Interrogative Simulation and Uncertainty Quantification of Multi-Disciplinary Systems

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Background

- Simulation, testing and prediction of the responses of multidisciplinary systems are extremely difficult
  - Nonlinear competing mechanisms (e.g. aeroelastic phenomena)
  - Multiple coexisting attractors, supercritical and subcritical bifurcations , limit cycles, etc.

- Global Metrics
  - Flutter speed, LCO amplitude
  - Lift and drag forces on bluff bodies
  - Pull-in voltage and displacement of MEMS

- Dependence on physical parameters is not straightforward.

Motivation

- Difficult and expensive to use brute- force computations to relate global response metrics to system parameters
- Need for reduced-order models
  - Reproduce results of high-fidelity simulations or experiments
  - Relate global metrics to system parameters
  - Quantify system response uncertainties
  - Implement control strategies (flow control)
- Physically motivated models
  - Make use of scientific principles, feasible for certain systems (simple structures, …)
- Black-box models
  - No clear relation between physics and system (networks, …)
  - Match the behavior with that of a mathematical model

Objective

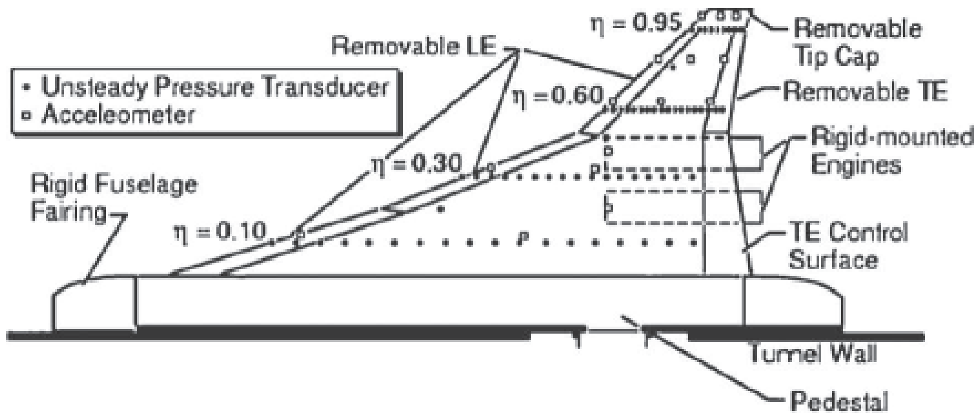
Interrogative Testing/Simulation and Uncertainty Quantification of Multi-Disciplinary Systems

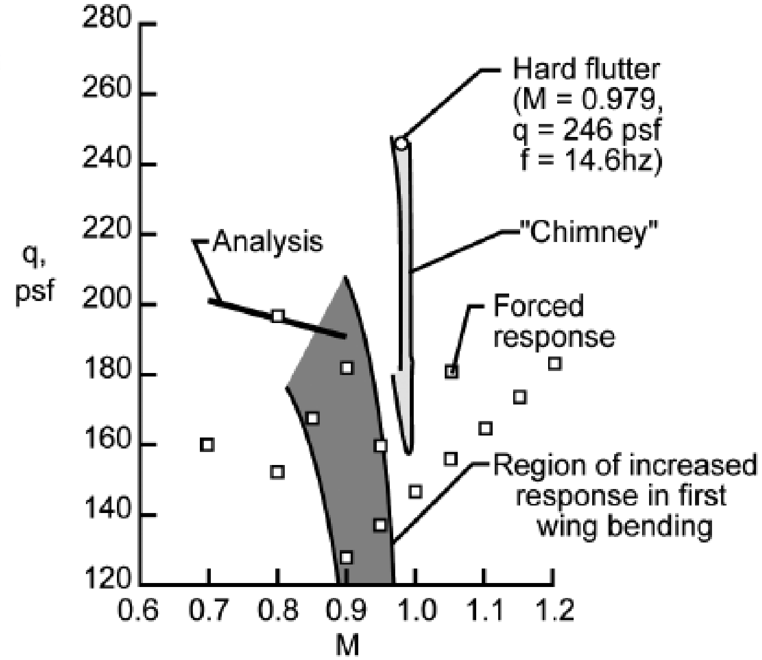
- Approach - Design the experiment/simulation to
  - Exploit specific behavior
  - Draw more information from data
  - Relate model features to system dynamics

- Examples
  - System identification
  - Global metrics
  - Uncertainty quantification
  - Control (flow control)

