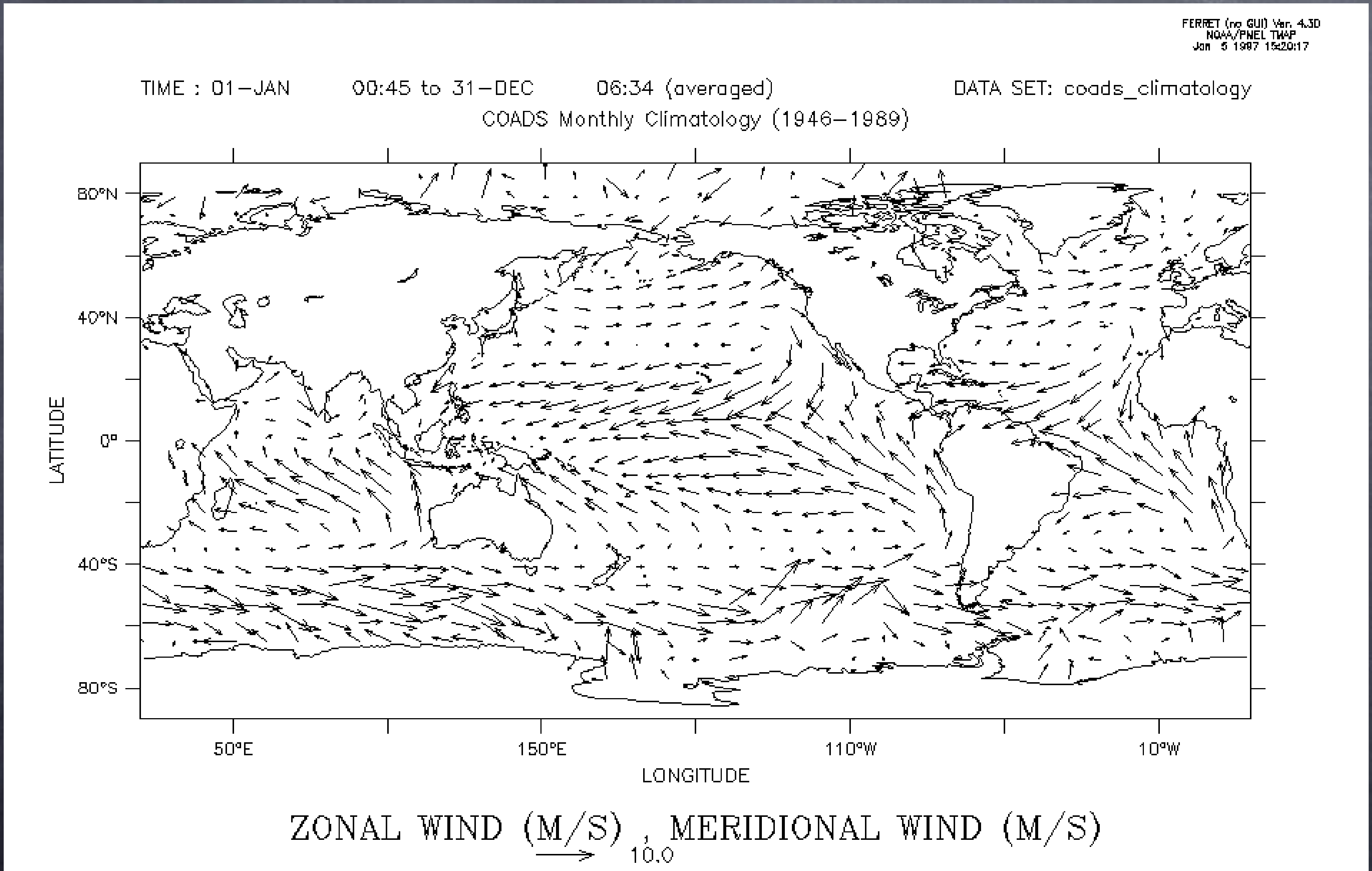
THE APPLICATION OF IMPLICIT TECHNIQUES IN OCEAN MODELING

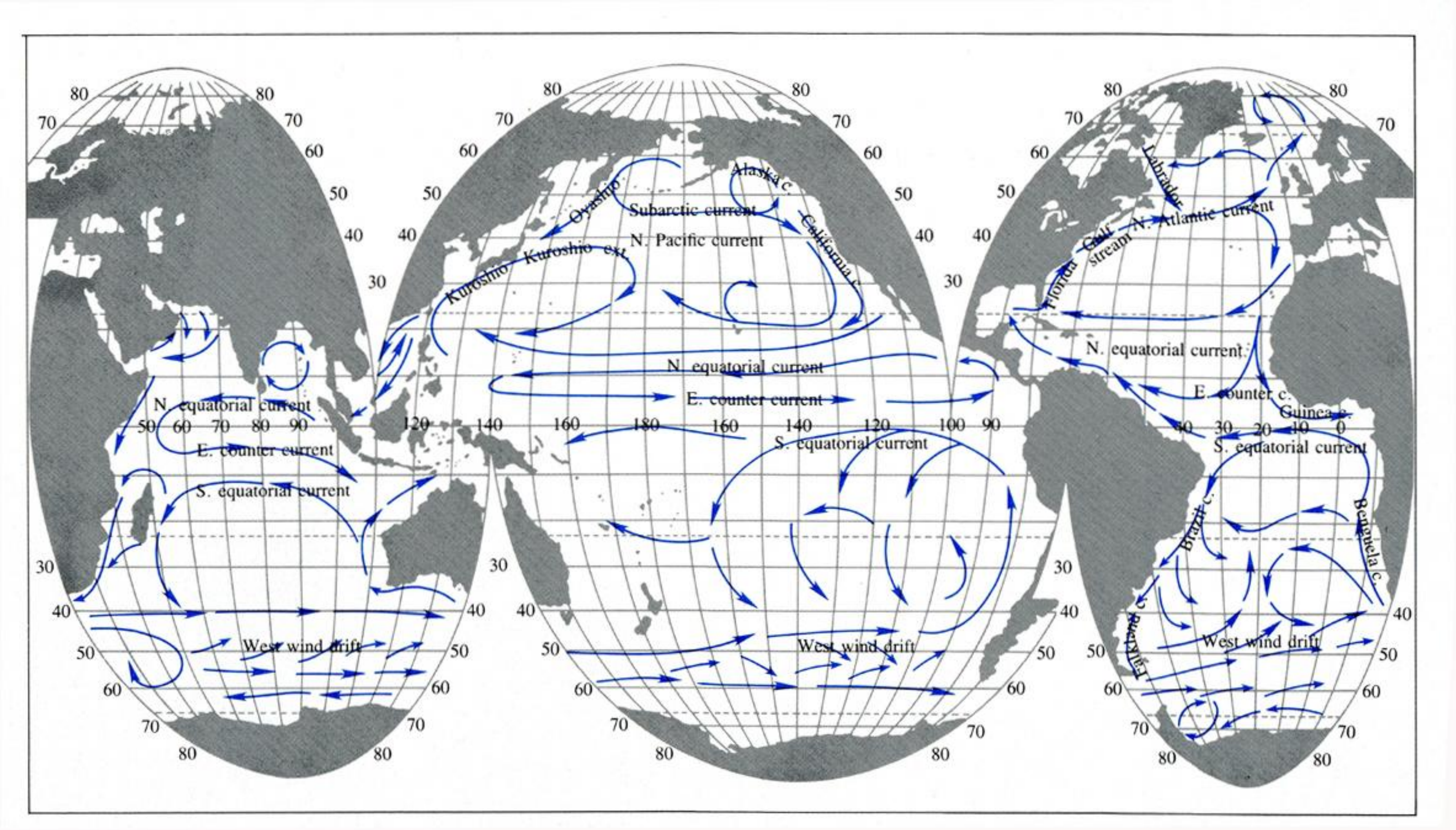
Erik Bernsen and Henk A. Dijkstra
IMAU, Department of Physics and Astronomy, Utrecht University, The Netherlands

Fred Wubs and Jonas Thies
Department of Mathematics and Computer Science
University of Groningen, The Netherlands

