APPLICATION OF PROPER ORTHOGONAL DECOMPOSITION FOR THE IDENTIFICATION OF WET MODES OF MARINE STRUCTURES

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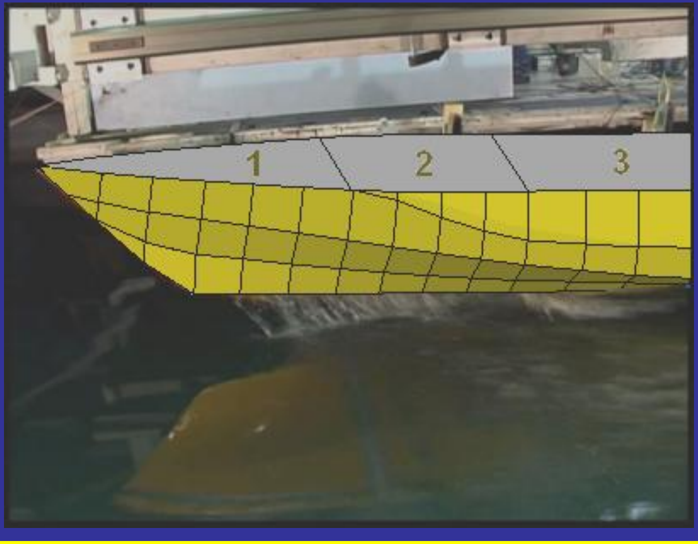
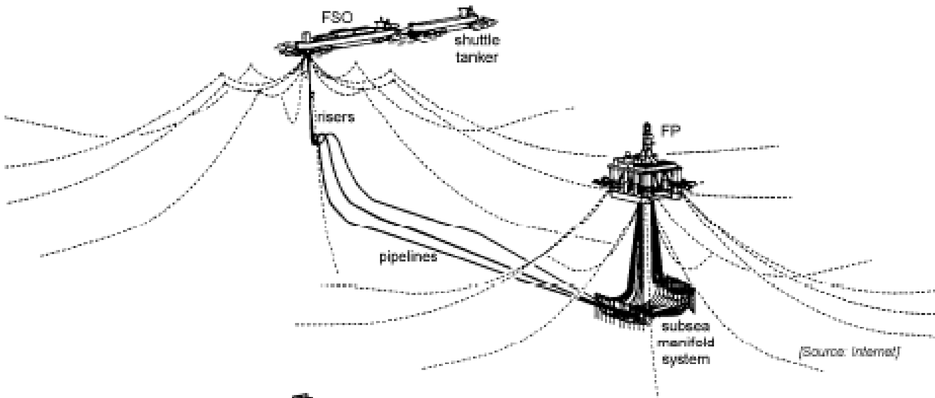


Summary

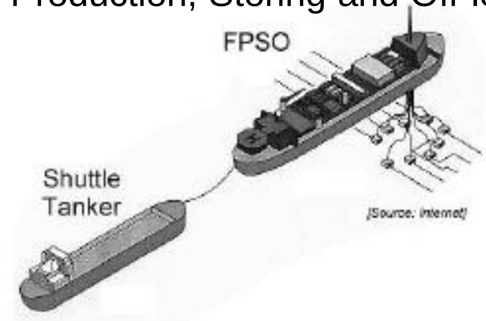
- This presentation concerns with the use of Proper Orthogonal Decomposition (POD) to obtain the "wet operational modes" of ships / floating marine structures
- The data analysed with POD are obtained with tests on a physical scaled model of a real – ship
- The convergence of Proper Orthogonal Modes (POMs) to Linear Normal Modes is addressed
- These modes constitute a basis to obtain a reduced order model

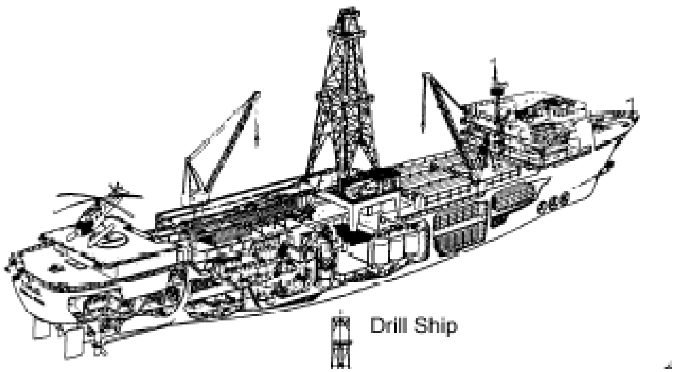
Multi disciplinary features of marine structures

Floating Storage and Off-loading unit

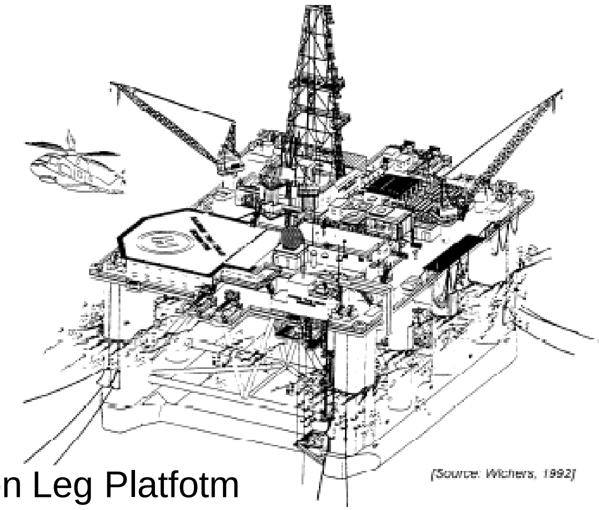


Floating Production, Storing and Off-loading unit





Marine Structures



Tension Leg Platfotm

Loads

Environment
water (waves, current)
winds

Links to
Ground / Sea Bottom

Nature of wet modes

$$(\mathbf{M}+\mathbf{A})\ddot{\mathbf{q}}+(\mathbf{C}+\mathbf{B})\dot{\mathbf{q}}+(\mathbf{K}+\mathbf{R})\mathbf{q}=\hat{\mathbf{f}}^{FK}+\hat{\mathbf{f}}^{d}$$

Loads

Environment
water (waves, current)
winds

Links to
Ground / Sea Bottom

Depend. on local draft and its time derivatives (nonlinear)

Added (linear) Mass, Damping, Stiffness

Marine Structures

Elastic dofs

Floating marine structure (ship wet modes)

Cables / Risers dynamics

Moored & floating marine Structures (multi-body elastic wetted – modes)

Motivation for dry / wet modes identification

From design to construction

• Marine structures are in general huge and topologically complex. Uncertainties in finite element models of the "dry" structure may be relevant.

Full-scale marine structures

Elastic modes in water may be significantly different. Frequency reduction and damping increase are usually important. Added damping is difficult to predict for elastic modes.

