A Singular Perturbation Approach in Nonlinear Aeroelasticity for Limit-Cycle Oscillations

D. Dessi
INSEAN
via di Vallerano, 139 – 00128 Rome – Italy
tel. +39-06-50299254 E-mail d.dessi@insean.it

F. Mastroddi
Dept. of Aerospace Eng. and Astronautics
University of Rome "La Sapienza"
via Eudossiana, 16 – 00184 Rome – Italy
tel. +39-06-44585945 E-mail franco.mastroddi@uniroma1.it

L. Morino
Department of Mechanical and Industrial Engineering
University of Rome "Roma Tre"
Via della Vasca Navale, 79 – 00146 Rome – Italy tel. +39-06-55173279
E-mail L.Morino@UniRoma3.it

Abstract

Aeroelastic systems are typically characterized by a strong coupling between flow and structure that requires a simultaneous description of both of them. Moreover, aeroelastic systems present a also a relevant complexity that demands for simplified mathematical models and/or reducing techniques. Reduced-order modelling, Refs. [1, 2], seems to provide a general approach to this effort. It is essentially based on performing high-fidelity simulations (numerical or experimental) of the complex system, thus providing data about the system behavior from which essential features are extracted. For instance, by projecting the model onto a reduced-space basis, a limited set of generalized coordinates and modes capable of describing the system dynamics is obtained. As outlined above, a rather different approach is that one based on the use of simplifying physical assumptions to reduce the intrinsic complexity of the problem. This approach is also commonly followed in linear fixed-wing aeroelasticity since the early formulations of the aeroelastic problem and it is based on the following considerations. First, the elastic motion of a cantilever wing can be described with sufficient accuracy by the first bending and torsional modes, thus reducing the structural degrees-of-freedom (dofs). Second, the load acting upon the wing is provided by the span-wise distribution of the lift and pitching moment, assuming that the flow around each wing section is two-dimensional, incompressible and potential. These concepts were well established at the early stage of aeroelasticity through the concept of typical section, the reduction of the original problem to the study of the equivalent 2-D section, placed for instance at 70% of the wing span. More recently, a further simplification has been provided by the finite-state formulation of the unsteady aerodynamic loads: few augmented states (and corresponding differential equations) demonstrated to be necessary to account satis-

factorily for the circulatory lift. In this way, the aeroelastic system can be finally recast in a pure differential form with a very limited set of unknowns. On this basis, in last decades nonlinear aeroelasticity has broaden the predictive capability of classical linear aeroelastic models via the inclusion of aerodynamic and structural nonlinearities, providing several interesting applications of both the concepts and numerical/analytical methods developed in the field of nonlinear dynamics.

The most relevant problem that has received a renovate attention within fixed-wing aeroelasticity is the prediction of the air-flow condition above which the wing-air system may become unstable. In fact, advanced wing configurations or the deterioration of the airplane control surfaces have required a generalization of the well-established concept of critical speed. Generally speaking, if the instability involves oscillations, the phenomenon is called flutter, otherwise it is called divergence. According to the linear stability analysis, the oscillations beyond the so-called (linear) flutter speed U_L are not damped and their amplitude exponentially grows, from a mathematical point of view, leading to the collapse of the wing structure. In the case of nonlinear aeroelastic systems, more attention must be paid to the effects that some kind of nonlinearities may induce on flutter. In Dowell (1997) an exhaustive review of the scenario of nonlinear aeroelastic phenomena was presented. Within this framework, nonlinear torsional stiffness and control-surface freeplay were extensively analyzed in the last decade technical literature [see Alighanbari and Price (1996), Lee (1998)]. In these papers the nonlinear aeroelastic vibration of a 2-dofs pitching and plunging airfoil or 3-dofs pitching, plunging airfoil with a control surface independent rotation was numerically studied and sometimes compared with experimental results.

Focusing on nonlinear aeroelastic systems exhibiting limit cycle oscillations (LCO), there is well known experimental evidence shown, in [Lacabanne (1997), Matsushita (1998), Chen (1998)], as well as numerical evidence, as shown in [Conner (1997), Dessi and Mastroddi (2004)], that a combination of *(i)* small-amplitude unstable limit cycles (LC), and *(ii)* large-amplitudes stable limit cycles may occur below the linear flutter speed. This implies the possibility, under suitable initial conditions, of *finite* amplitude limit-cycle oscillations even below the linear flutter speed.

More recently, aeroelastic modelling has considered the combination of nonlinear and stochastic responses via the inclusion of the effects due to flow random perturbations, as done in Poirel and Price (2001). In general, two distinct effects may be identified for an airfoil undergoing a randomly perturbed inflow. In the first case, the perturbation velocity components are orthogonal to the undisturbed flow (vertical gust) and are independent on the state-space variables, whereas, in the second case, the perturbation involves only the flow-wise component of the velocity, thus generating aerodynamic forces that are dependent on the state-space variables.

Indeed, the inclusion of vertical gust effects in the aeroelastic modelling provides the physical mechanism by which the wing is actually perturbed in the rest condition [Dessi and Mastroddi, (2008)]. In particular, in the knee-bifurcation scenario, a vertical gust of adequate intensity might induce LCOs of relevant amplitude even below the linear flutter speed. A basin of attraction of the limit cycle solution in terms of the gust parameters reveals to be more actually interesting from a physical point of view than that one obtained by varying the system initial conditions.

In the present work, the oscillations of an aeroelastic typical section (described in terms of plunge, pitch and control surface deflection) are analyzed with the use of both numerical simulations and a perturbation technique (the normal form method and multiple time scale method). The nonlinear analysis concerns the determination of the steady-state solutions (fixed points and both stable and unstable limit cycles) and the prediction of transient behavior of slave modes as well. Investigation of the physical mechanism that causes the onset of flow-induced vibrations, that is, the vertical gust excitation, is also accounted for in the proposed results. In fact, the use of nonlinear techniques like perturbation methods can help to simplify the governing equations and, in this way, to reduce also the numerical task connected to ‘draw’ the boundaries of the basins of attraction in the space of initial conditions. It is finally worth pointing out that, whenever the nonlinear modal description for an actual 3-D aeroelastic system were available, namely, identified by a possible numerical or experimental approach, the proposed singular-perturbation approach would be quite applicable demonstrating its generality as tool to study and describe the local bifurcation of aeroelastic systems.

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