System Identification

HSCT – FSM



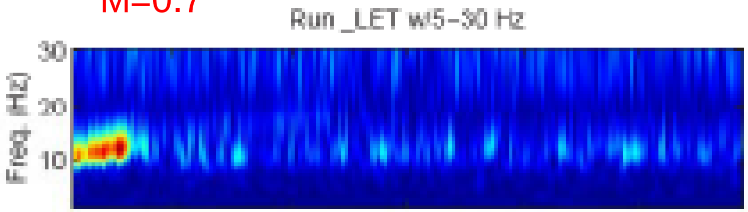


System Identification

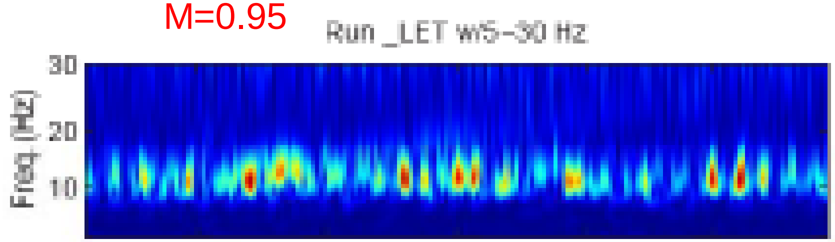
HSCT – FSM

Wing response to sweep excitation of control surface

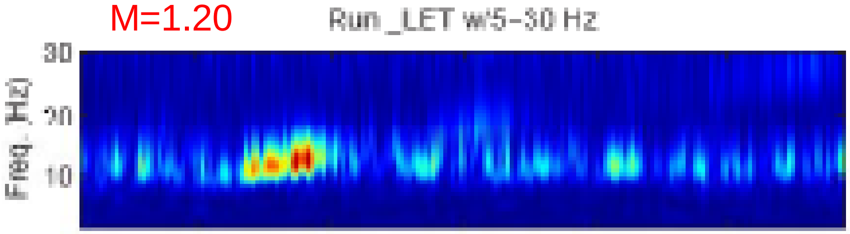
M=0.7



M=0.95



M=1.20

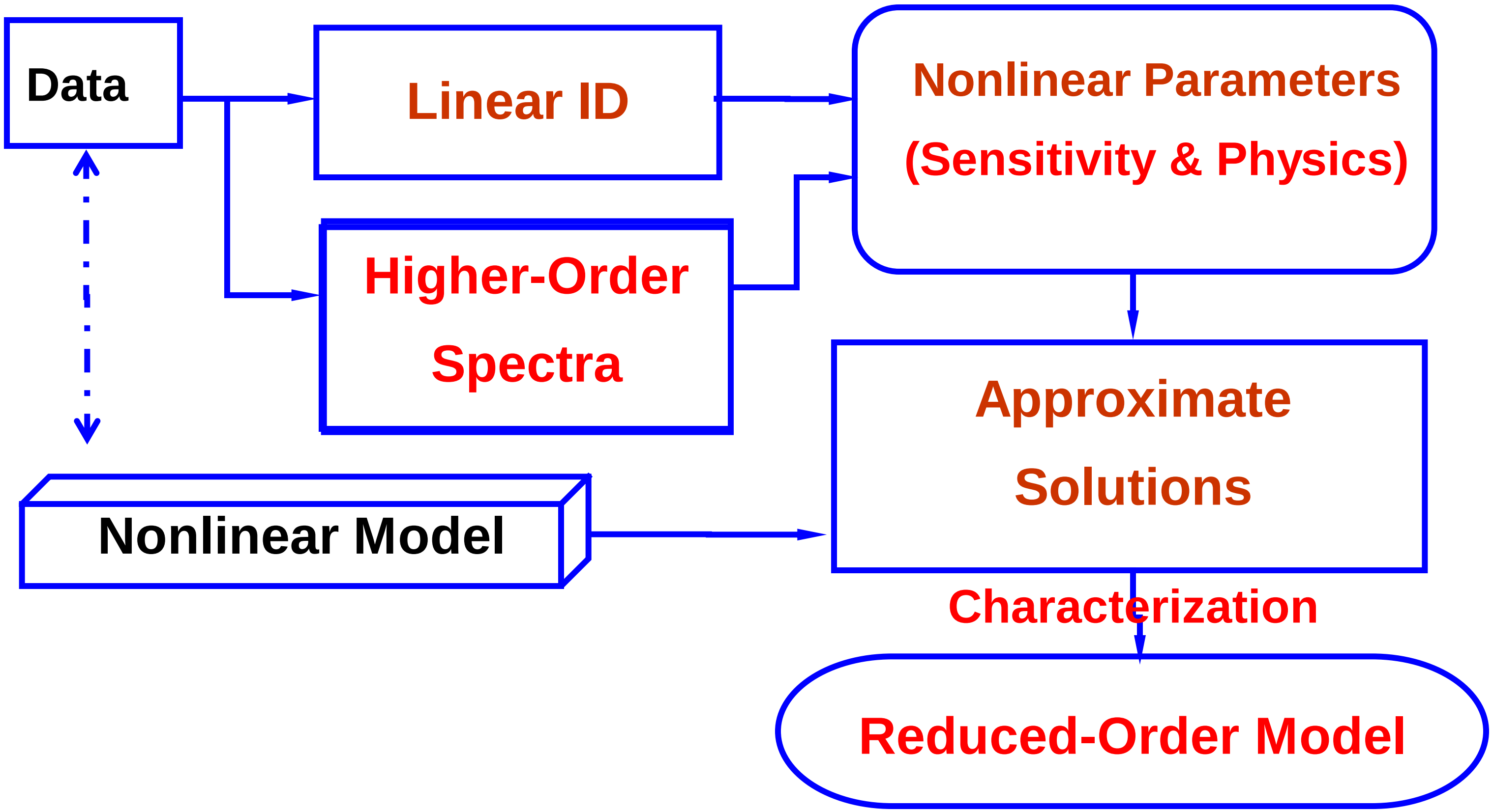


Response is different in subsonic, transonic and supersonic flow regimes.

Exploitation of specific behavior in different regimes

Exploitation of Nonlinearities

Nonlinear Response

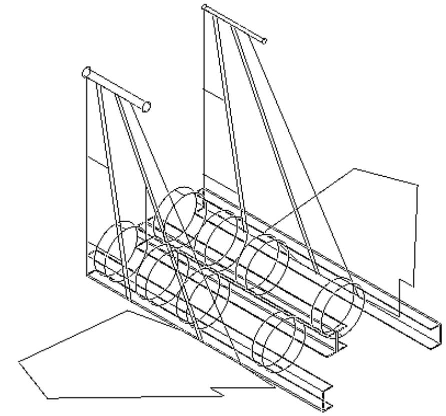


Structure Vibrations, Ship Motions, Lift and Drag on Cylinders

System Identification

F-15 tail Assembly Model

- Model
  - 1/16 dynamically scaled model
  - 0.355 *m* x 0.28 *m* x 0.482 *m*
  - Series of aluminum channels, brass rings, composite plates, metal masses,...



- Excitation
  - Mounted on a 250-lb shaker
  - Exploit parametric resonance to maximize influence of nonlinearities
- Identification Procedure
  - Series of experiments
  - Combination of approximate solutions for governing equations and data analysis
- Control and effects of uncertainties

System Identification

Governing Equations

u_1, u_2 : modal coordinates (micro strains (μs))

ω_1, ω_2 : natural frequencies (*radian / s*)

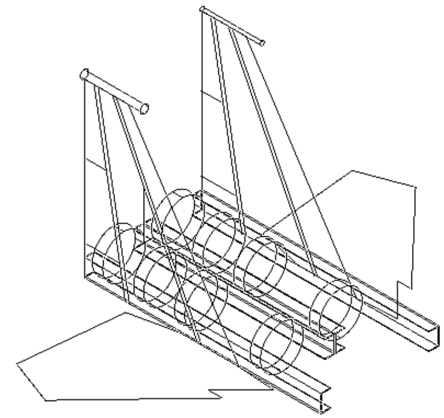
$_1$, $_2$: linear damping coefficients (*radian / s*)

$_3$, $_4$: aerodynamic damping coefficients.(*1/μs*)

$_1$, $_2$: coefficients of cubic nonlinearity (*1/μs^2*s^2*)

