LABORATORY

Prediction of Bifurcation Onset of Large Order Aeroelastic Models

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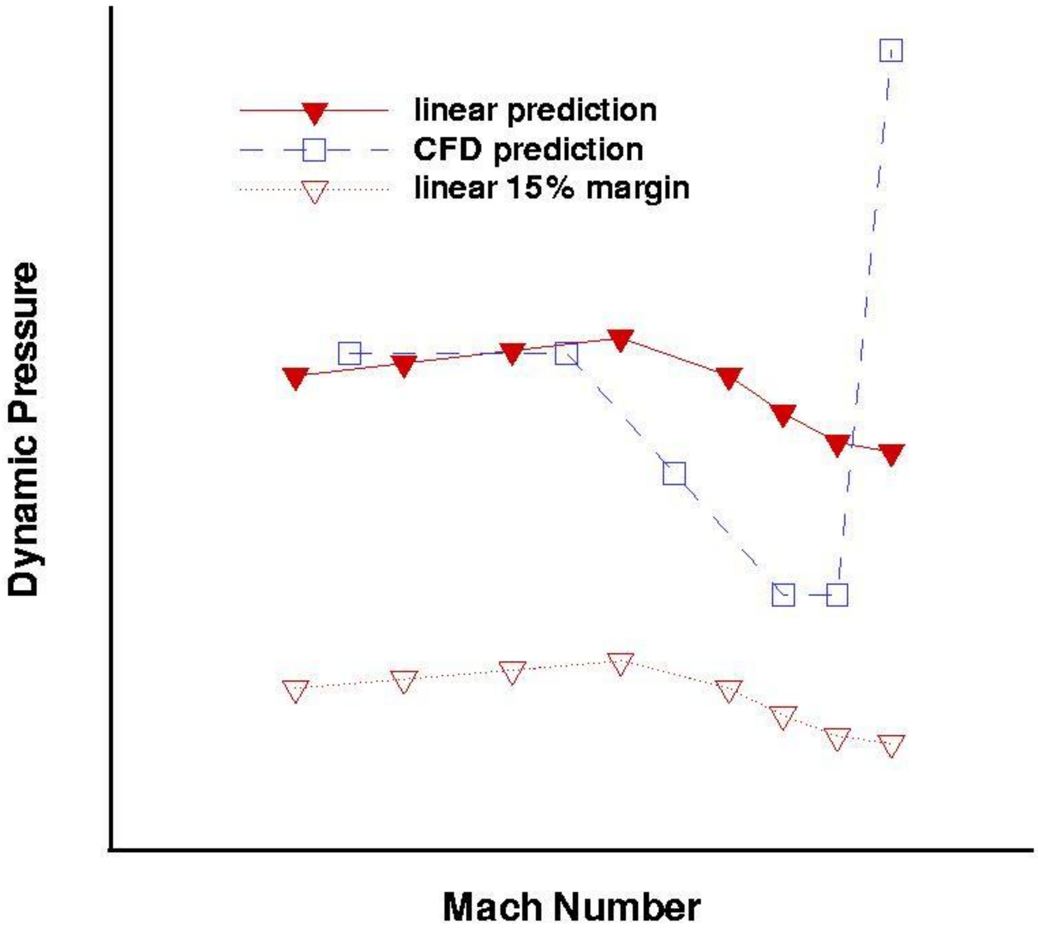
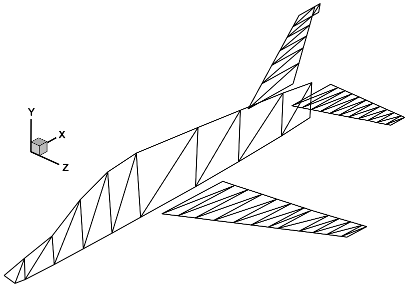
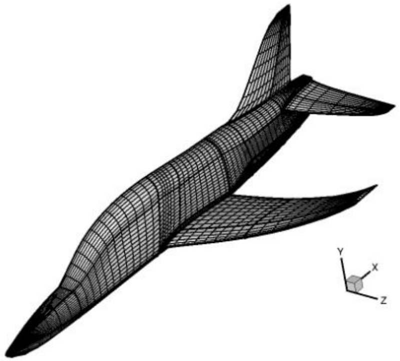
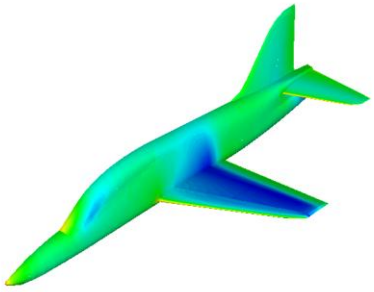
Marie Curie Excellence Team

Enabling Certification by Analysis









Woodgate, M. Badcock, K.J. Rampurawala, A.M. Richards, B.E. Nardini D. and Henshaw M. Aeroelastic calculations for the Hawk aircraft using the Euler equations, **Journal of Aircraft, 42(4),** 2005, 1005-1012.