Annual mean surface wind velocity



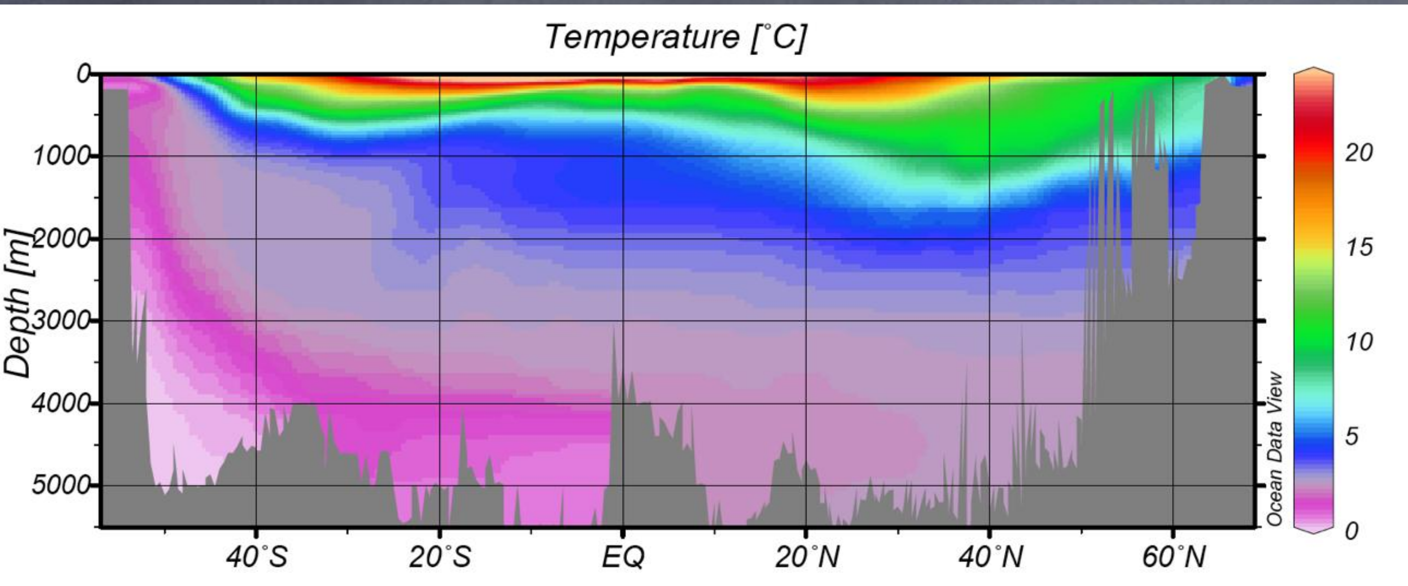
Surface Ocean Circulation

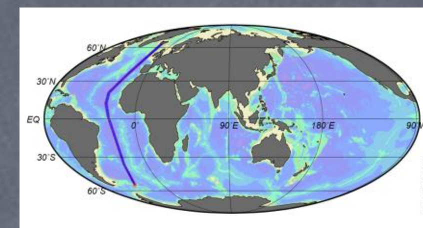


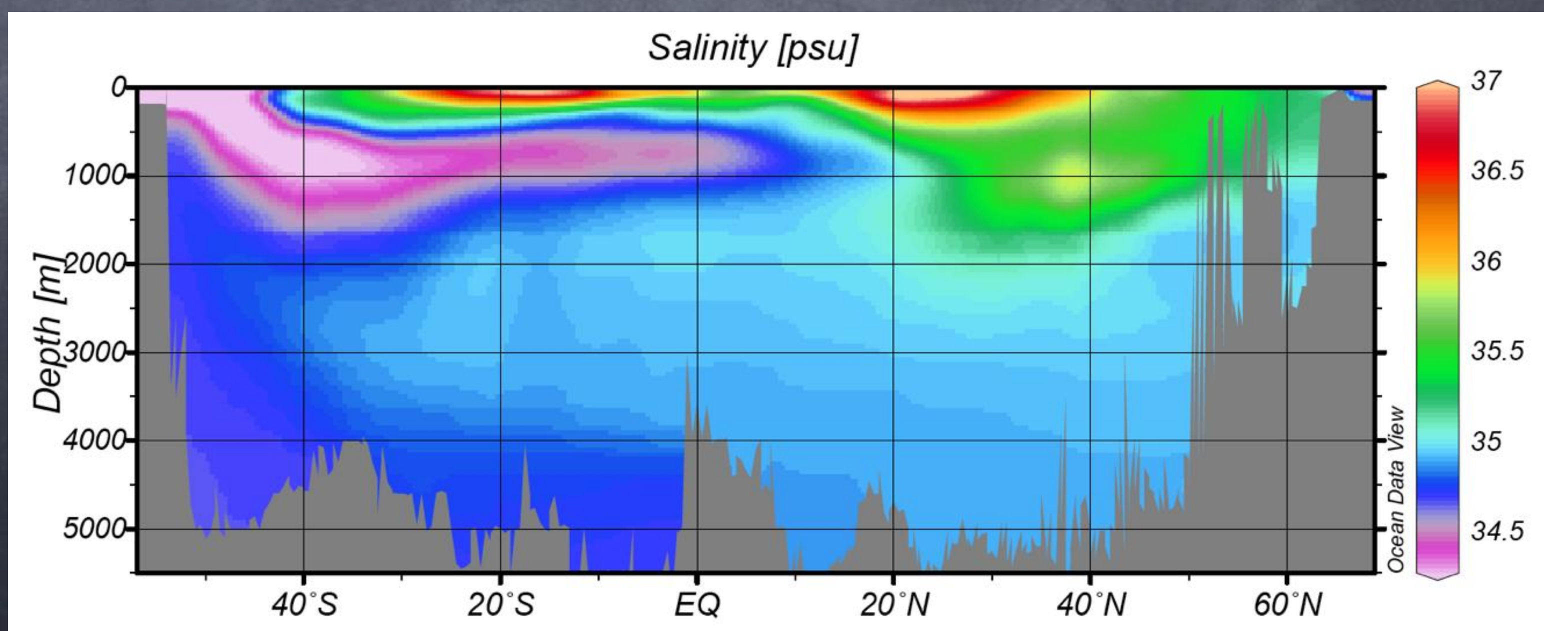
Wind-driven circulation: circulation associated with direct forcing of the wind

After: Sverdrup, H.U. et al. (1942)