$_1$, $_2$: transmissibility terms (*1/g*s^2*)

k : coupling term. (*1/s^2*)



$$\ddot{u}_1 + \omega_1^2 u_1 + 2\mu_1\dot{u}_1 + \alpha_1 u_1^3 + \mu_3\dot{u}_1|\dot{u}_1| - k(u_2 - u_1) = u_1 \eta_1 F \cos(\Omega t + \tau_1)$$

$$\ddot{u}_2 + \omega_2^2 u_2 + 2\mu_2\dot{u}_2 + \alpha_2 u_2^3 + \mu_4\dot{u}_2|\dot{u}_2| - k(u_1 - u_2) = u_2 \eta_2 F \cos(\Omega t + \tau_2)$$

Linear Identification: Frequency Response Functions

f_1, f_2 : 9.135 Hz, 9.05 Hz

damping ratios: 0.014 and 0.019 - will also be identified from parametric excitation experiments

System Identification

Nonlinear Identification

$$\ddot{u}_1 + \omega_1^2 u_1 + 2\mu_1\dot{u}_1 + \alpha_1 u_1^3 + \mu_3\dot{u}_1|\dot{u}_1| - k(u_2 - u_1) = u_1\, \eta_1 F \cos(\Omega t + \tau_1)$$

$$\ddot{u}_2 + \omega_2^2 u_2 + 2\mu_2\dot{u}_2 + \alpha_2 u_2^3 + \mu_4\dot{u}_2|\dot{u}_2| - k(u_1 - u_2) = u_2\, \eta_2 F \cos(\Omega t + \tau_2)$$

Uncoupled experiments – one tail fixed (*k*=0); $\Omega = 2\omega_1 + \varepsilon\sigma_1$

Multiple scales $u_1 = A_1(\varepsilon t)e^{i\omega_1 t} + \varepsilon(.....) + ... + cc$ $\quad A_1 = \frac{1}{2} a e^{i\beta}$

$$a' = -\mu_1 a - \boxed{\frac{4\mu_3}{3\pi}\omega_1 a^2} + \frac{\eta_1 F a}{4\omega_1}\sin\gamma$$

$$a\beta' = \frac{3\alpha_1}{8\omega_1}a^3 - \frac{\eta_1 F a}{4\omega_1}\cos\gamma$$

$$\gamma = \sigma_1 T_1 - 2\beta + \tau_1$$

Steady state solution

$a' = 0$ and $\gamma' = 0$

$$\varsigma_1 = 0.01357, \quad \mu_3 = 3.157\times 10^{-4}\,\mu\varepsilon^{-1},$$

$$\alpha_1 = -3.675\times 10^{-2}\frac{1}{s^2\mu\varepsilon^2}, \quad \eta_1 = 161.54\frac{1}{gs^2}$$

$$\varsigma_2 = 0.01856, \quad \mu_4 = 1.9588\times 10^{-4}\,\mu\varepsilon^{-1},$$

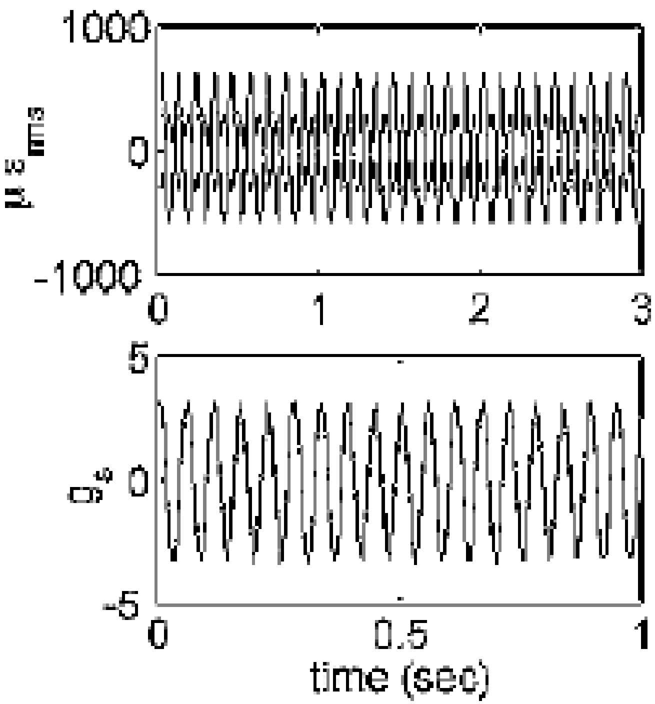
$$\alpha_2 = -2.977\times 10^{-3}\frac{1}{s^2\mu\varepsilon^2}, \quad \eta_2 = 275.12\frac{1}{gs^2}$$

System Identification

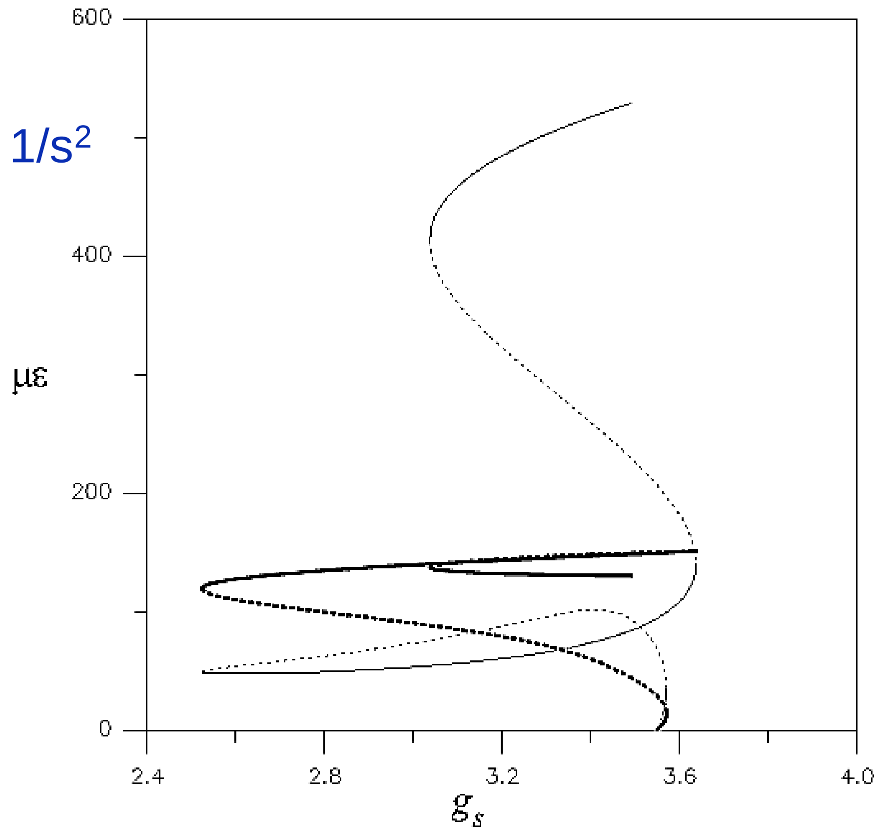
Nonlinear Identification

$$\ddot{u}_1 + \omega_1^2 u_1 + 2\mu_1\dot{u}_1 + \alpha_1 u_1^3 + \mu_3\dot{u}_1|\dot{u}_1| - k(u_2 - u_1) = u_1\ \eta_1 F \cos(\Omega t + \tau_1)$$