Full-scale marine structures in actual service / operational conditions

- Lack of physical modeling of the dependence of vibration modes on
    - ship speed
    - waves added damping
    - strong nonlinear interactions (i.e.,water exit / slamming events)
- Damage identification related to changes in modal properties

Motivation for dry / wet modes identification

Scaled model of marine structures for testing

- Experimental Fluid-Structure Interaction demands for feasible exp. set-up, that is physical simplification of the more complex problem to retain only the sought after behaviour
- On the other hand, hydroelastic scaling accordingly to Froude similarity must be assessed and verified. This may be done as in the present case on the basis of modal correspondence
- Thus, experimental reduced – order scaled physical models to reproduce full-scale dynamics of ships:
    - ship bending behavior with 1-D beam model
    - continuous fluid loading through a limited set of forces

Model reduction

Real Ship

3D Finite Elements Model

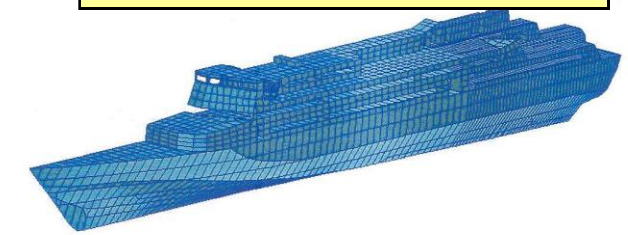
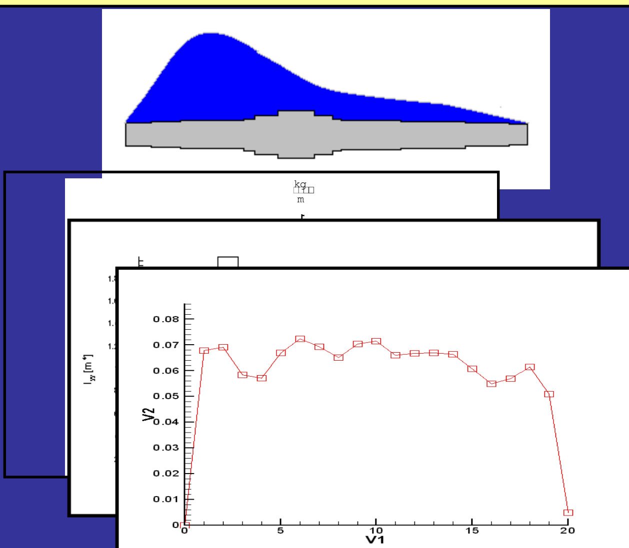


Figura 2.1 – *Modello FEM MDV-3000*

Reference Ship Data

- ❑ mass distribution
- ❑ bending stiffness distribution
- ❑ shear area distribution
- ❑ natural frequencies, mode shapes

1D Equivalent Beam (Timoshenko Beam)

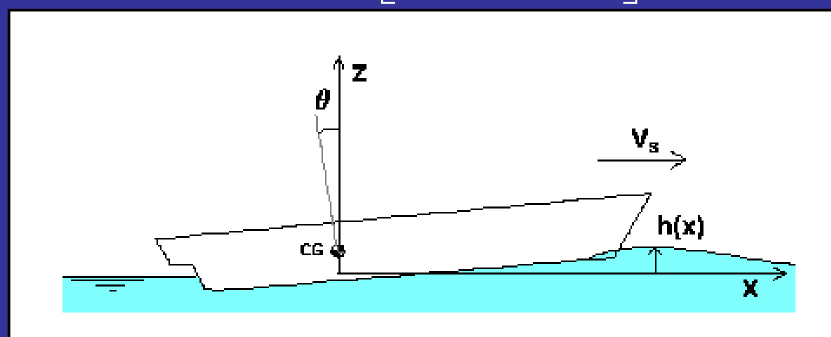


Governing equations

Equivalent beam Structure
(Euler beam for sake of simplicity)

$$\mu(x)\frac{\partial^2 w(x,t)}{\partial t^2}+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]=f(t;z;\theta;w)$$

Strip-theory for loads



ADDED Mass ADDED Damping

Governing model:
Lighthill equation based on mass conservation)

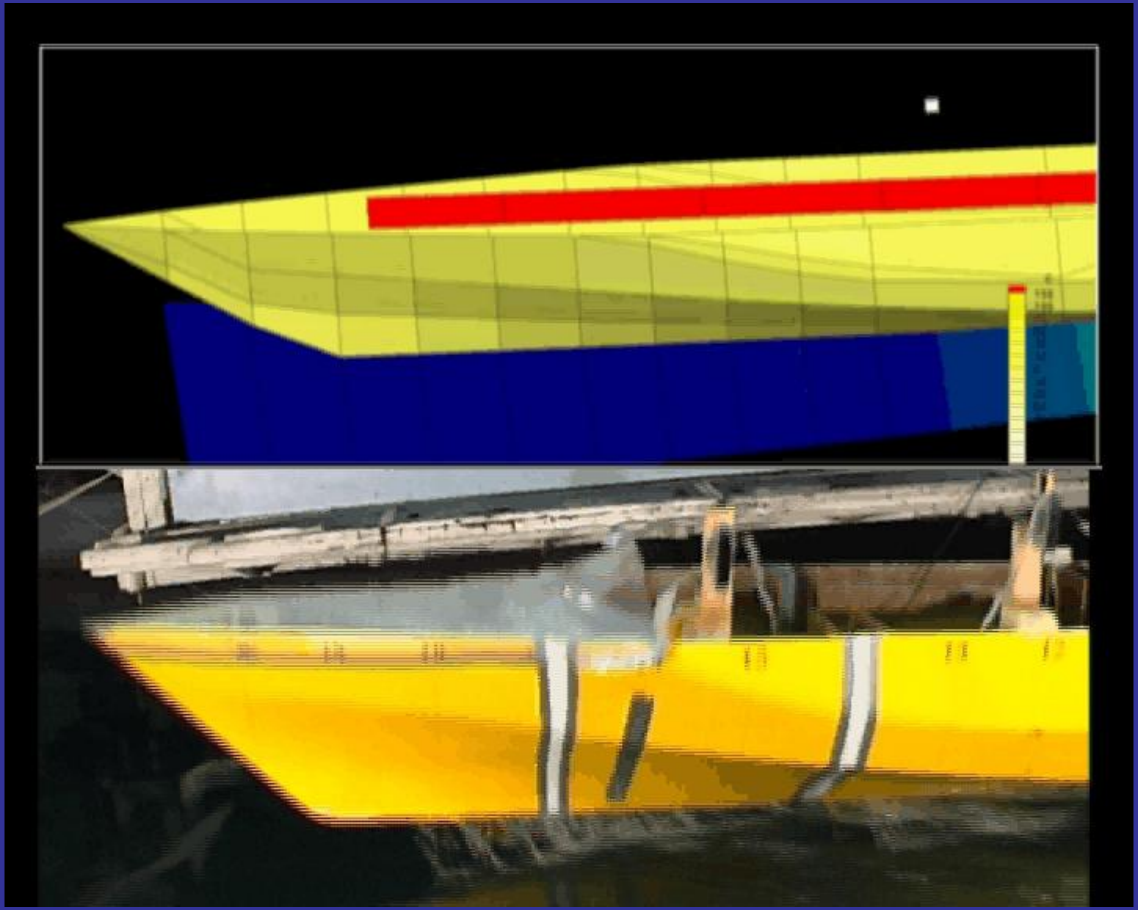
$$\mu(x)\frac{\partial^2 w(x,t)}{\partial t^2}+a_{33}(\xi;x)\frac{D^2 w(x,t)}{Dt^2}+\frac{Da_{33}(\xi;x)}{Dt}\frac{Dw(x,t)}{Dt}+$$
$$+\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2 w(x,t)}{\partial x^2}\right]+\rho g b(\xi;x)w(x,t)=f(t;z;\theta)$$

ADDED Stiffness

$D/Dt(.)=\partial/\partial t+U\,\partial/\partial x$ *is the material derivative*

Identification of scaled ship wet - modes

Experiment virtual reconstruction



Sea-keeping test in model basin

The aims of the present analysis are:

- validate reduced – order physical test model with head waves excitation on the basis of displayed modes and associate frequencies

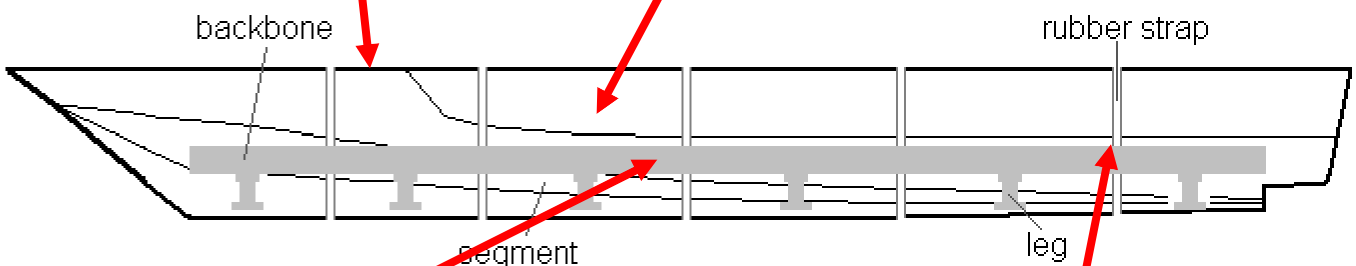
- explore the possibility to identify damping for the bending vibration modes

Elastic model concept for vibration analysis

Full scale tests on ships, though necessary for final validation, present some drawbacks that still motivate extensive measures in laboratory (even if quite large like a model basin).