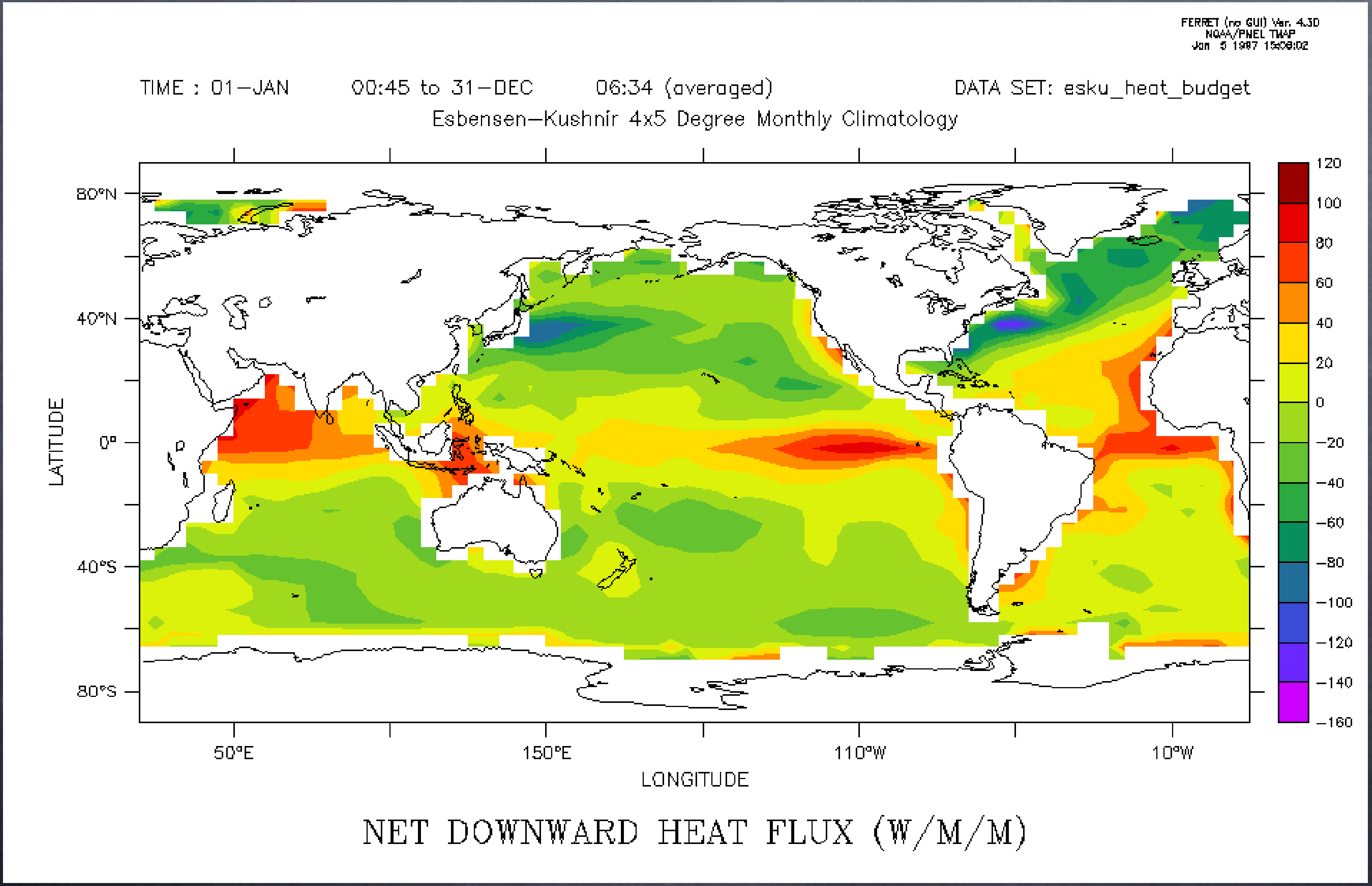
Temperature + Salinity section



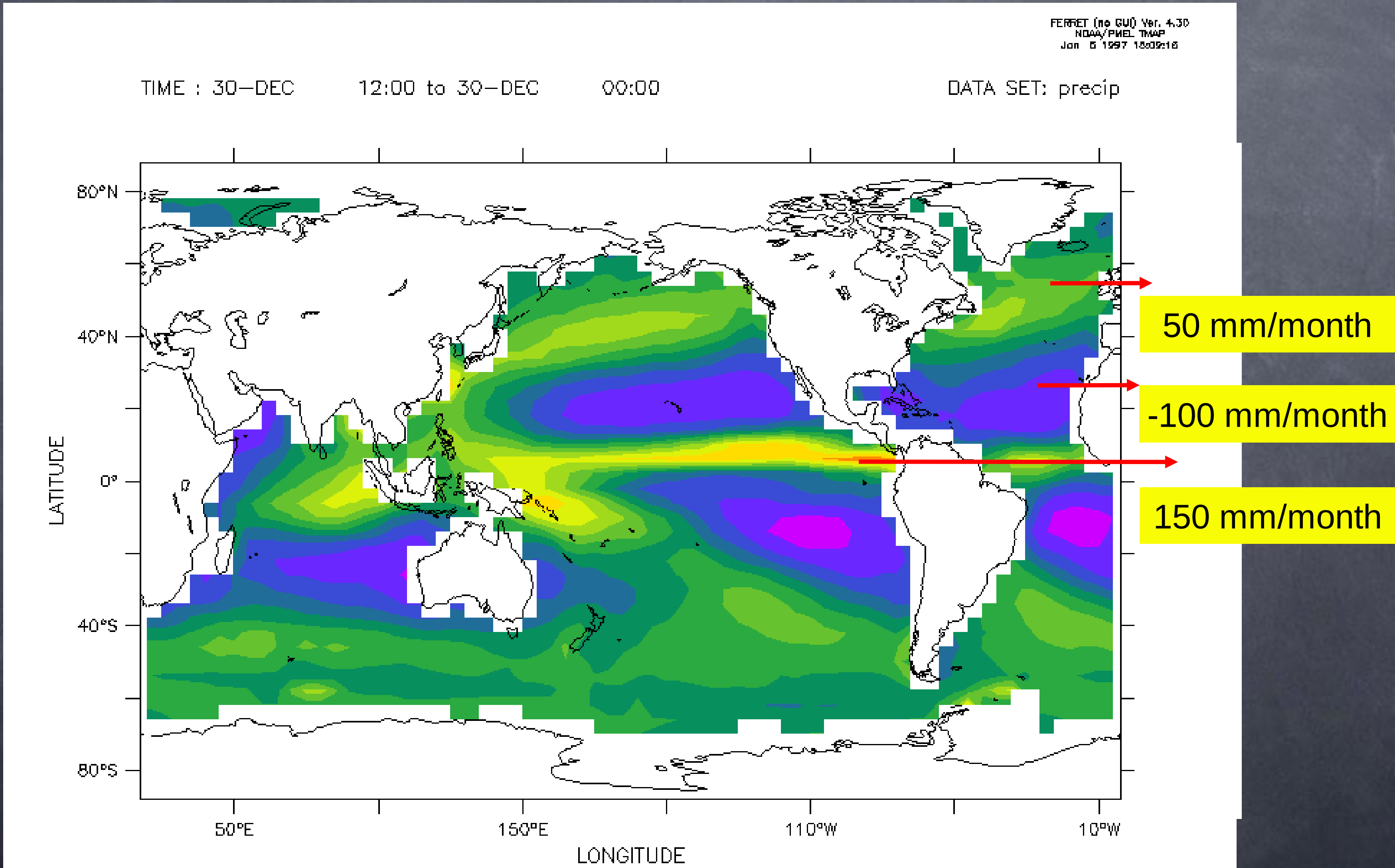




Annual mean surface heat flux

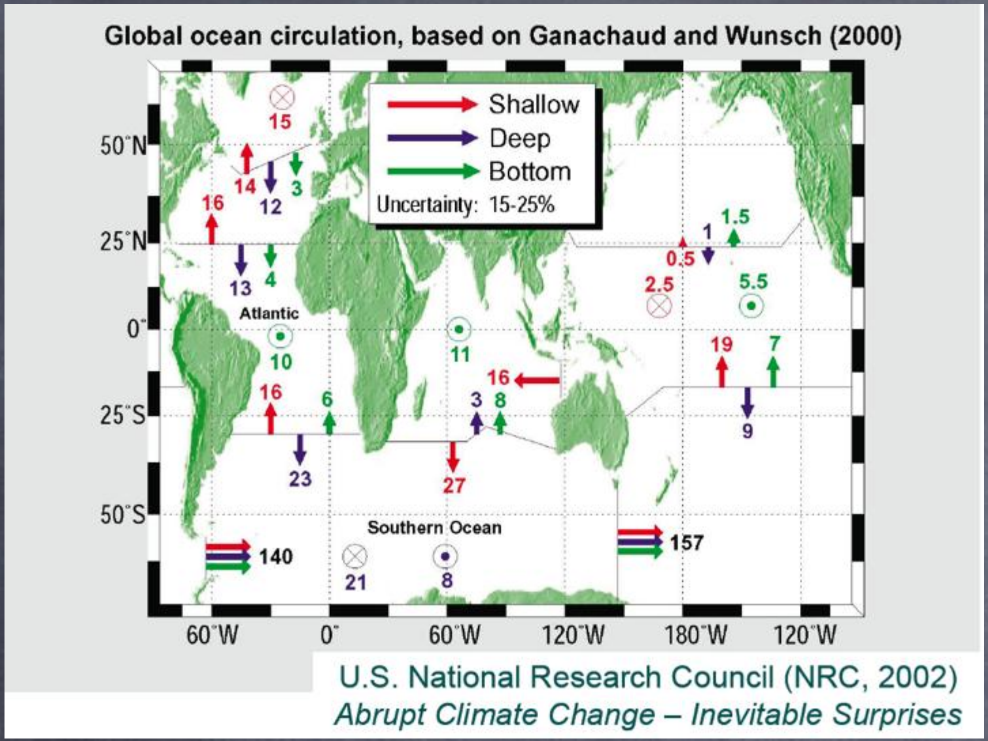


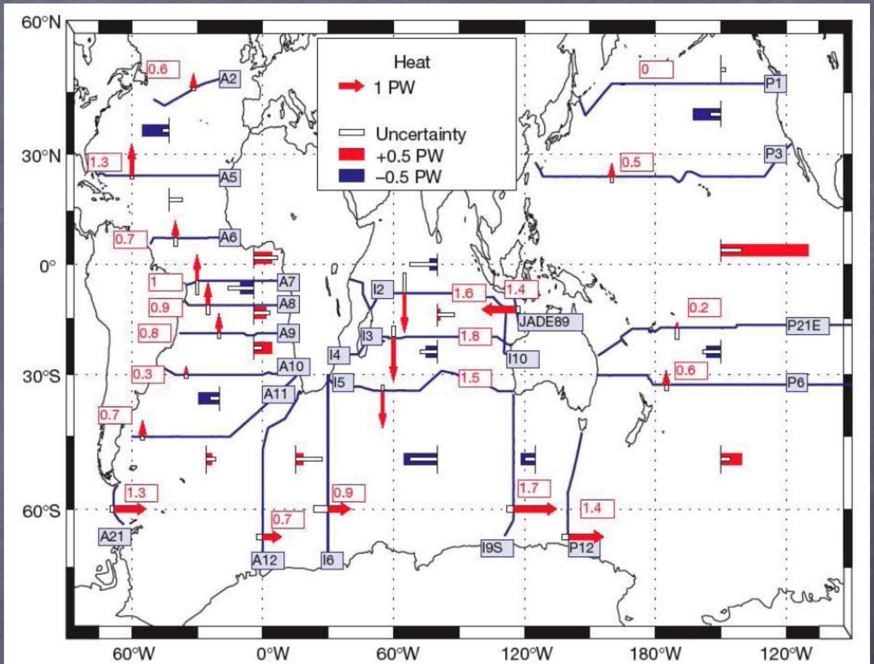
Annual mean freshwater flux



Annual net precipitation – evaporation

Global Conveyor Circulation





Ganachaud & Wunsch, Nature, 408, 453, (2000).

1 Sv = 10^6 m^3/s

QuickTime™ and a
Cinepak decompressor
are needed to see this picture.

Thermohaline circulation: circulation associated with the transport of heat and salt

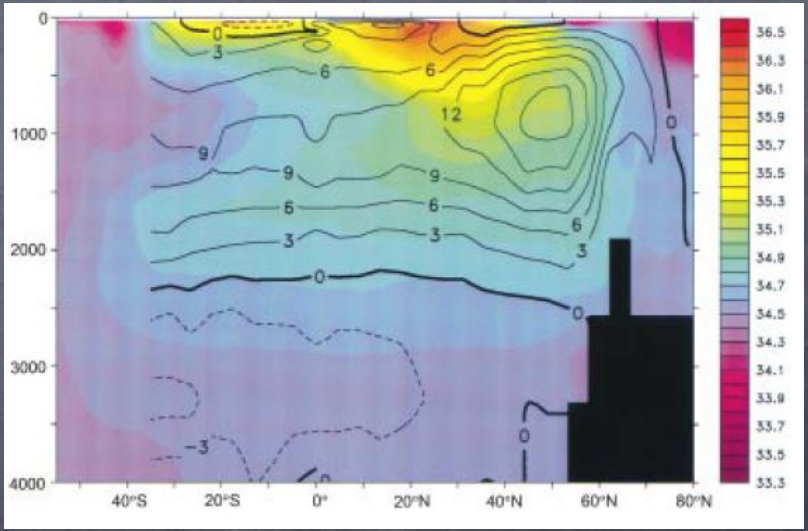
Meridional Overturning Circulation (MOC) & ThermoHaline Circulation (THC)

QuickTime™ and a
Sorenson Video decompressor
are needed to see this picture.