$$\ddot{u}_2 + \omega_2^2 u_2 + 2\mu_2\dot{u}_2 + \alpha_2 u_2^3 + \mu_4\dot{u}_2|\dot{u}_2| - k(u_1 - u_2) = u_2\ \eta_2 F \cos(\Omega t + \tau_2)$$

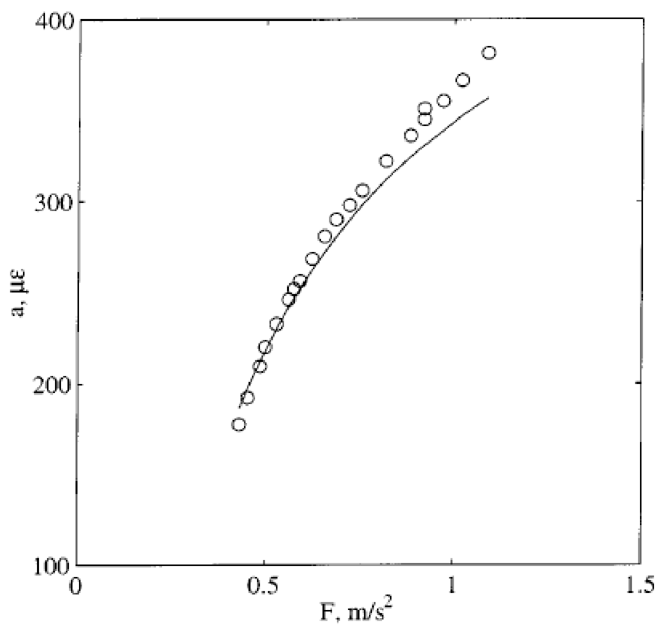


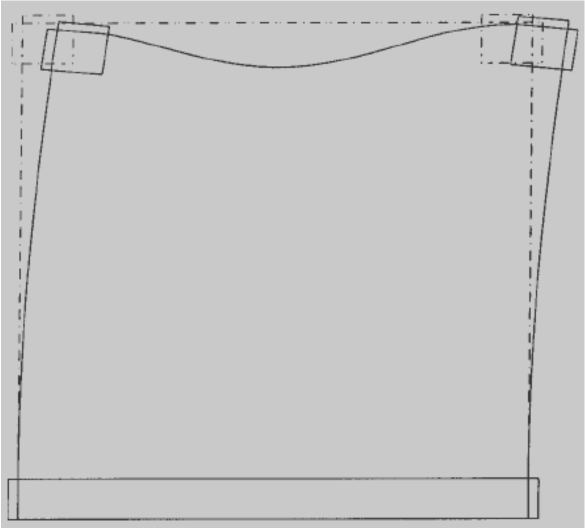
K = 87 1/s^2



Three-beam Frame

$$\ddot{u}+\omega^2 u+2\varepsilon^2\mu_1\dot{u}+\boxed{\varepsilon^2\mu_2\dot{u}\,|\,\dot{u}\,|}$$
$$\varepsilon\alpha_2 u^2+\varepsilon^2\alpha_3 u^3+\varepsilon^2\delta\ddot{u}^2 u=$$
$$\varepsilon\eta_1 f\cos(\Omega t+\tau_e)+\varepsilon^2\eta_2 f u\cos(\Omega t+\tau_e)$$





Roll Instabilities in Ship Motions

- Data Source: Large-Amplitude-Motions Program (LAMP)
- Nonlinear Model: 3 DOF: heave, pitch, roll
- Approximate Solution: Method of Multiple Scales
- Higher-order spectral analysis: spectral parameters and in particular phase measurement
- Account for multiple phase quantities

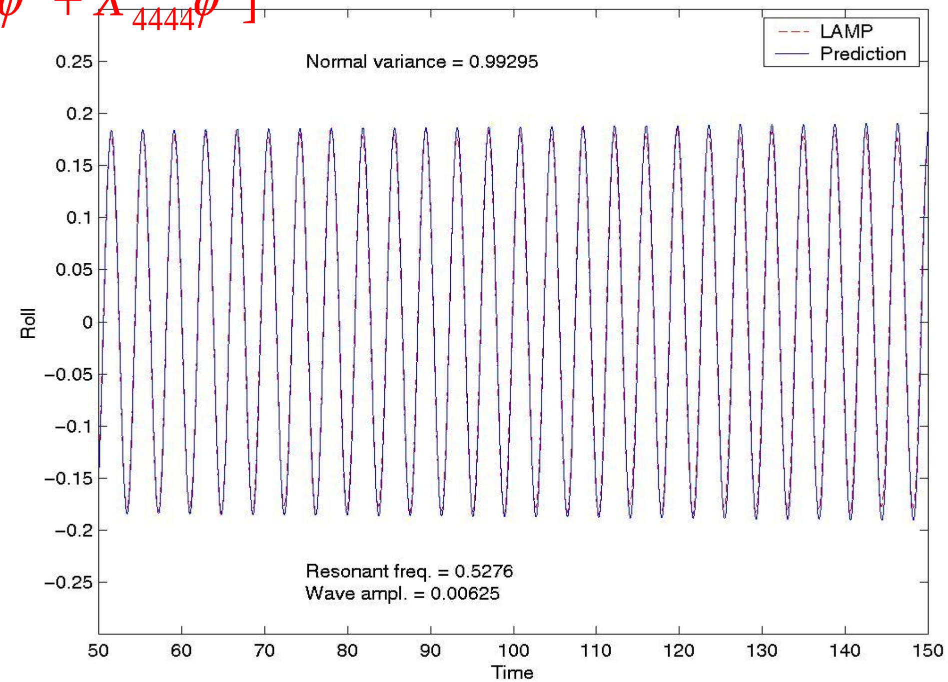
Roll Instabilities in Ship Motions

$$m_{33}\ddot{z} + m_{35}\ddot{\theta} + k_{33}z + k_{35}\theta = -c_{33}\dot{z} - c_{35}\dot{\theta} + \varepsilon f_{zp}\cos(\Omega_p t + \tau_{zp})$$