The hull is built in fiber glass. It is supposed to be rigid.

The hull is divided into segments.



The beam is the elastic backbone of the model.

Rubber straps provide water tight feature.

Segmented – model response

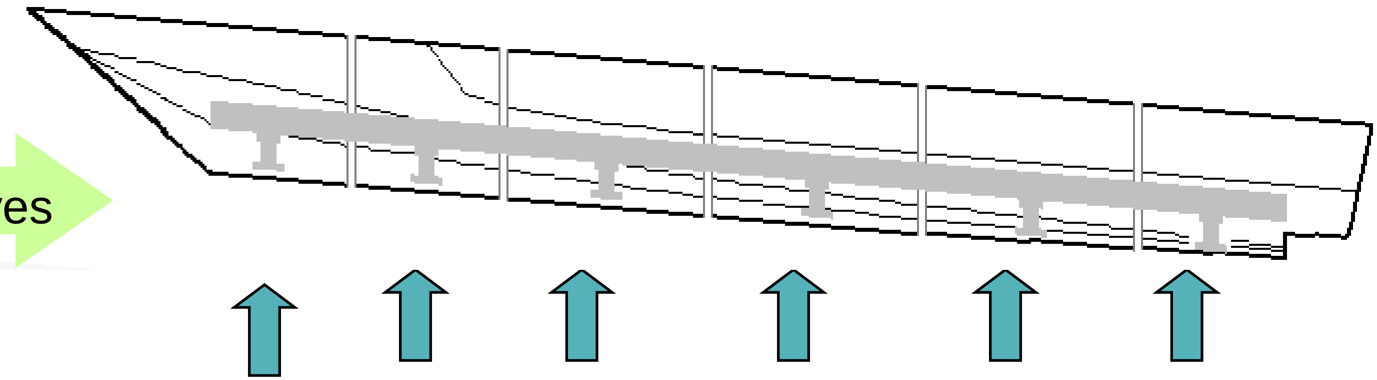
Optical system

Heave and pitch motion

Beam deformation

Strain gauges

Head waves



Wave loads exerted through the legs

How to “measure” wet-modes?

- Ships may be huge structure not easy to be excited, particularly when they are floating

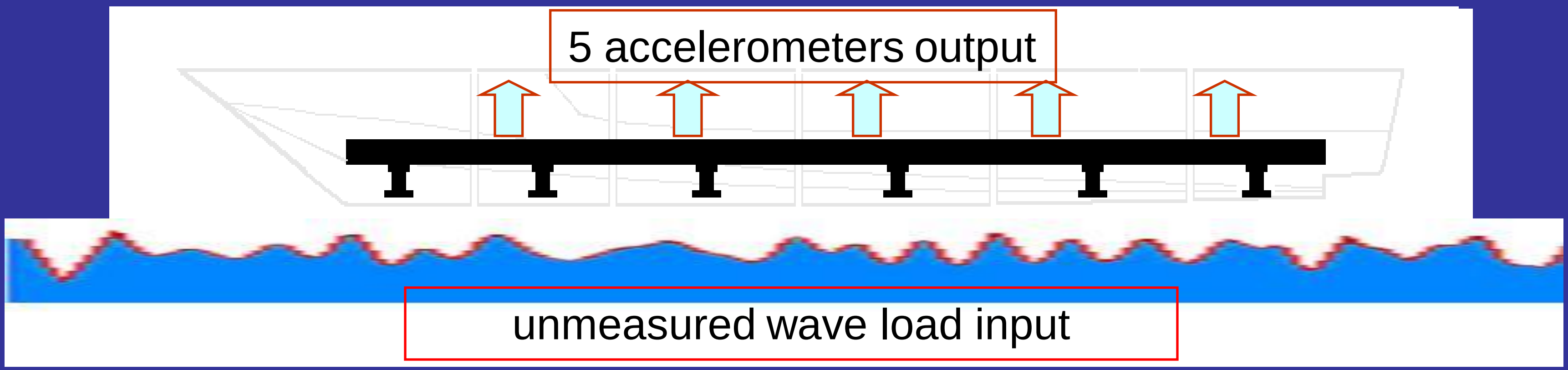
- Also at model scale tests, input / output techniques like FRF technique did not demonstrated to be nor easy nor sensitive in providing vibration modes

- Output – only techniques, largely applied in the past in other engineering fields, avoid this difficulty

- Moreover, in the case of marine structures, they exploit the natural source of excitation provided by the waves, benefiting of their intensity and randomness.

Acceleration analysis

The ambient (noise) excitation exploits the natural random excitation to which the structures may be subjected, like in this case the random wave excitation



If the ambient noise has a broad-band frequency representation (white noise), it is possible to determine the modal properties without measuring the excitation input.

From frequency domain to time domain…

- Output – only analysis has been developed in the field of civil engineer to provide operational modes for bridges, wind-turbines, buildings and so on.

- The work on operational modal analysis, promoted by Brincker et al. [2000] using frequency domain methods as that shown in the previous slides, has then meet the interests of a community of scientists and engineers as recent participations to IOMAC demonstrates.

- Nevertheless, a different approach (and a research community) is moving on a different path, combining output – only approach with a time – domain technique, i.e., the proper orthogonal decomposition.

- Starting from the original intuition of Lumley [1993], the works of Feeney [1998,…], Kirshen [2002,…] and their colleagues the proper orthogonal decomposition has been applied, interpreted and shown to be useful for the modal analysis of linear responding structures.

- Very recently, due to Chelidze et al. [2006], a variant of the POD, called smooth orthogonal decomposition (SOD), has been proposed to overcome some limitations.

Proper Orthogonal Decomposition

Consider a random scalar field of zero mean (e.g., w is displacement)

$$w(x,t) = \sum_{l=1}^{\infty} w_l(x) \boxed{\psi_l(x)}$$

← Basis functions

satisfying the PDE where L is the structural operator, ρ is mass density, f load

$$\rho(x,t)\,\ddot{w}(x,t) + L\,w(x,t) = f(x,t)$$

The POD provides the approximate representation of the field w that accounts for more energy than any other orthogonal function representation

$$\left(\int_0^{\ell} w(x,t)\,\psi(x)\,dx \right)^2 = \max \quad \text{with} \quad \int_0^{\ell} \rho(x)\,\psi^2(x)\,dx = 1$$

⇕

$$\int_0^{\ell} \rho(y) R(x,y)\,\psi(y)\,dy = \lambda \psi(x) \quad \text{with} \quad R(x,y) = \frac{1}{T}\int_0^{T} w(x,t)\,w(y,t)\,dt$$

Proper Orthogonal Decomposition

The discretization of the integral eigenvalue problem leads to

$$\sum_{j=1}^{M} c_j \, \rho(y_j) R(x_i, y_j) \psi(y_j) = \lambda \psi(x_i), \qquad i = 1,\dots, M$$

where the correlation matrix is

$$R(x_i, y_j) = 1/N \sum_{n=1}^{N} w(x_i, t_n)\, w(x_j, t_n) = 1/N \, \mathbf{w}^{(i)} \cdot \mathbf{w}^{(j)}$$