Model determined

Meridional overturning streamfunction

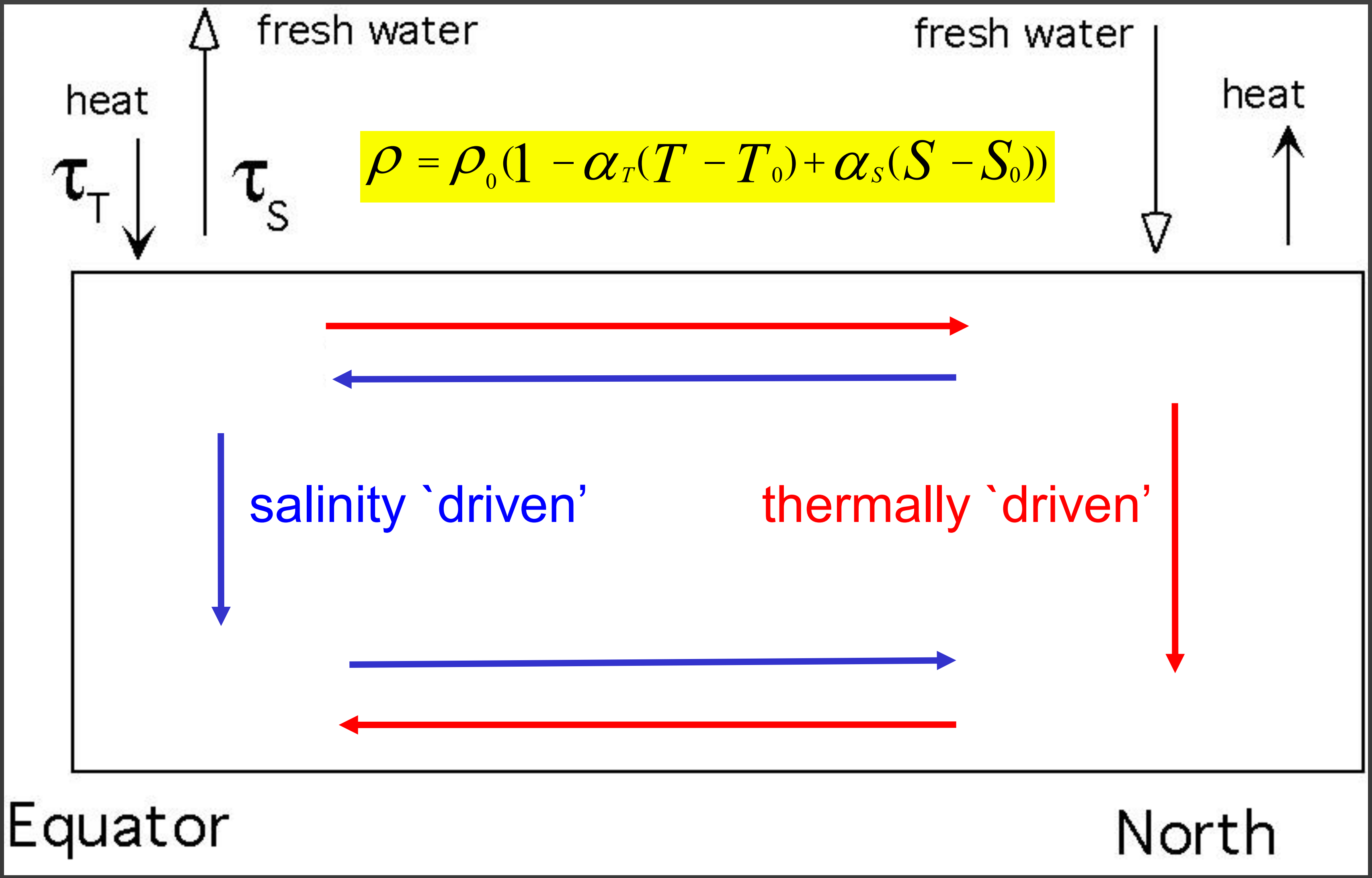
Depth



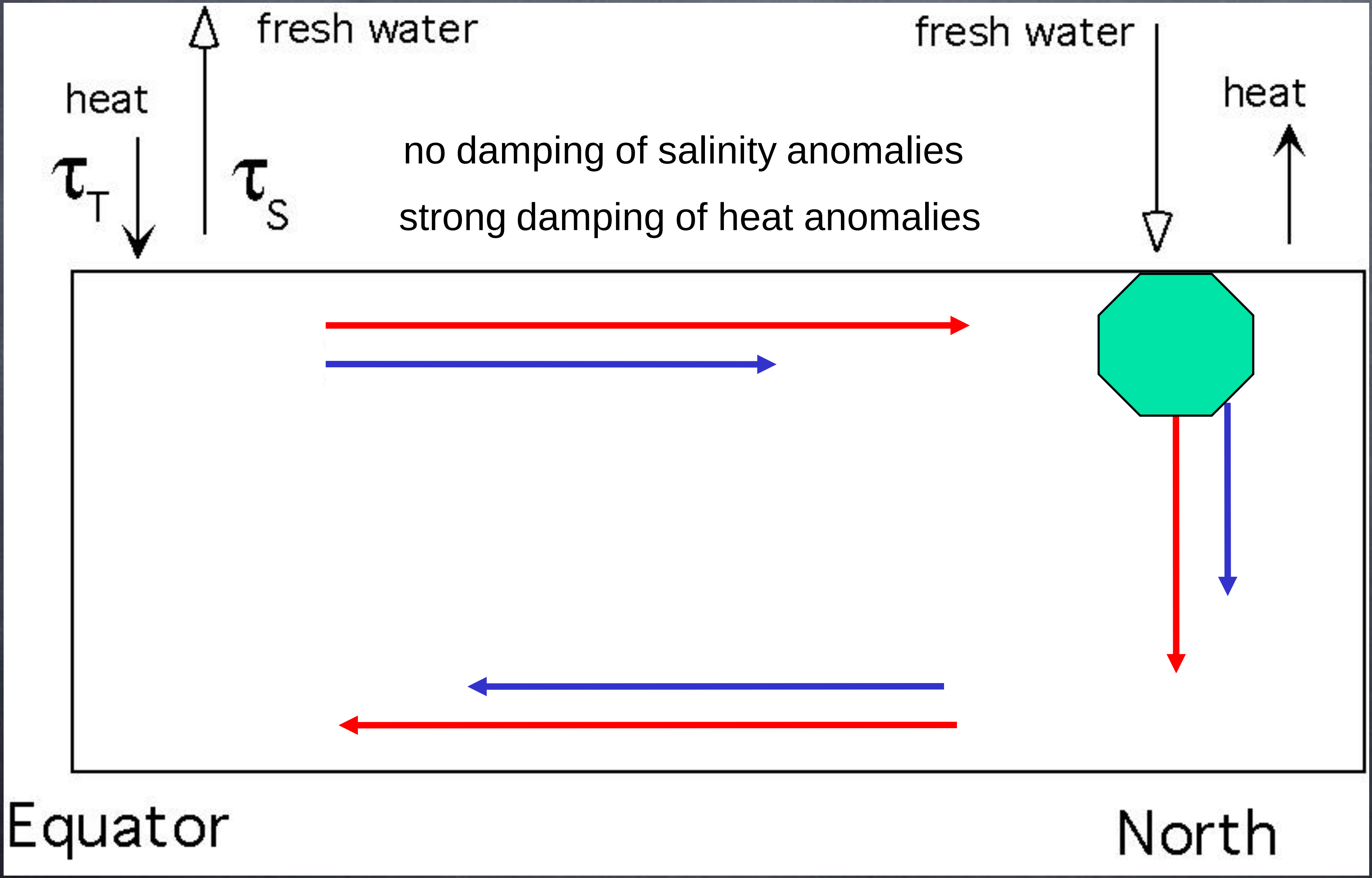
40S *Contours in Sv* 80N

MOC: Total northward/southward transport in latitude/depth (is observable)
THC: Part of MOC driven by heat/freshwater exchange at the surface and subsequent vertical mixing (is an interpretation)