$$m_{53}\ddot{z} + m_{55}\ddot{\theta} + k_{53}z + k_{55}\theta = -c_{53}\dot{z} - c_{55}\dot{\theta} + \varepsilon f_{\theta p}\cos(\Omega_p t + \tau_{\theta p})$$

$$m_{44}\ddot{\phi} + k_{44}\dot{\phi} = -c_{44}\dot{\phi} + \varepsilon f_{\phi p}\cos(\Omega_p t + \tau_{\phi p})$$

$$\varepsilon[2R_{434}z\phi + 2R_{454}\theta\phi] - \varepsilon^2[V_{4444}\phi^3 + X_{4444}\dot{\phi}^3]$$

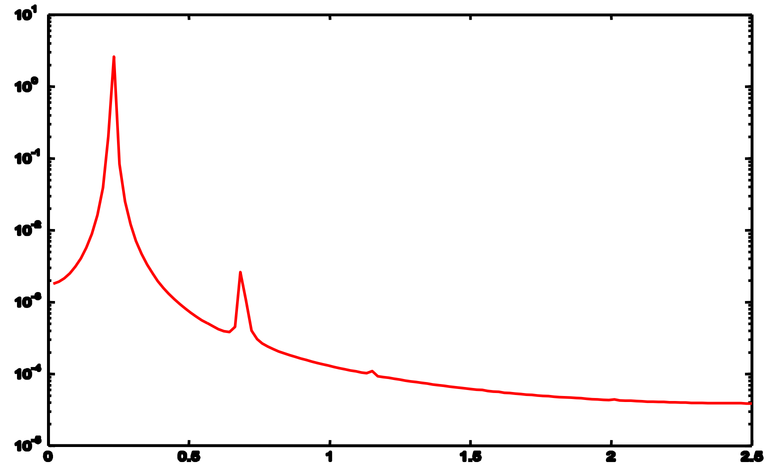


Lift on Stationary Circular Cylinders

- Data Source: Numerical simulation

- Nonlinear Model

Represent response behavior



- Approximate Solution: Method of Multiple Scales

- Higher-order spectral analysis (trispectrum): spectral parameters and in particular phase measurement

Lift on Stationary Circular Cylinders

Lift Modeling and Approximate Solution

- Rayleigh Equation

$$\ddot{l} + \omega_s^2 l - \mu_r \dot{l} + \alpha_r \dot{l}^3 = 0$$

$$l = a\cos(\omega_s t + \beta) + \frac{\alpha_r \omega_s}{32} a^3 \cos(3\omega_s t + 3\beta - \frac{1}{2}\pi)$$

$$\phi(3\omega_s) - 3\phi(\omega_s) = -\pi/2$$

- van der Pol Equation

$$\ddot{l} + \omega_s^2 l - \mu_v \dot{l} + \alpha_v l^2 \dot{l} = 0$$

$$l = a\cos(\omega_s t + \beta) + \frac{\alpha_v}{32\omega_s} a^3 \cos(3\omega_s t + 3\beta + \frac{1}{2}\pi)$$

$$\phi(3\omega_s) - 3\phi(\omega_s) = \pi/2$$

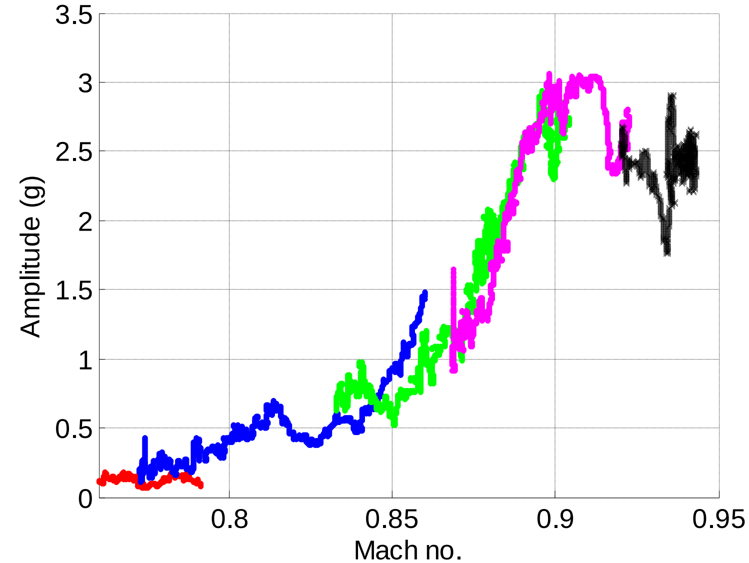
F16 flight testing

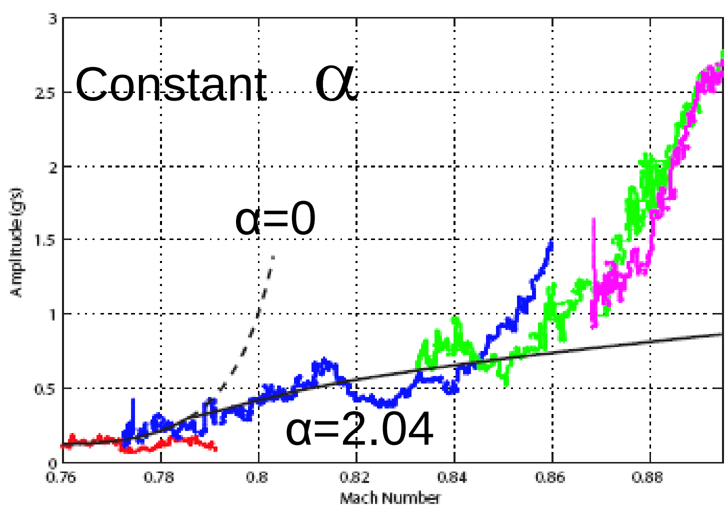
Relate global metrics to system parameters