With $\mathbf{w}^{(i)}$ the vector of N time samples of the i dof.
By defining $\mathbf{W} = [\mathbf{w}^{(1)}, \mathbf{w}^{(2)}, \dots, \mathbf{w}^{(M)}]$ and using lumped masses, the discrete form of the eigenvalue problem becomes:

$$1/N \, [\mathbf{W}^T \cdot \mathbf{W}] \cdot \mathbf{M}\,\mathbf{z} = \lambda\, \mathbf{z}$$

where z are the POMs, $\mathbf{M}$ is the mass matrix obtained by condensation of the mass distribution and the POC are given by

$$\mathbf{A} = \mathbf{W} \cdot \mathbf{Z}$$

Proper Orthogonal Decomposition

Random excitation given by stochastic sea → Dynam. System →

Response in *m* pts and *n* time instants

$$\mathbf{W} = \begin{bmatrix} w_{11} & \cdots & w_{1\,m-1} & w_{1\,m} \\ w_{21} & \cdots & \cdots & \cdots \\ \cdots & \cdots & w_{n-1\,m-1} & w_{n-1\,m} \\ w_{n1} & \cdots & w_{n\,m-1} & w_{n\,m} \end{bmatrix}$$

$$E_{rs} = \sum_{i=1}^{n} w_{i\,r} \cdot w_{i\,s}$$

Covariance matrix $\mathbf{R}_W = (1/N)\,\mathbf{W}^{\mathrm{T}}\mathbf{W} = \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{m1} \\ E_{21} & E_{22} & \cdots & E_{m2} \\ \cdots & \cdots & \cdots & \cdots \\ E_{m1} & E_{m2} & \cdots & E_{mm} \end{bmatrix}$

Proper orthogonal values

Linear normal modes

$$\left(\mathbf{R}_W \hat{\mathbf{M}} - \lambda\,\mathbf{I}\right)\mathbf{z} = 0 \qquad \left(\mathbf{K} - \omega^2 \boldsymbol{M}\right)\boldsymbol{\varphi} = 0$$

Proper orthogonal modes

$$m \leq N \qquad n \to \infty$$

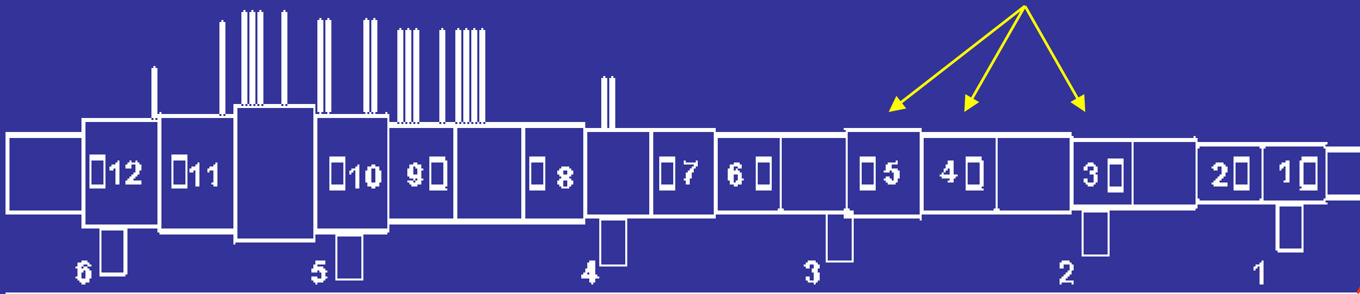
Application of POD to experimental data

Several aspects may be of concern in the application of classical POD to data obtained with measurements:

- The number of measuring points may be quite small limiting the number of observable POMs
- If convergence to LNM need to be assessed, the mass matrix need to be known
- But, even the mass matrix is known but the measuring point are few, the projection mass matrix may be roughly approximated thus limiting this convergence

Displacement analysis

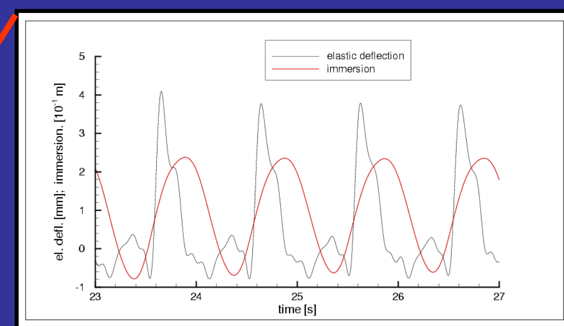
The bending response was measured using 12 strain gauges

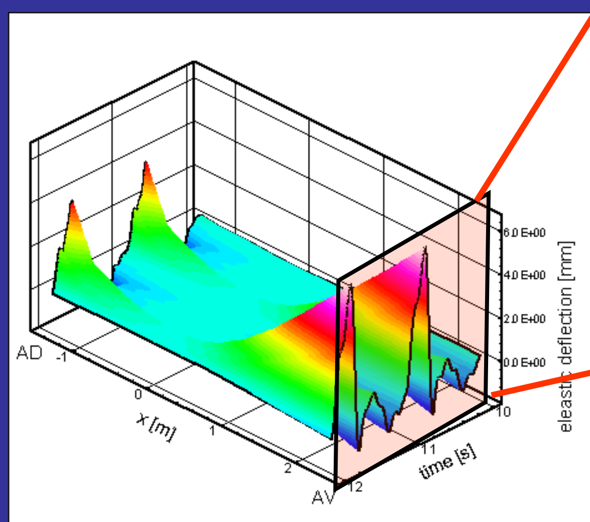


$H_{1/3} = 2\ m$

$T_1 = 7.5\ s$

$V_S = 20\ kn$



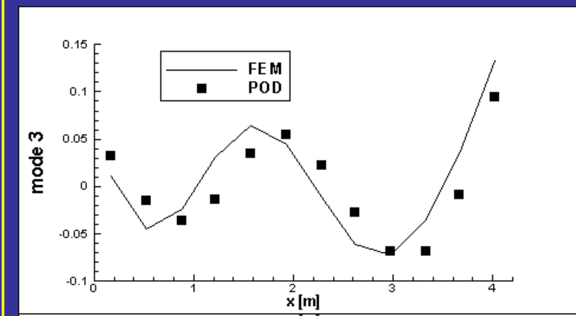


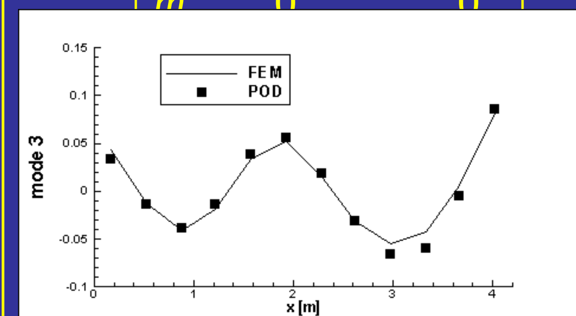
Beam elastic deflection

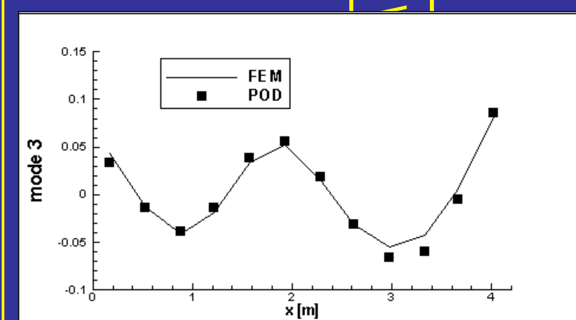
Sensitivity of the POD to the mass input (dry case)