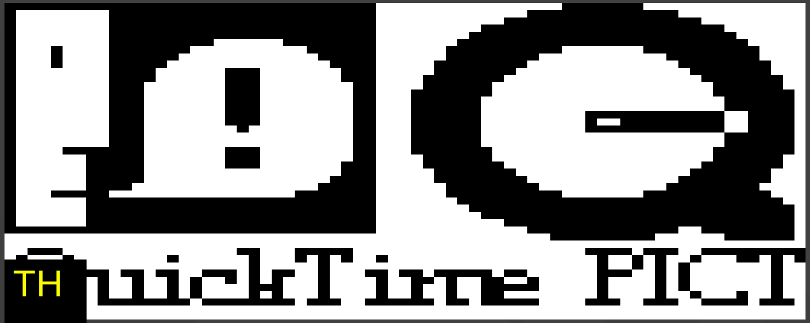
Conceptual Model of the Atlantic MOC



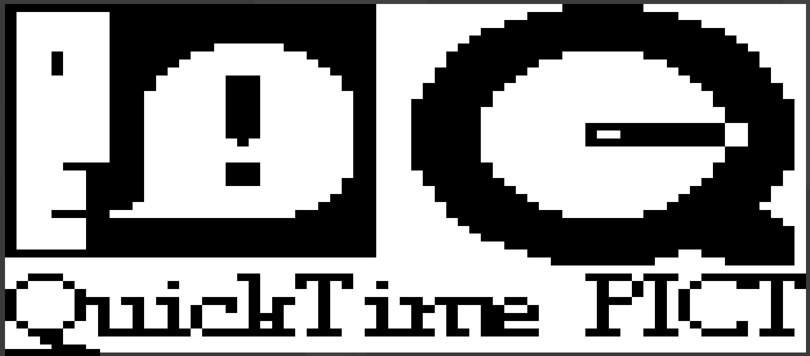
The salt-advection feedback



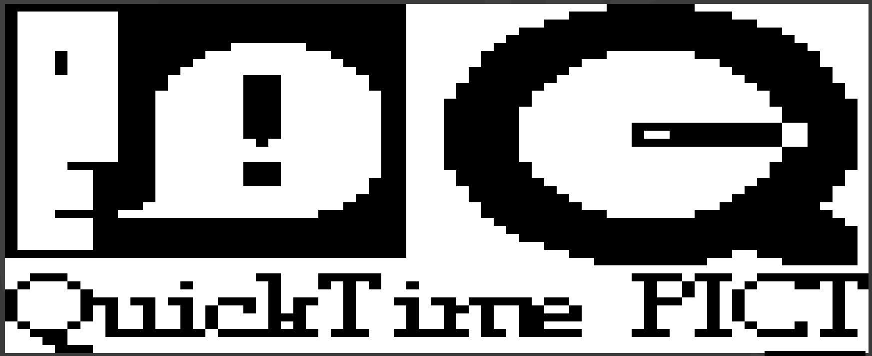
Multiple equilibria in an Atlantic ocean model …



TH



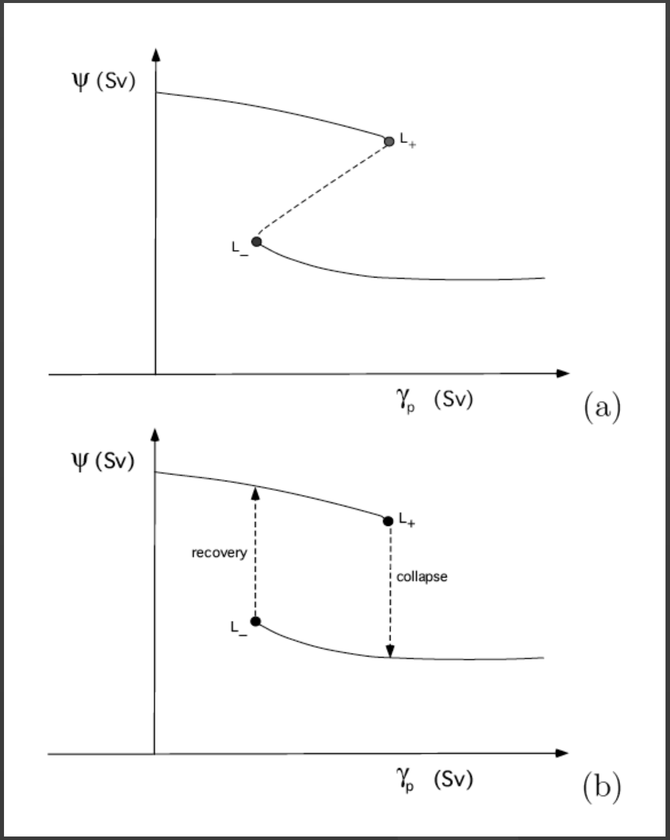
SPP

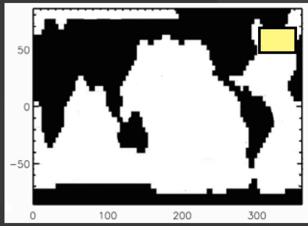


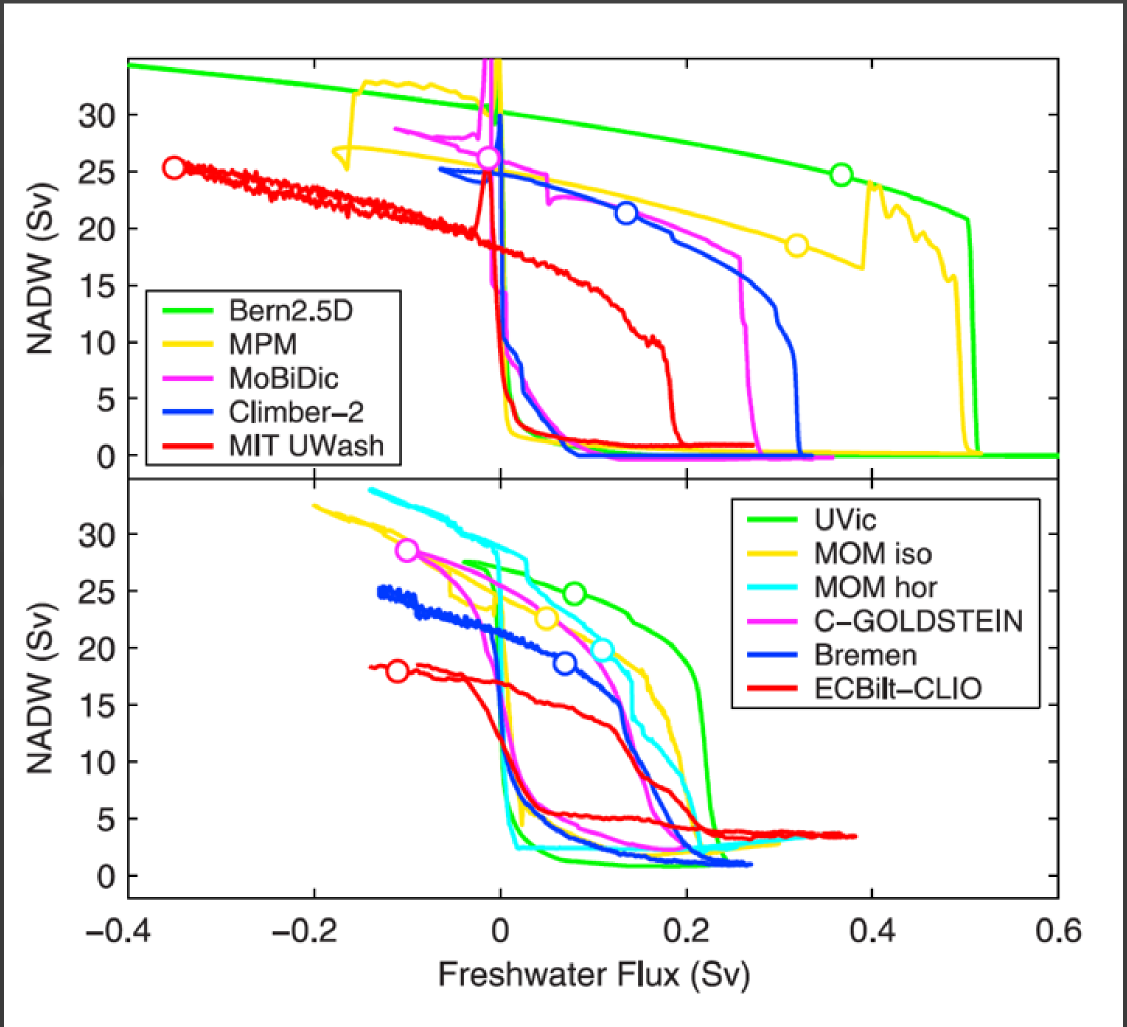
NPP

Source: Bryan, F. High-latitude salinity effects and interhemispheric thermohaline circulations, Nature, 323, 301-30

... and in global ocean models and EMICs







Rahmstorf et al., GRL, (2005)

Use of implicit methods

- I: Equilibria versus parameters

★ Steady state solvers and bifurcation analysis

✦ continuation: no transient integration

- II: Spin-up

★ Use of Newton-Raphson like techniques in explicit models

✦ equilibration time scale ~ 5000 yr
time step 1 hr

Ocean model formulation

Local conservation of mass, momentum, heat and salt:

$$\mathcal{M}_\lambda \frac{\partial \mathbf{u}}{\partial t} + \mathcal{L}_\lambda \mathbf{u} + \mathcal{N}_\lambda(\mathbf{u})\mathbf{u} = \mathbf{F}_\lambda$$