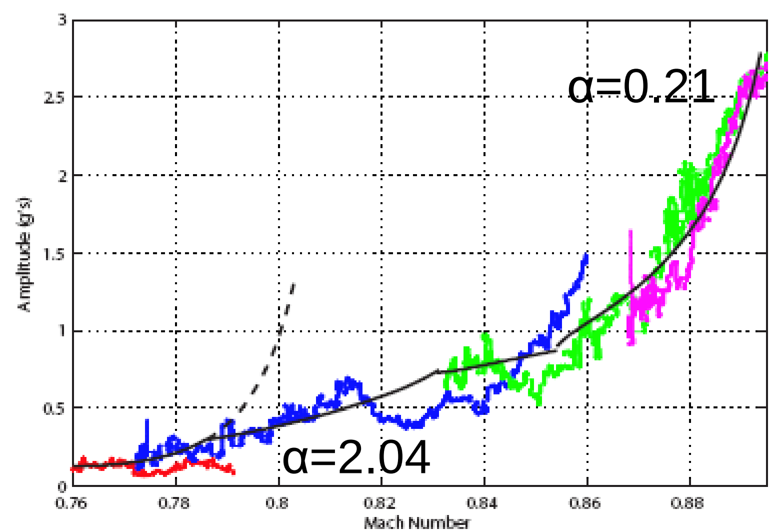
LCO amplitude vs. Mach number

$$\dot{a} = \kappa(M - M_c)a - \alpha a^3$$

$$\dot{\theta} = \omega + \chi a^2$$





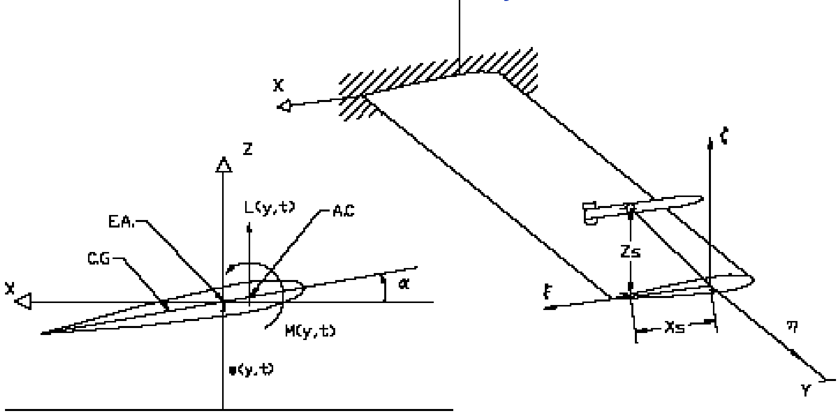


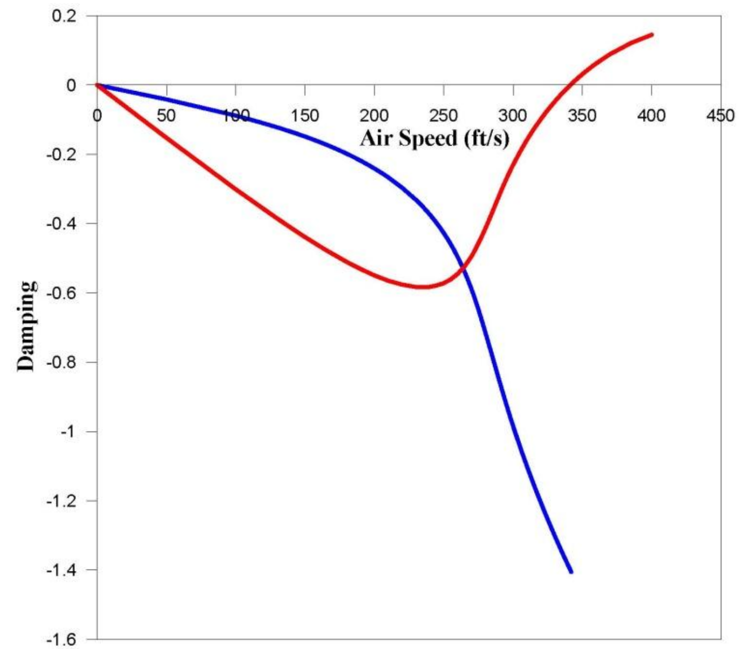
Goland wing with Store

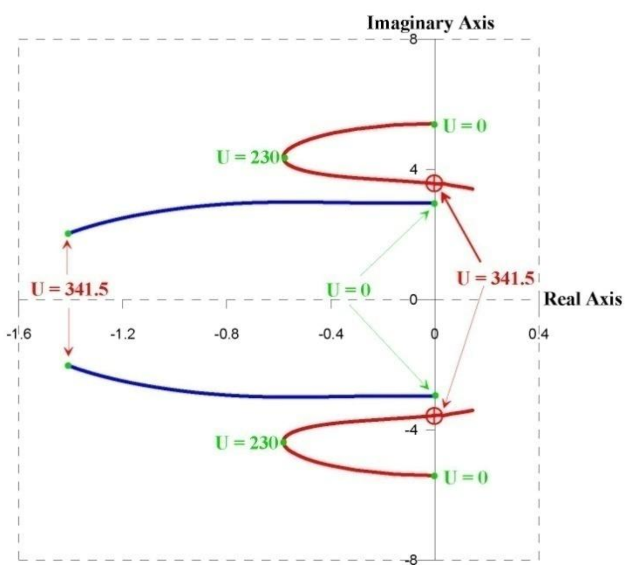
Flutter Speed

Virginia Tech

Beran et al, Nonlinear Dynamics 37: 323-339,







The ***complex conjugate*** representing the second mode cross the imaginary axis transversely, and the instability is due to a ***Hopf bifurcation***.