Considered mass dist’n for

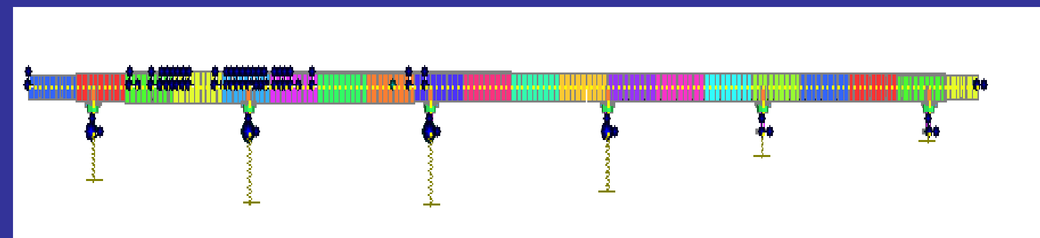
“Dry” POMs



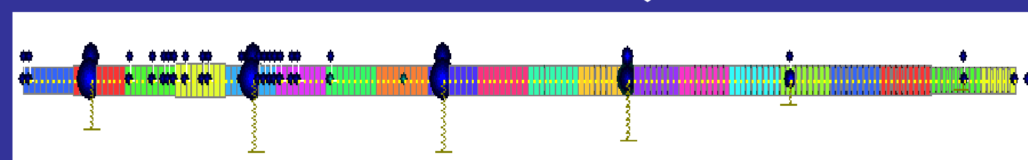




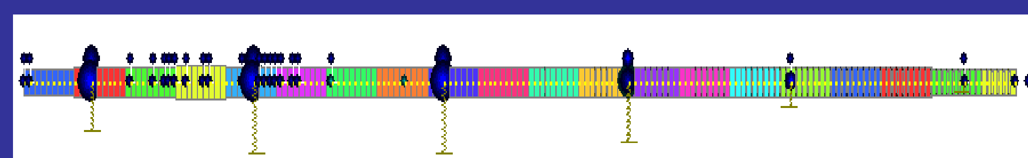
Considered model for simulations



+ simple: no legs



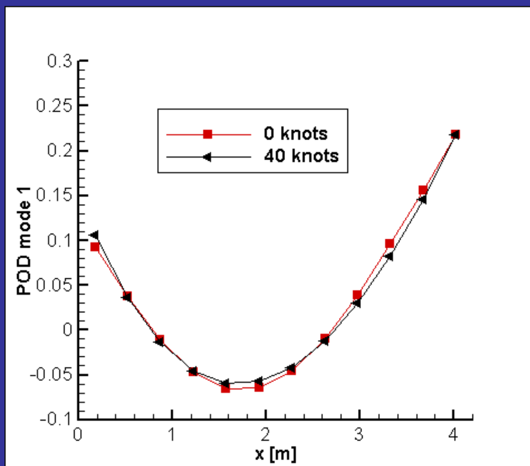
+ simple: no rot. in. for the mass

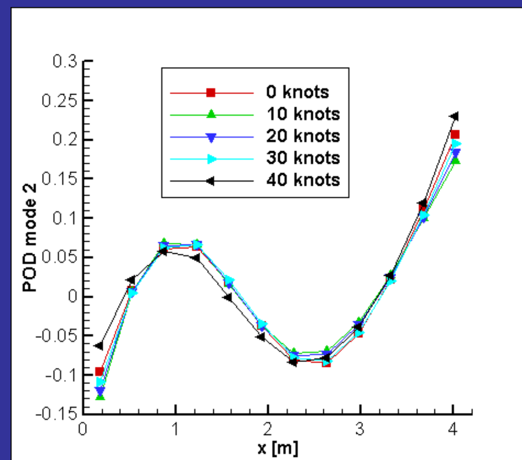


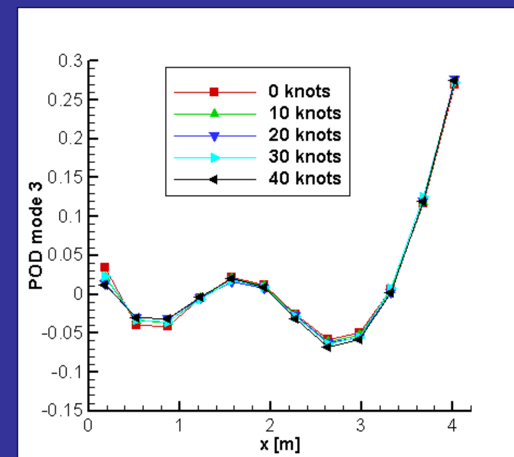
Variation of the wet modes with speed

Sea spectrum characteristics: $H_{1/3}=2m$, $T_1=7.5s$

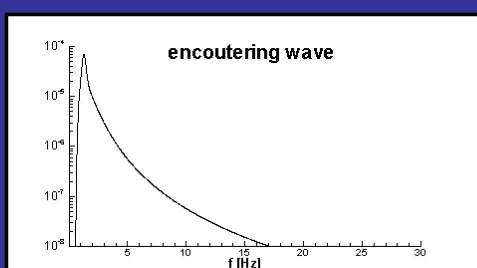
Forward speed V_S: 0, 10, 20, 30, 40 kn

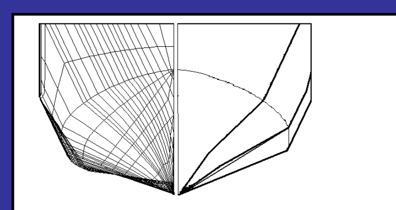


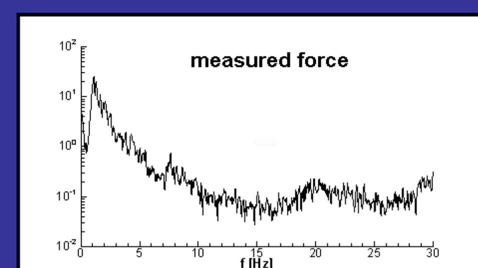




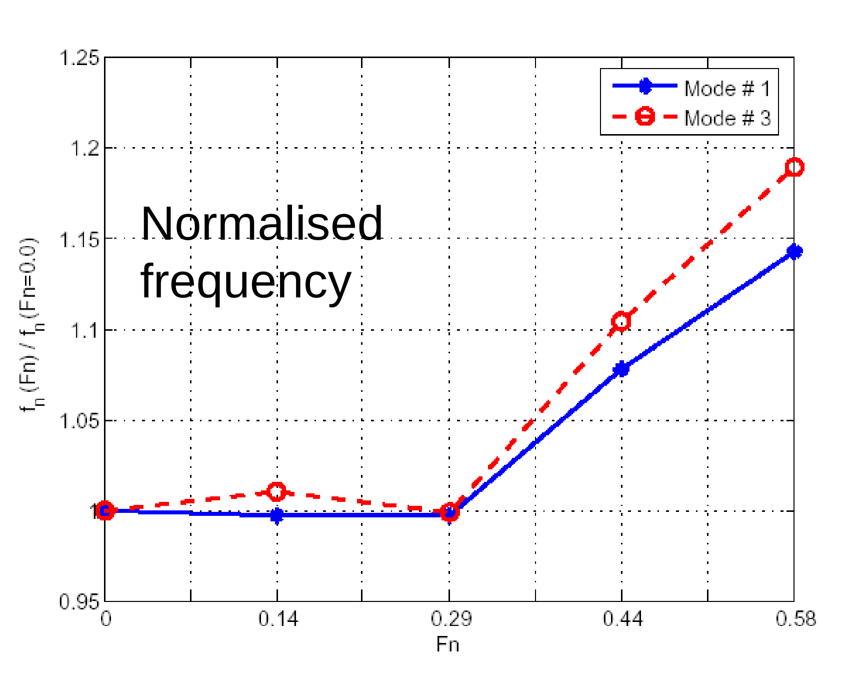
Remark: the excitation band of the sea spectrum is enlarged up to the low-freq. mode bending modes range via the nonlinear load transfer function