$\mathcal{M}_\lambda, \mathcal{L}_\lambda, \mathcal{N}_\lambda$: Operators

λ: Parameter Vector

$\mathbf{u}$: State Vector

+ Boundary conditions & Initial conditions

$$\mathbf{u} = \begin{pmatrix} u \\ v \\ w \\ p \\ T \\ S \end{pmatrix}$$

Zonal velocity

Meridional velocity

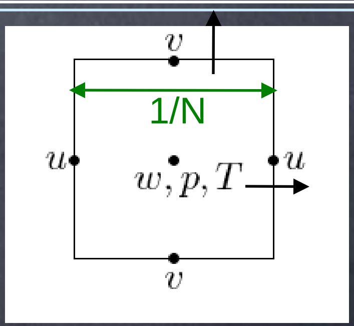
Vertical velocity

Pressure

Temperature

Salinity

Discretization on grid: N x M x L



degrees of freedom: d = N x M x L x 6

Governing equations

$$
\begin{aligned}
\frac{Du}{dt} - uv\tan\theta - 2\Omega\; v\sin\theta + \frac{1}{\rho_0\, r_0\cos\theta}\frac{\partial p}{\partial\phi} &= \\
A_V\frac{\partial^2 u}{\partial z^2} + \boxed{A_H} L_u(u,v) &+ \frac{\tau_0}{\rho_0 H_m}\tau^{\phi}\mathcal{G}(z) \\
\frac{Dv}{dt} + u^2\tan\theta + 2\Omega\; u\sin\theta + \frac{1}{\rho_0\, r_0}\frac{\partial p}{\partial\theta} &= \\
A_V\frac{\partial^2 v}{\partial z^2} + \boxed{A_H} L_v(u,v) &+ \frac{\tau_0}{\rho_0 H_m}\tau^{\theta}\mathcal{G}(z) \\
\frac{\partial p}{\partial z} &= \rho_0\, g(\alpha_T T - \alpha_S S) \\
\frac{\partial w}{\partial z} + \frac{1}{r_0\cos\theta}\left(\frac{\partial u}{\partial\phi} + \frac{\partial(v\cos\theta)}{\partial\theta}\right) &= 0 \\
\frac{DT}{dt} - \nabla_H\cdot(K_H\nabla_H T) - \frac{\partial}{\partial z}\left(\boxed{K_V}\frac{\partial T}{\partial z}\right) &= \frac{(T_S - T)}{\tau_T}\mathcal{G}(z) \\
\frac{DS}{dt} - \nabla_H\cdot(K_H\nabla_H S) - \frac{\partial}{\partial z}\left(\boxed{K_V}\frac{\partial S}{\partial z}\right) &= F_0 F_S\mathcal{G}(z)
\end{aligned}
$$

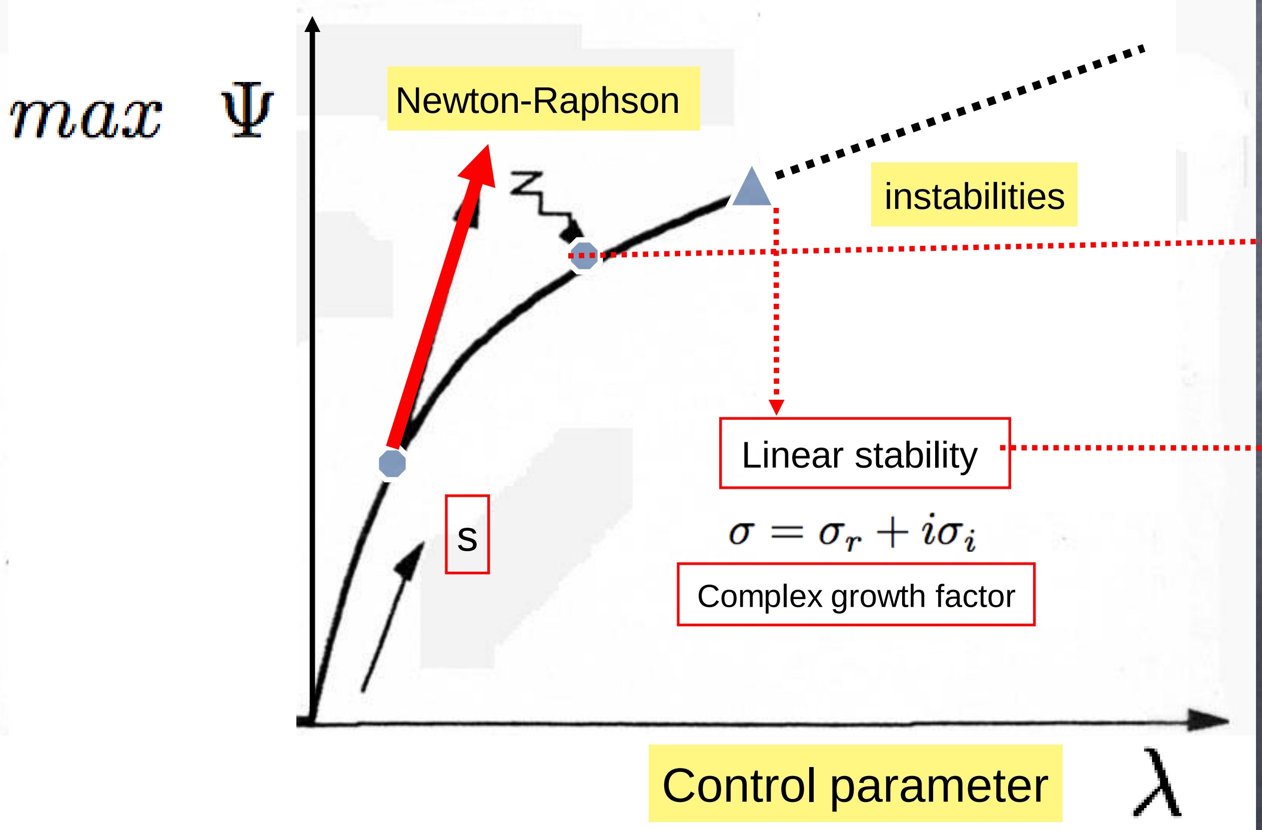
Continuation Methods

$$M\frac{dx}{dt} + G(x,\lambda) = 0, x \in R^d$$

d: # degrees of freedom

Steady states

$$G(x,\lambda) = 0 \Rightarrow$$
$$J(x^k,\lambda^k)\Delta x^{k+1} = -G(x^k,\lambda^k)$$



Numerics

$Ax = b$

$A : d \times d$

MRILU

GMRES

$Ax = \sigma Bx$

$A, B : d \times d$

JDQZ

Status UU group ~ 2008

A tailored solver for bifurcation analysis of ocean-climate models

Arie de Niet [a], Fred Wubs [a,*], Arjen Terwisscha van Scheltinga [b], Henk A. Dijkstra [b]

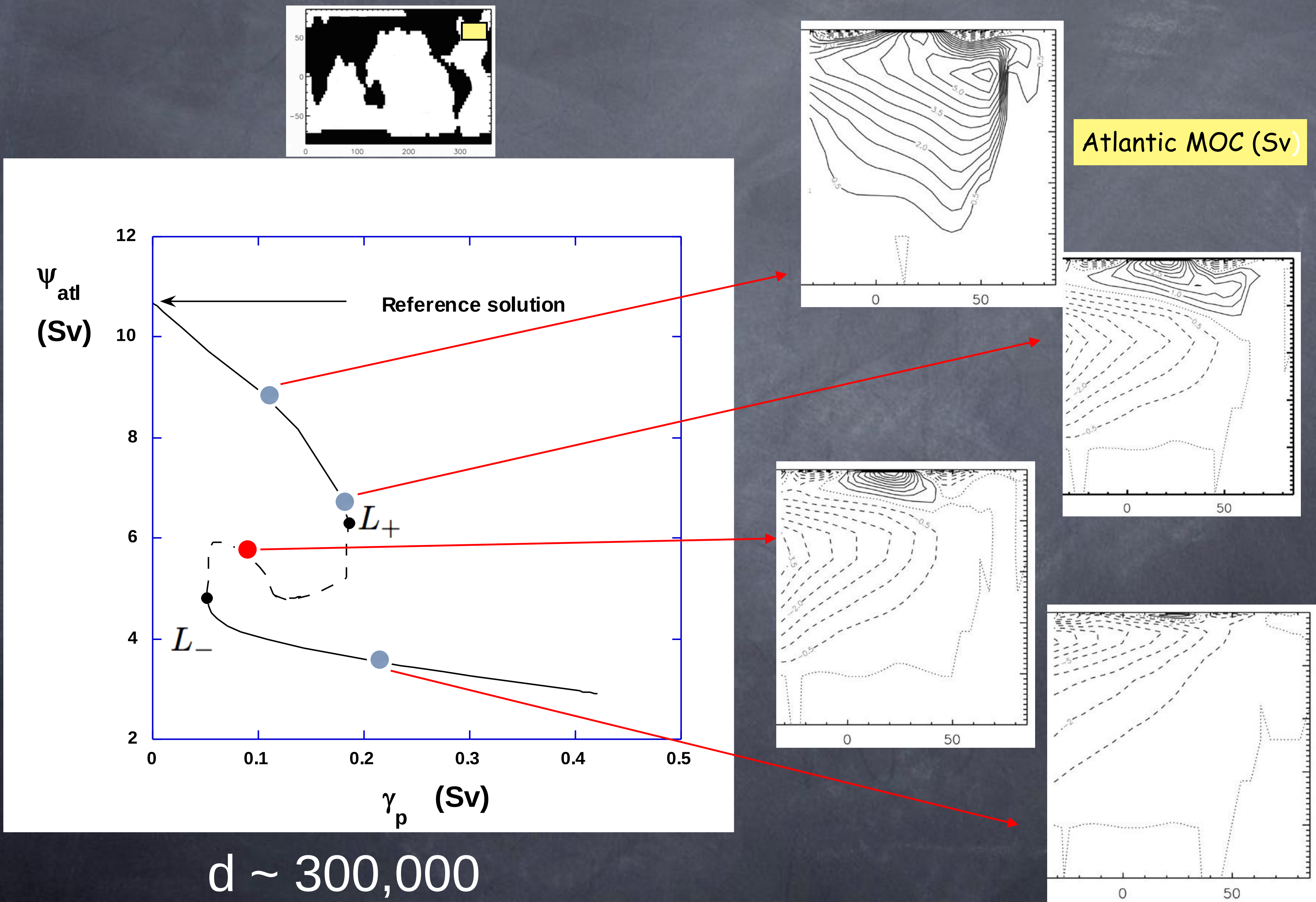
Journal of Computational Physics 227 (2007) 654–679