Goland wing with Store

Flutter speed variations with uncertainties in damping

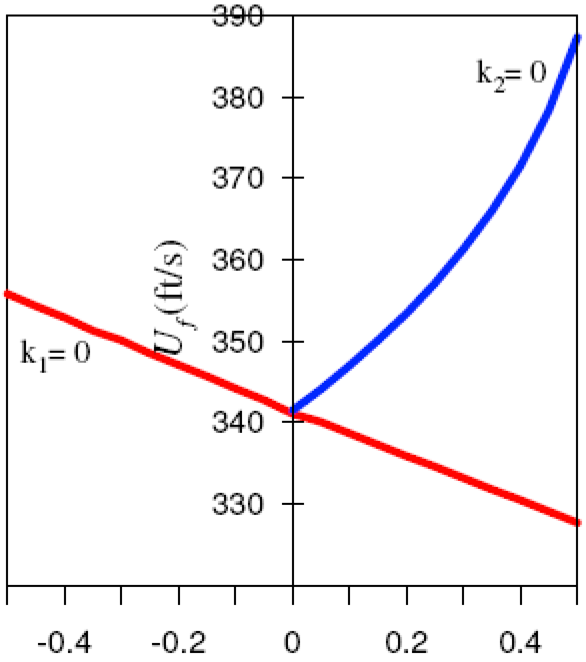
$$M\ddot{q}(t) + \frac{UL}{\tau}(D + k_1 D_1 + k_2 D_2)\dot{q}(t) + (K_s + K_e U^2)q = 0$$

$$U = U_f + \epsilon\, U_1$$

$$q(t;\epsilon) = q_1(T_0, T_1) + \epsilon\, q_2(T_0, T_1) + \cdots$$

$$q_1 = A(T_1) v_f e^{i\omega_f T_0} + \bar{A}(T_1)\bar{v}_f e^{-i\omega_f T_0}$$

$$A(T_1) = \frac{1}{2} r(T_1) e^{i\beta(T_1)}$$



$$\frac{d\,r}{d\,t} = [0.00190782(U - U_f) - 0.00218554k_1 + 0.00218554k_2]\, r$$

$$\frac{d\,\beta}{d\,t} = [0.0376153(U - U_f) - 0.00104746k_1 + 0.0013755k_2]$$

Variations in flutter speed due to uncertainties in the damping parameters k_1 and k_2

$$U_1 = U - U_f = 1.14558k_1 - 0.345961k_2$$

Goland wing with Store

Nonlinear parameter uncertainty

Variations in the flutter speed due to uncertainties in other parameters

$$U_1 = U - U_f = \Gamma_1 k_1 - \Gamma_2 k_2 + \Gamma_3 \delta M_s + \Gamma_4 \delta D_x + \Gamma_5 \delta D_y$$

Parameters Uncertainty and Effects on Global Measures – Nonlinear analysis

$$M\ddot{q}(t) + \frac{UL}{\tau}(D + k_1 D_1 + k_2 D_2)\dot{q}(t) + (K_s + K_e U^2)q = N_s(q) + N_{store}(q) + N_{aero}(q)$$

$$v_f^*(2i\omega_f M + \frac{U_f L}{\tau} D) v_f A' + v_f^* \left(\frac{i\omega_f L}{\tau} D + 2U_f K_e \right) v_f U_1 A$$
$$+ \frac{i\omega_f U_f L}{\tau} v_f^* (k_1 D_1 + k_2 D_2) v_f A = (\Lambda_s + \Lambda_{store} + \Lambda_{aero}) A^2 \bar{A}$$

$$\frac{d\,r}{d\,t} = -\left[\kappa_1 (U - U_f) + \gamma_{11} k_1 + \gamma_{12} k_2\right] r - \alpha r^3$$
$$\frac{d\,\beta}{d\,t} = -\left[\kappa_2 (U - U_f) + \gamma_{21} k_1 + \gamma_{22} k_2\right] + \chi r^2$$

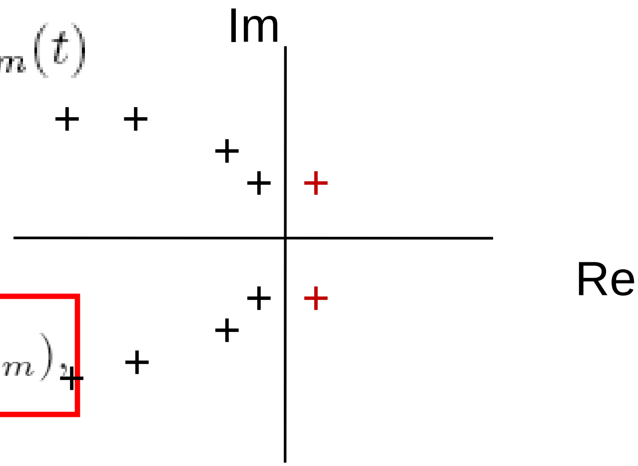
ROM of the Velocity Field

- Project Navier-Stokes equation onto the POD modes
- ROM (M =10, Re=100)
  - Ordinary-differential equations
  - Nonlinear system

Linear stability – eigenvalues
- Pair in right-half plane
- Hopf bifurcation

$$\mathcal{B} = \mathcal{B}^{NL} + \frac{1}{\mathrm{Re}_D}\mathcal{B}^{L}$$

$$\dot{q}_k(t) = \mathcal{A}_k + \sum_{m=1}^{M} \mathcal{B}_{km} q_m(t) + \sum_{m=1}^{M}\sum_{n=1}^{M} \mathcal{C}_{kmn} q_n(t) q_m(t)$$



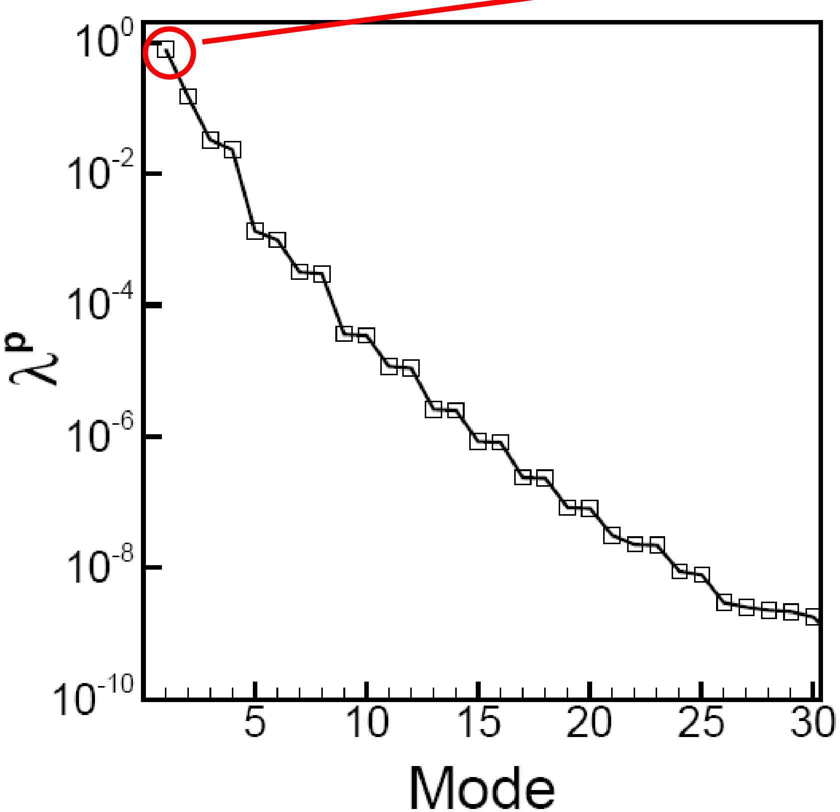
$$\mathcal{A}_k = \frac{1}{\mathrm{Re}_D}(\Phi_k, \nabla^2 \bar{\mathbf{u}}) - (\Phi_k, \bar{\mathbf{u}}.\nabla\bar{\mathbf{u}}),$$