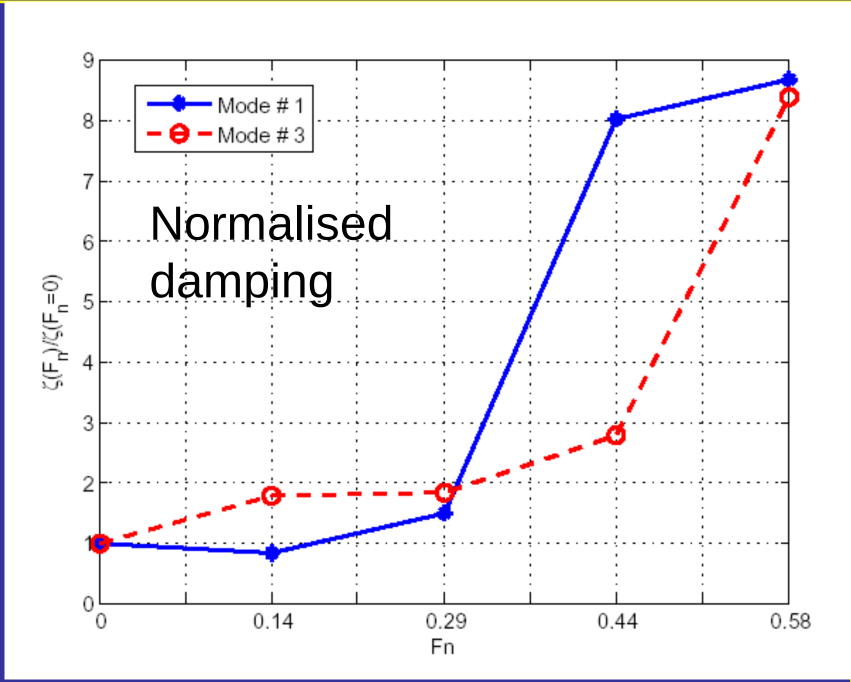






Dependency on ship forward speed



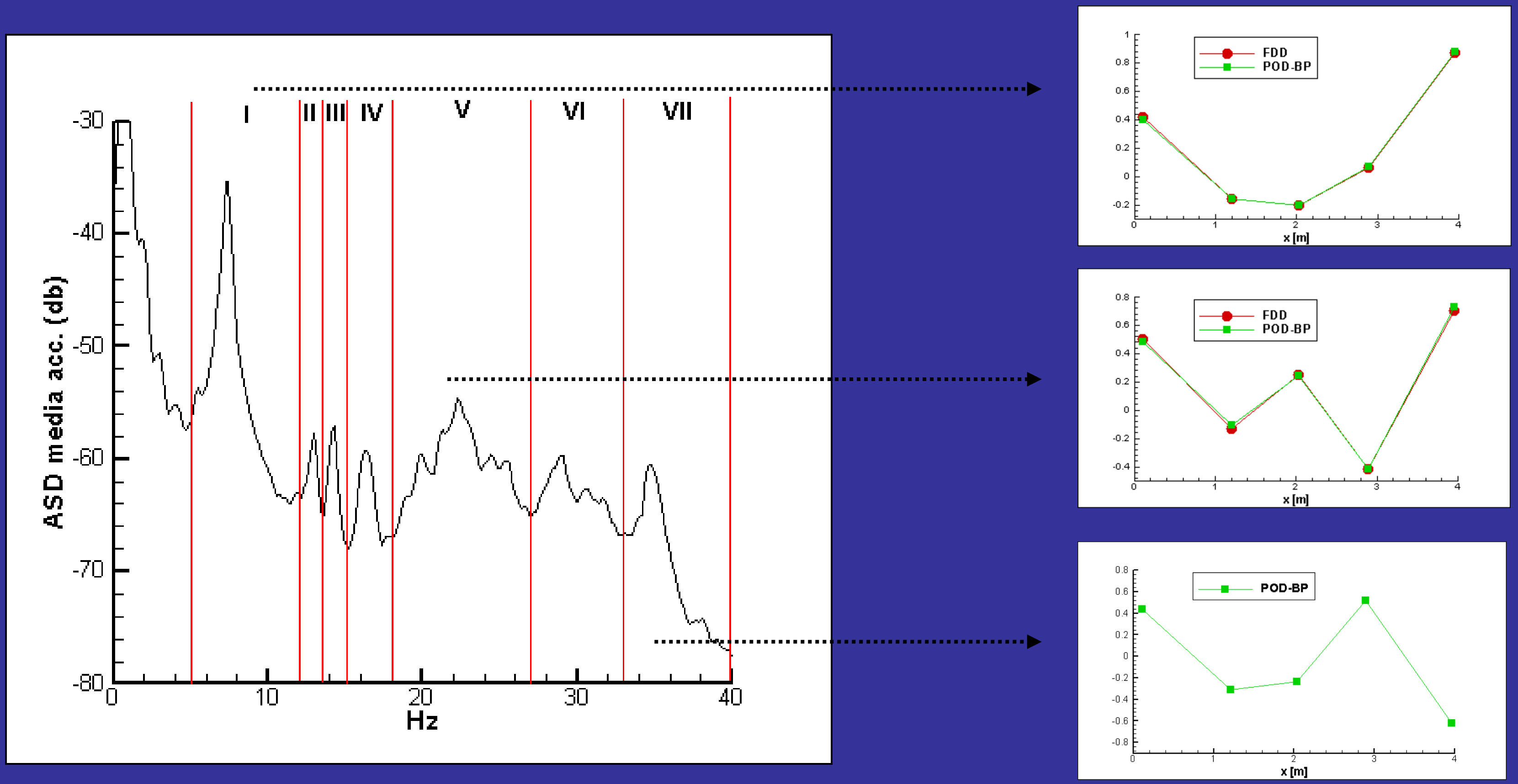


Observations

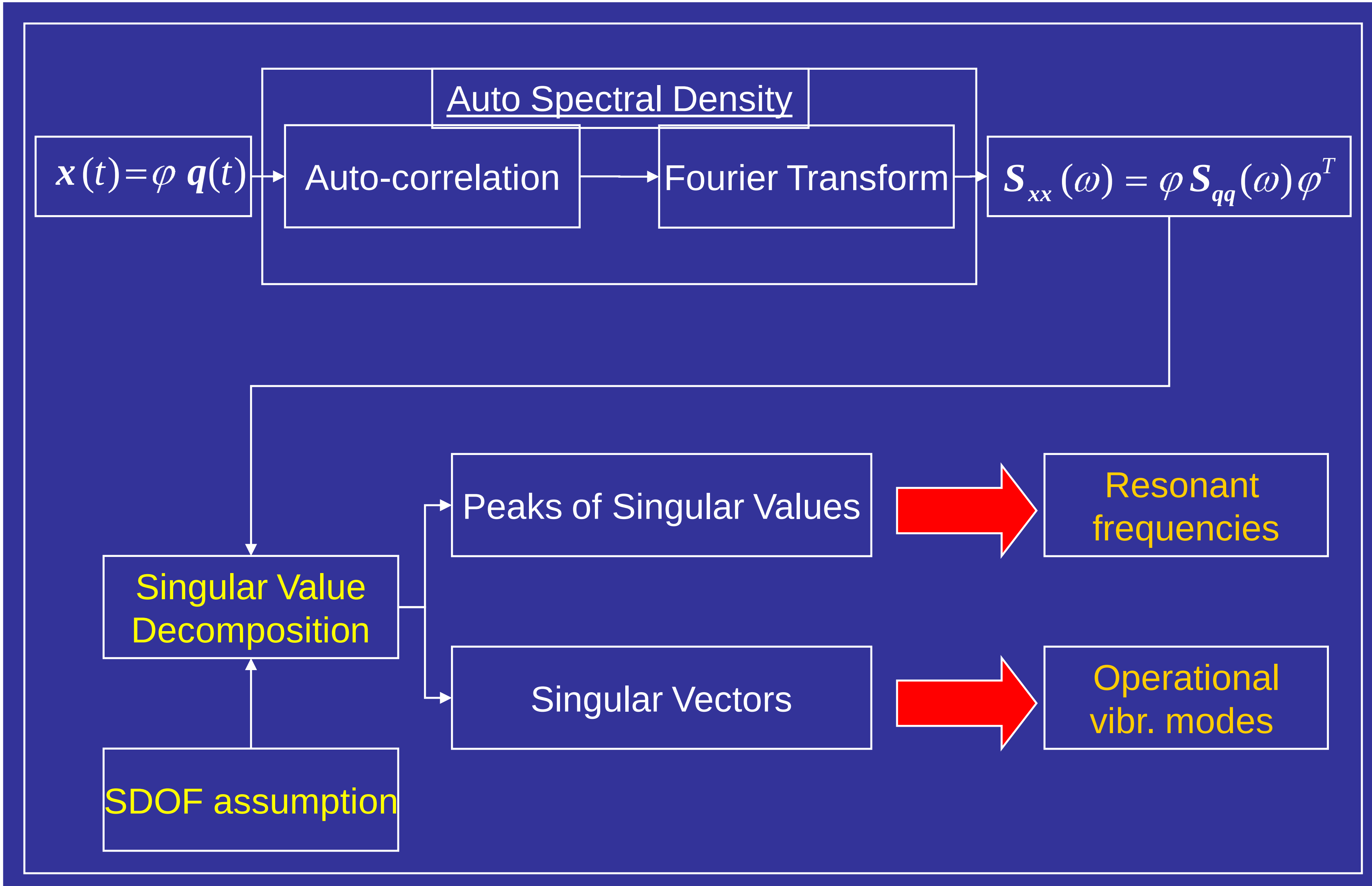
- Frequency and damping ratio grow as long as speed is increased.
- They seem to depend on the amplitude of the excitation (possible nonlinear effects).
- The dramatic increase for the damping is strictly related to the segmented hull configuration as well (increase of model structural damping).

Avoiding the mass input

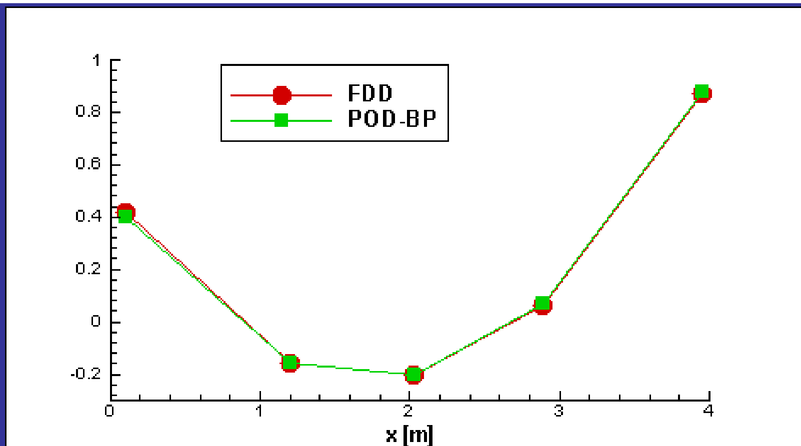
If the aim is to obtain operational modes as close as possible to the linear normal modes, a modification of the original technique exploits band pass filtering in case of well-spaced frequencies.