- Full 3D primitive equations
- Solutions for ocean models with d ~ 1,250,000
- Efficient handling of bathymetry
- Implementation of state of the art mixing schemes, such as neutral physics and GM

Recent developments:

Parallel implementation into TRILINOS

Bifurcation diagram (global ocean model)

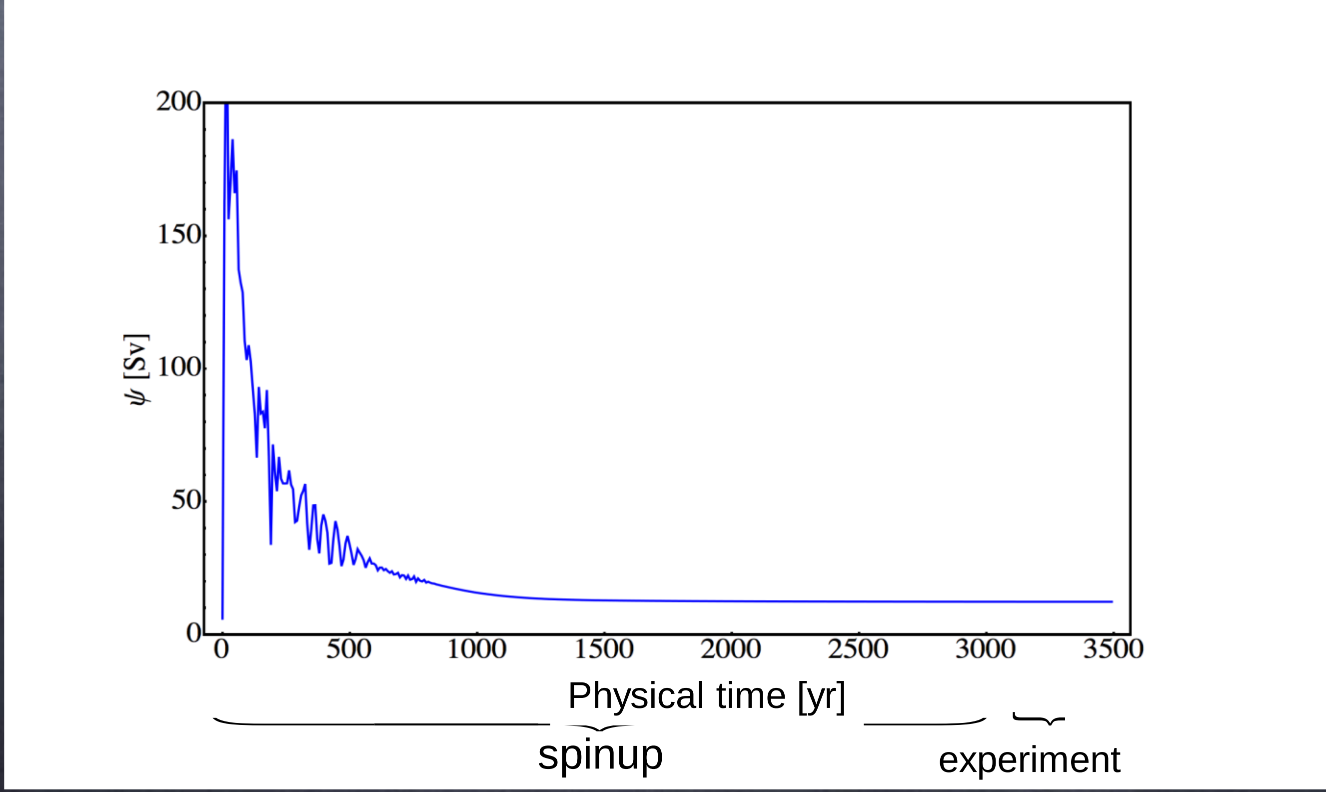


d ~ 300,000

II. Spin-up

Spinup time: The (CPU) time needed for the ocean model to reach an equilibrium.

Spinup timescale (physical time): Set by vertical diffusivity K and mean depth of the ocean basin D



Jacobian Free Newton-Krylov (JFNK) methods

Goal: Find a steady state of: $\mathrm{d}\vec{x}/\mathrm{dt} = F(\vec{x})$

Timestepper: Iterate $\vec{x}_{k+1} = \vec{x}_k + \Delta t F(\vec{x})$

until $\|\vec{x}_{k+1} - \vec{x}_k\|$ small.

Newton-Raphson: Iterate $0 = F(\vec{x}_k) + J\ (\vec{x}_{k+1} - \vec{x}_k)$

until $\|F(\vec{x}_k)\|$ small .

During GMRES method:

$$J\,\vec{v} \approx \frac{\vec{F}(\vec{x}_k + \varepsilon\vec{v}) - \vec{F}(\vec{x}_k)}{\varepsilon} \qquad (\text{with } \varepsilon \text{ small})$$

Status UU group ~ 2008

A method to reduce the spin-up time of ocean models

Erik Bernsen[a,*], Henk A. Dijkstra[a], Fred W. Wubs[b]

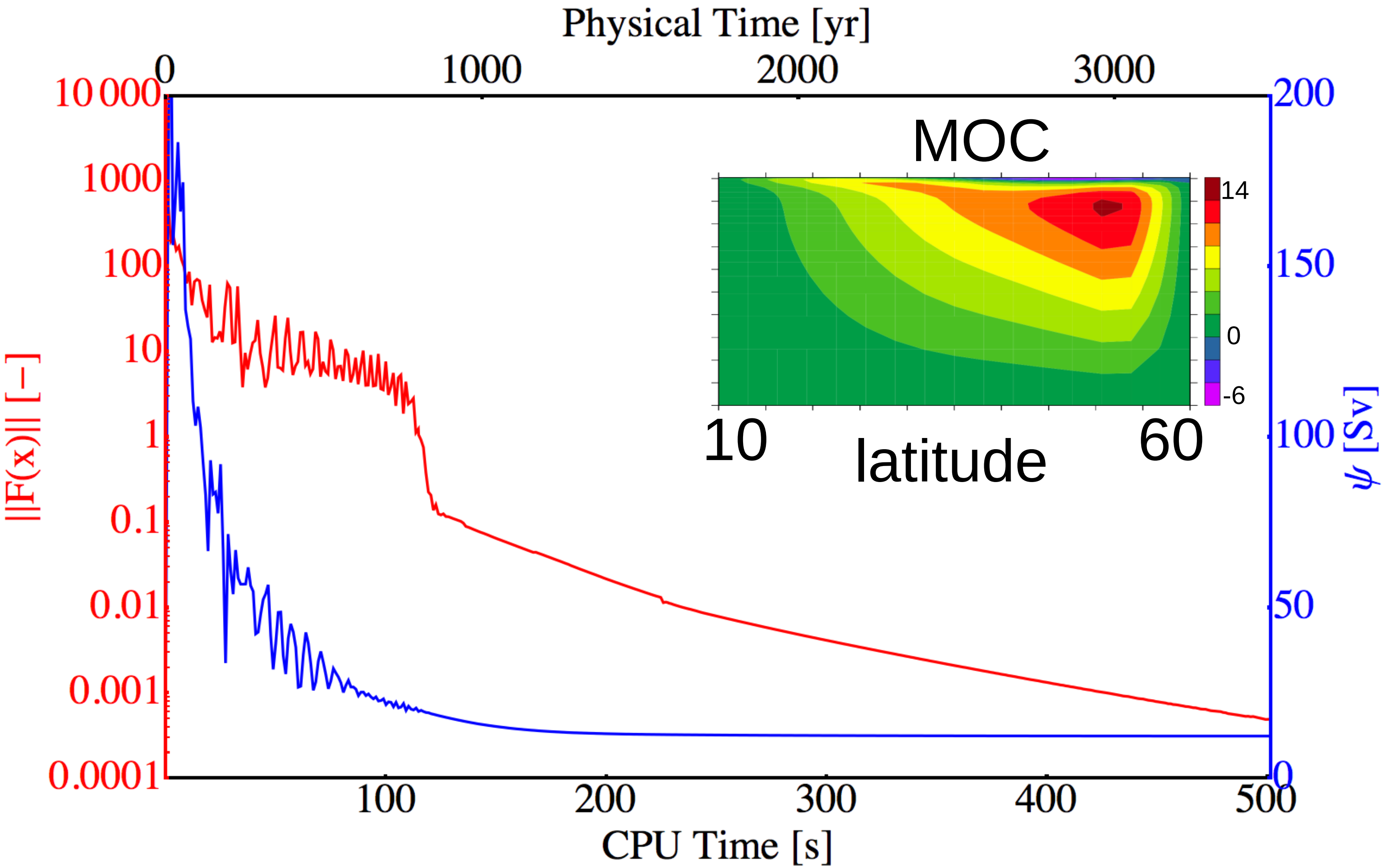
Ocean Modelling 20 (2008) 380–392

- 3D planetary geostrophic model; T and S only prognostic variables (Samelson & Vallis, 1997)
- Idealized wind-stress and restoring temperature and salinity forcing
- Preconditioner based on dependencies in Jacobian
- Continuation technique in model forcing to avoid problems in Newton-Raphson convergence