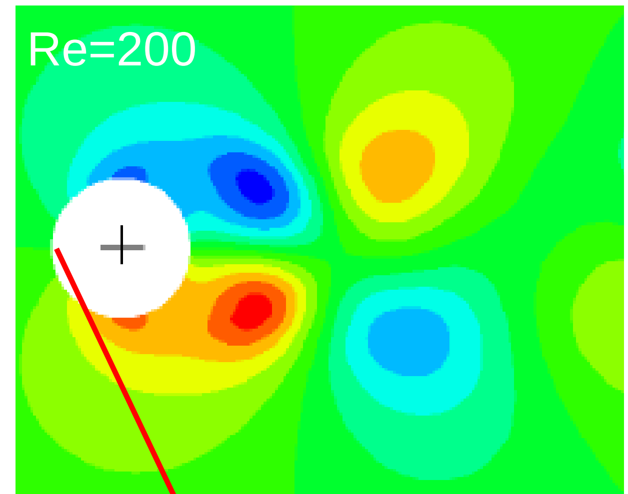
$$\mathcal{B}_{km} = -(\Phi_k, \bar{\mathbf{u}}.\nabla\Phi_m) - (\Phi_k, \phi_m.\nabla\bar{\mathbf{u}}) + \frac{1}{\mathrm{Re}_D}(\Phi_k, \nabla^2\phi_m),$$

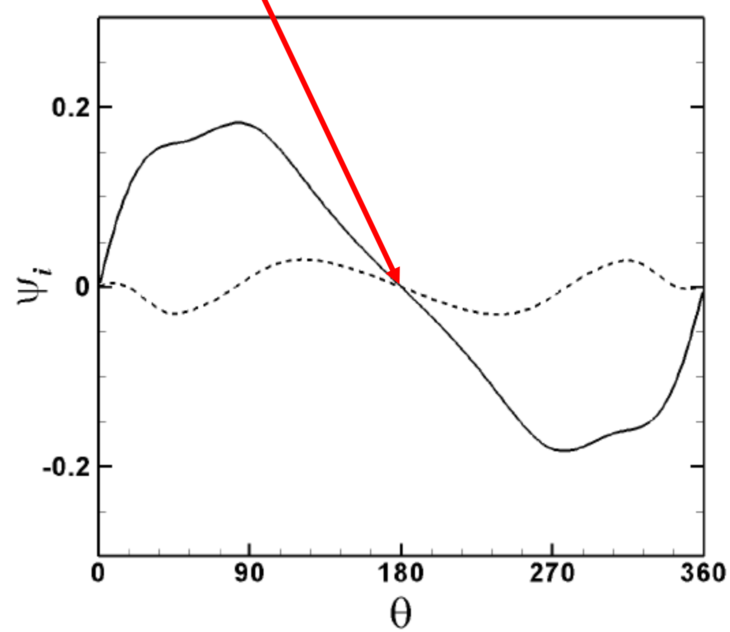
$$\mathcal{C}_{kmn} = -(\Phi_k, \phi_m.\nabla\Phi_n),$$

Actuators Placement

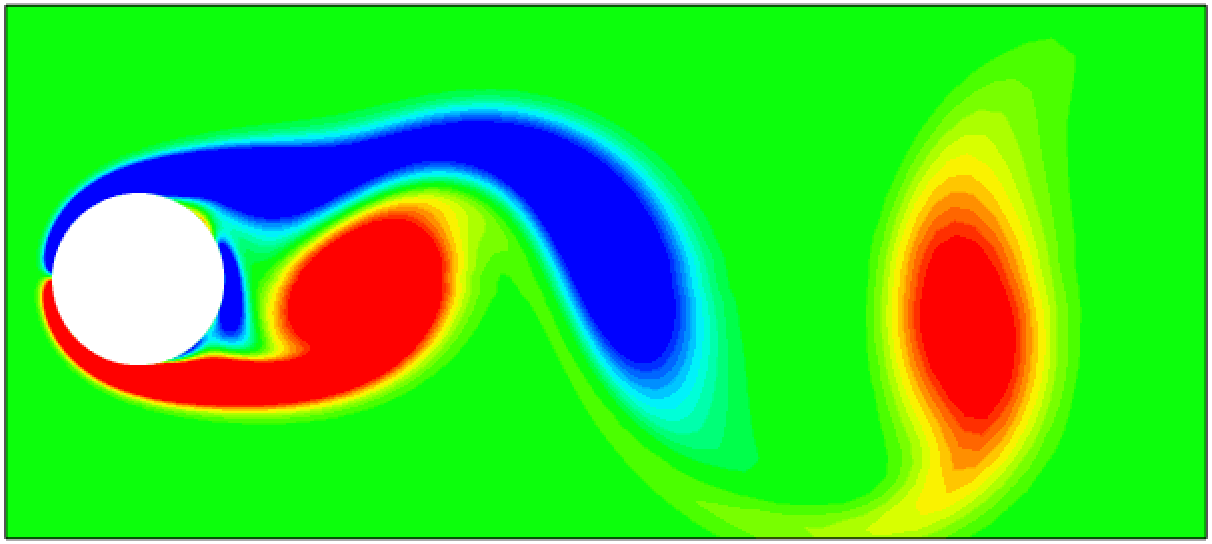
- Pressure POD - Eigenvalues
  - Use dominant mode
  - Optimum location







Control of vortex shedding



Summary

- System characterization and ROM development through exploitation of physical behavior
- Flight testing: Combination of understanding of physical behavior and data analysis to develop a model with parameters that depend on system variables and system characterization
- Goland wing + store
  - Modeling stages
  - Uncertainty quantification framework
- Use of ROM derived physical characteristics to control vortex shedding