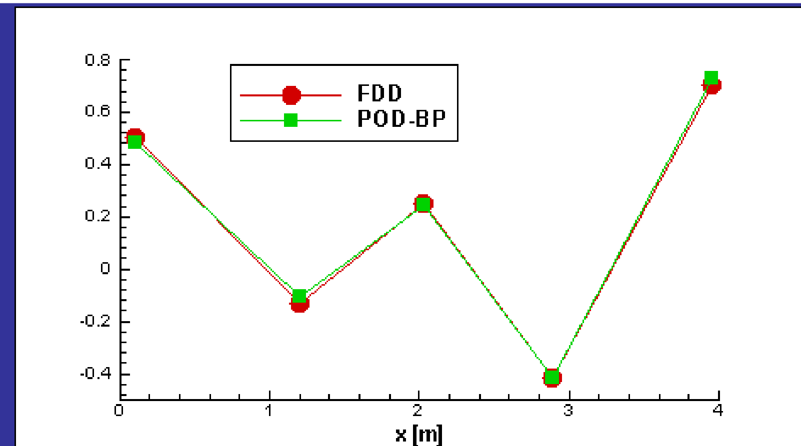
SVD – FDD technique



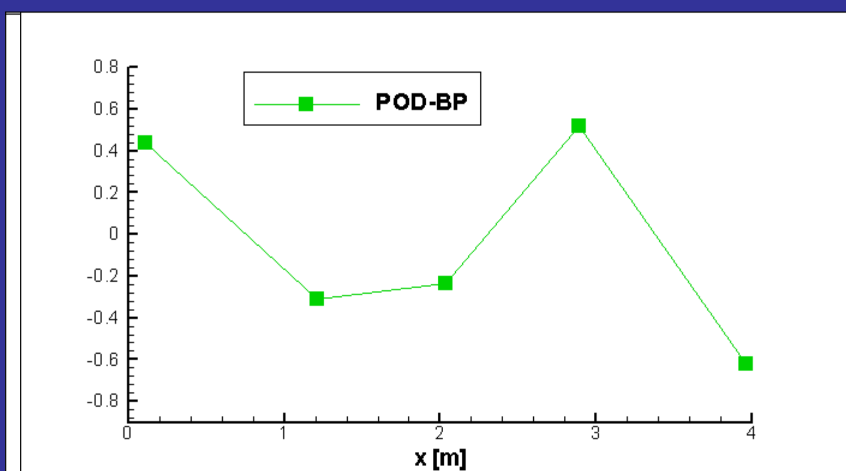
Comparison between SVD-FDD and POD modes



2-nodes mode



4-nodes mode.



3-nodes mode

modo	freq. [*Hz*]	MAC (2-1)	MAC (2-
3 nodi – a	12.99	0.467	0.341
3 nodi – b	14.16	0.157	0.090
3 nodi – c	16.21	0.612	0.566
3 nodi – d	22.36	0.002	0.005
3 nodi – e	29.00	0.055	0.229

MAC

Final remarks

- The POD has been applied to the extraction of the operational modes of a floating beam-like structure representing the hydroelastic behaviour of a true ship in usual conditions.