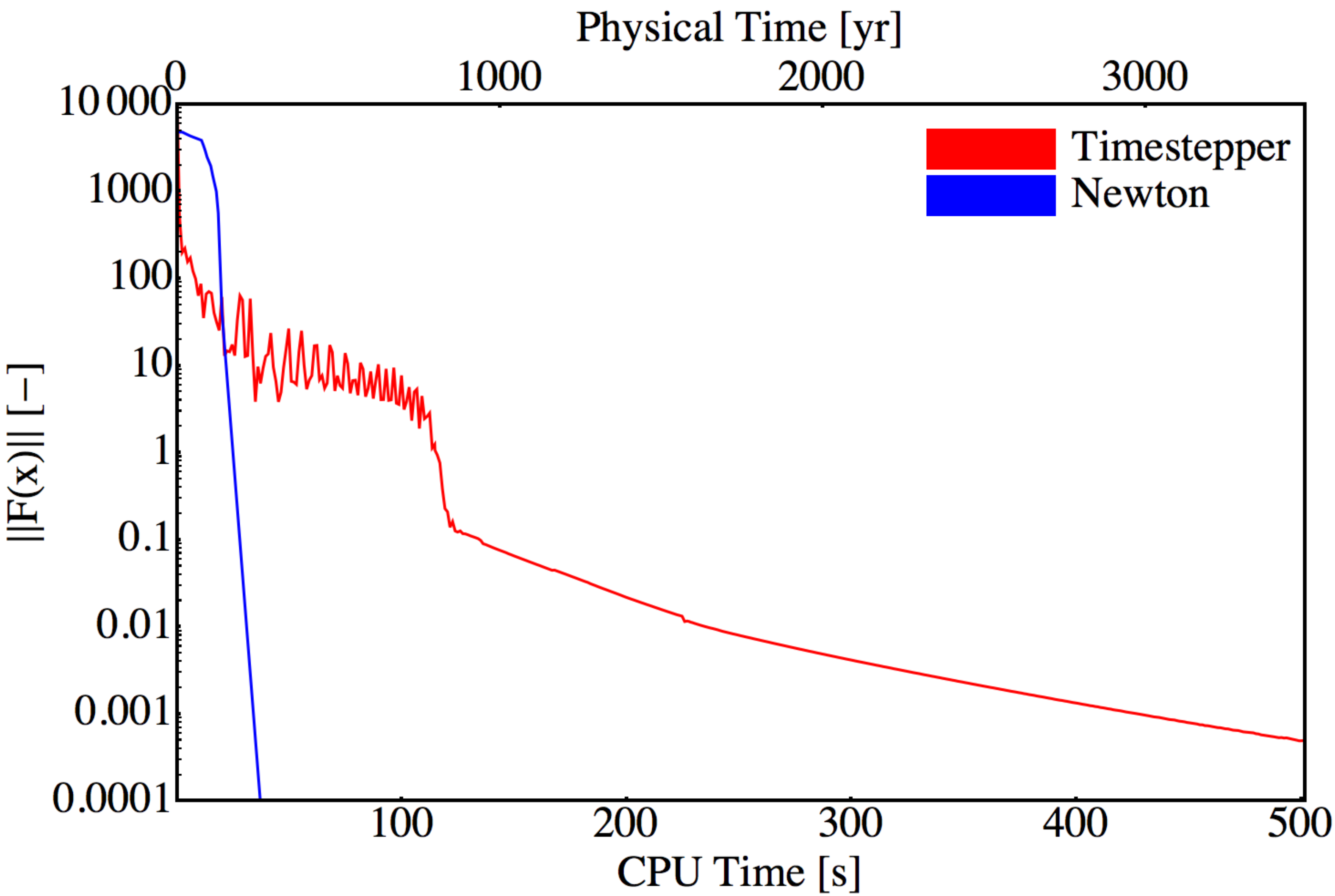
Recent developments:

Implementation into ocean GCM (MOM4)

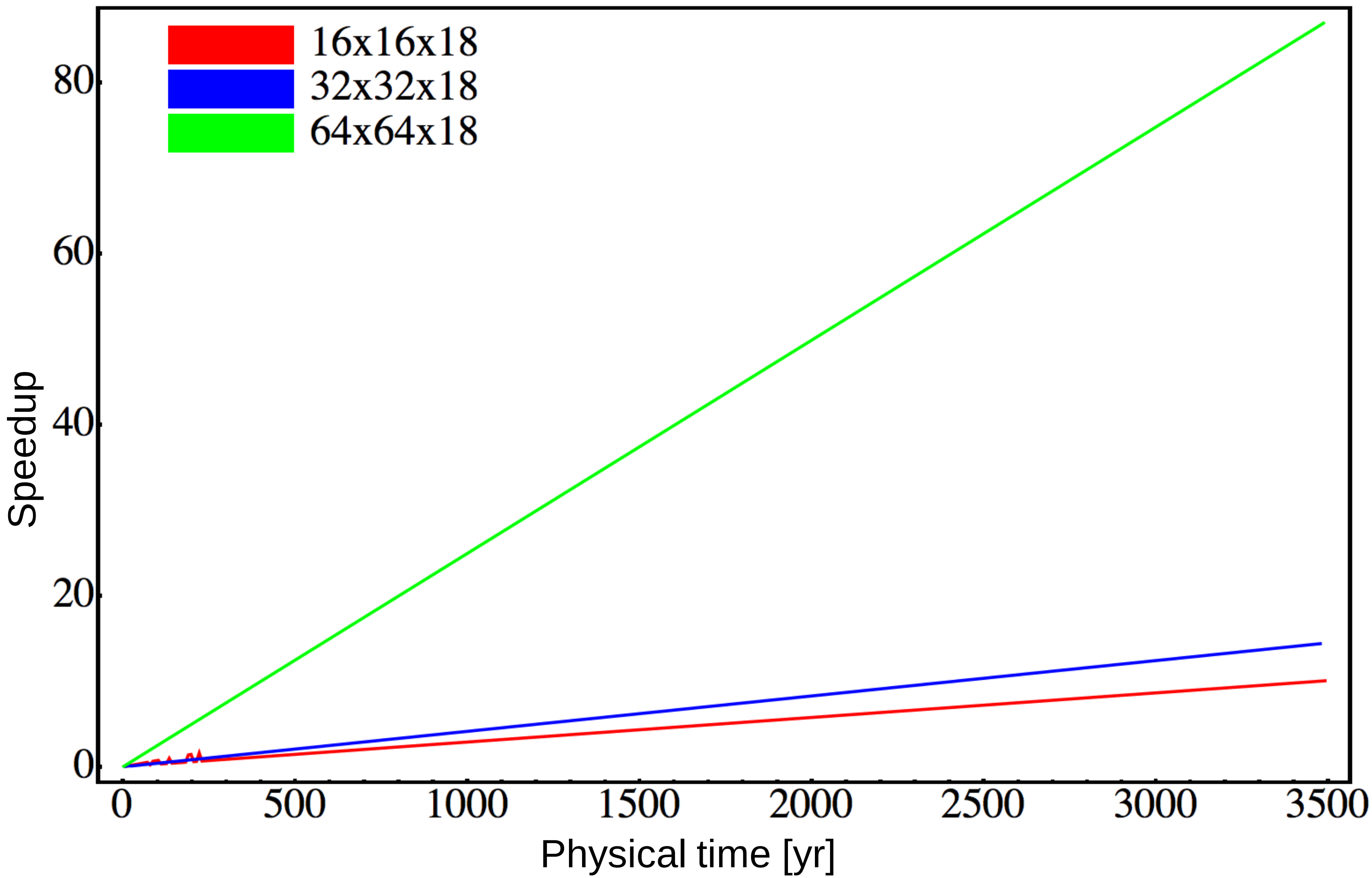
‘Normal’ Spin-up run (16x16x18, PGOM)



Comparison of methods (16x16x18)



Speedup of JFNK method



Conclusion and future work

Implicit techniques have a great application potential in ocean modeling:

- Bifurcation analyses possible for global ocean-climate models with hor 2° ontal resolution.

  ★ Future: Improved tailored parallel preconditioners

- JFNK methods can provide efficiency for spin-up problems using explicit models

  ★ Future: Application to statistical steady states