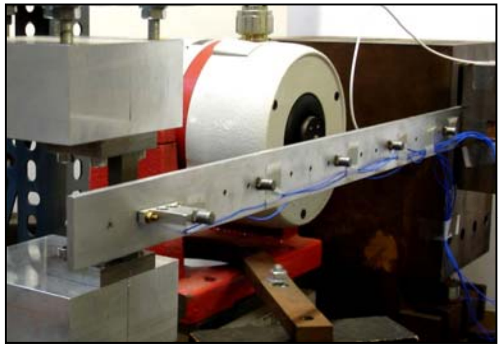

reduced
full
normalised probability
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
α

# Test Cases

Isogai Aerofoil
Case





→ **More discussion tomorrow**

# Outcomes

- Paper in preparation for Progress in Aerospace Sciences

- PhD’s for Prandina, Timme
- 24 papers so far
- Test cases on web
- Start on new collaborations
- Software being consolidated for wider use
- Ideas for new research

Marie Curie Excellence Team

Enabling Certification by Analysis