- The results seem encouraging also for the present application, because convergence to LNM has been demonstrated with use of SVD-FDD

- The POD can be used to reduce the model of a true ship (full-scale trials) in order to design the scaled elastic model or to verify the correspondence with a theoretical model

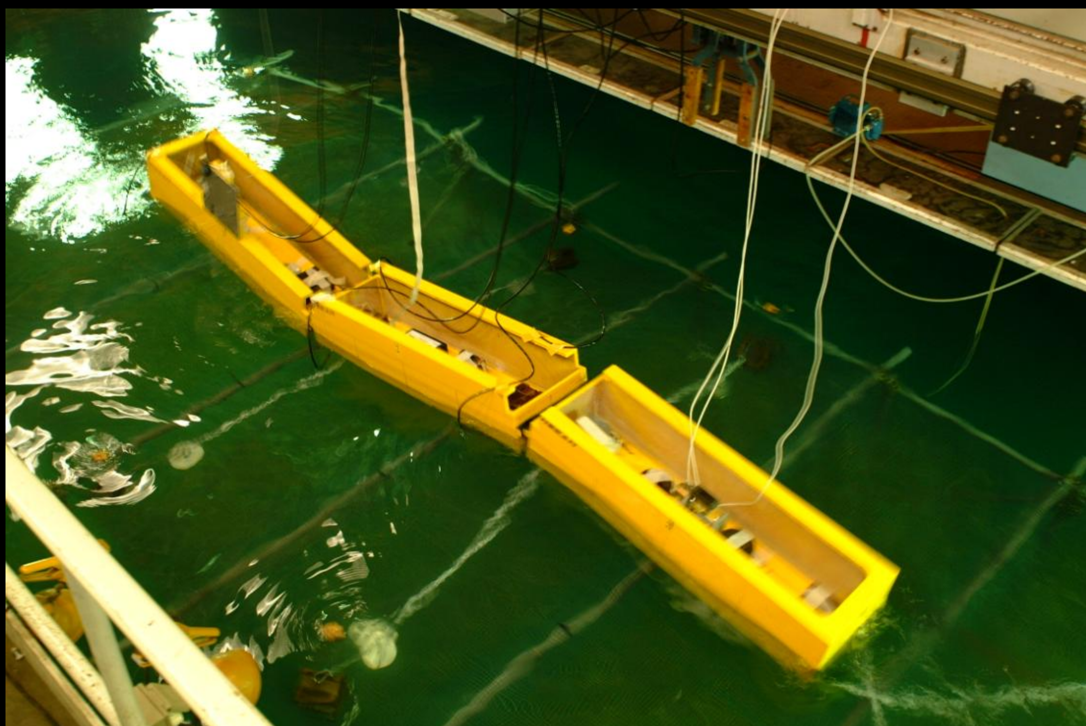
Final remarks

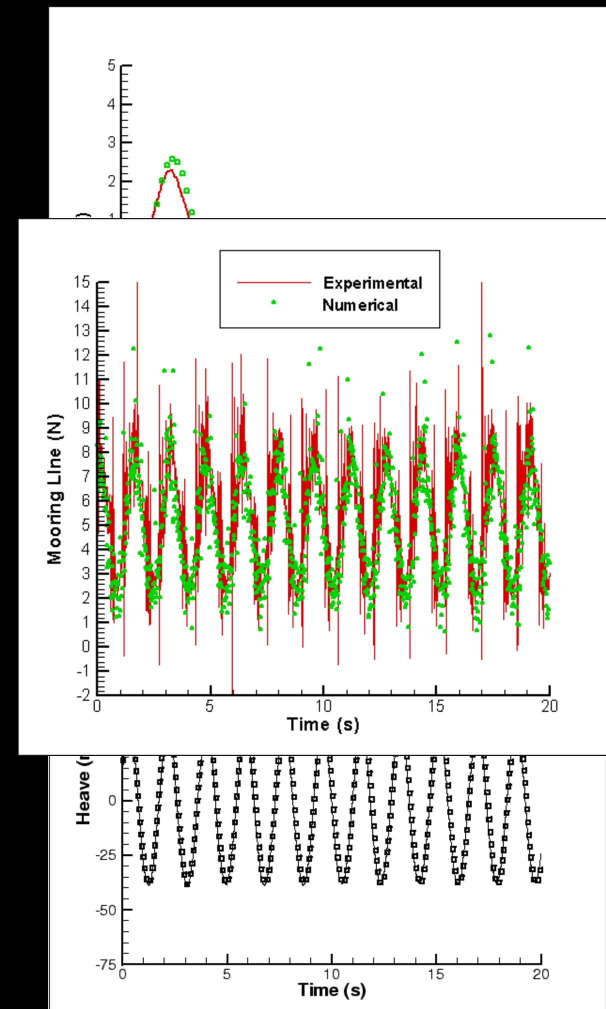
- The advantage of POD is that for the study of nonlinear behaviour the signal processing tool remain the same

Sequence of a slamming impact during a regular wave test.

Other challenging problems under investigation 1/2

Provide a reduced order model of the systems of moored pontoons (simulations & experiments

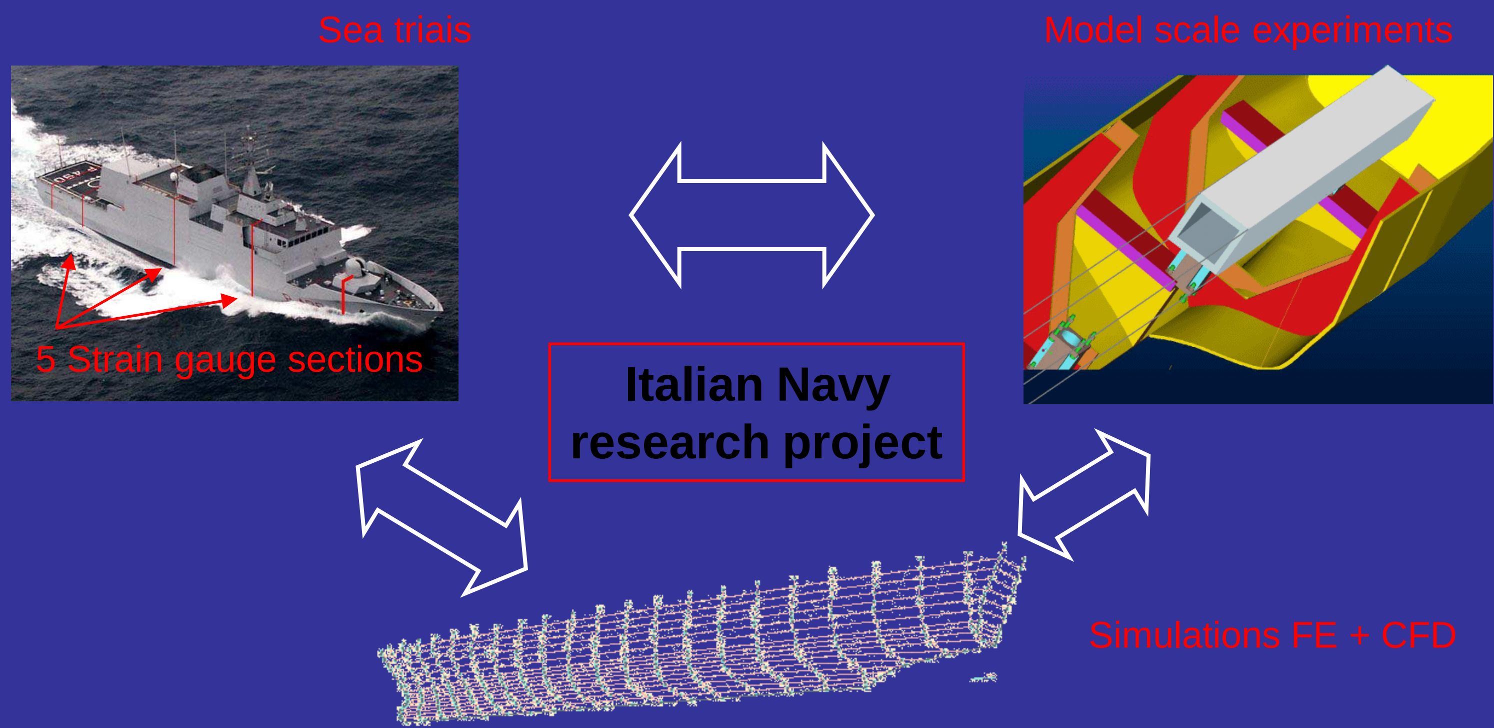




Other challenging problems under investigation

- Certification and international rules (IMO-SOLAS IMO-MARPOL rules)

- Accurate predictive models for whipping



- Hull stiffness deterioration due to collisions, grounding or aging (damage detection or localization seems still far to be practicable).