Denley, C.J., Eccles, T.A., Cross, A.G.T., Practical Unsteady CFD Application to Aircraft Flutter and Limit Cycle Oscillation, **RTO-AVT-152**, Loen, May, 2008

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \mathbf{p} = \lambda \mathbf{p}$$

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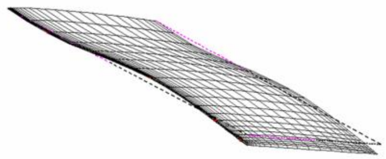
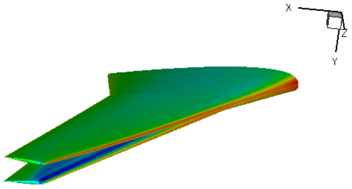
Badcock et al, **J Aircraft, 42(3), 731-737, 2005**

- solved an augmented system for the critical eigenvalue
- Assumptions/Limitations
  - good initial guess at flutter frequency
  - Symmetric problem
  - Sequential calculation



Badcock et al, **AIAA J, 45(6), 1370-1381,2007**

- Aeroelastic eigenvalues using the Inverse Power Method
- Assumptions/Limitations
  - Sequential calculation
  - Mode tracking



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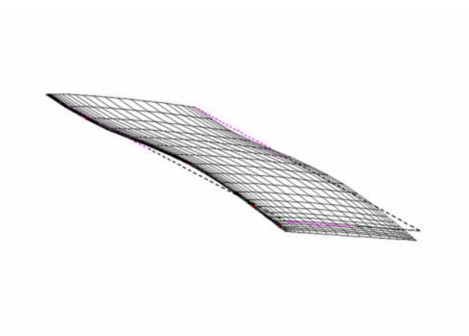

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- Aeroelastic eigenvalues using the Inverse Power Method
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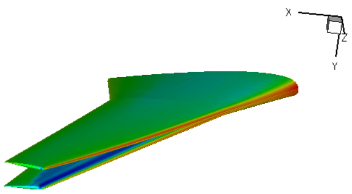
Addressed in current work



Inverse Power Method

$$z_k = \begin{bmatrix} A_{ff} - \lambda_0 I & A_{fs} \\ A_{sf} & A_{ss} - \lambda_0 I \end{bmatrix}^{-1} x_{k-1}$$

Inverse Power Method

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shift

Inverse Power Method

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shift

- good shift needed for convergence
    - Better shift - iteration matrix becomes more singular
- Iterative solvers thrown at solving this problem in parallel
    - good methods available but none has really done the job

Inverse Power Method

$$z_k = \begin{bmatrix} A_{ff} - \lambda_0 I & A_{fs} \\ A_{sf} & A_{ss} - \lambda_0 I \end{bmatrix}^{-1} x_{k-1}$$

New thinking needed

- good shift needed for convergence
    - Better shift - iteration matrix becomes more singular
- Iterative solvers thrown at solving this problem in parallel
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Eigenvalue Problem

$$\begin{bmatrix} A_{ff} & A_{fs} \\ A_{sf} & A_{ss} \end{bmatrix} \begin{bmatrix} p_f \\ p_s \end{bmatrix} = \lambda \begin{bmatrix} p_f \\ p_s \end{bmatrix}$$

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Schur complement version

(Bekas and Saad, SIAM Journal of Scientific Computing 27(2) 458, 2005)

$$S(\lambda)\, p_s = \lambda p_s$$

$$S(\lambda) = A_{ss} - A_{sf}\,(A_{ff} - \lambda I)^{-1} A_{fs}$$

λ Not an eigenvalue of A_{ff}

Eigenvalue Problem

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Good! Small order nonlinear eigenvalue problem

Eigenvalue Problem

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Good! – solves ill conditioning problem

Eigenvalue Problem

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Bad! – re-evaluate S at every nonlinear step

But

$$(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1}$$

Eigenvalue Problem

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λ Not an eigenvalue of A_{ff}

Bad! – re-evaluate S at every nonlinear step

But $$(A_{ff} - \lambda I)^{-1} \approx A_{ff}^{-1} + \lambda A_{ff}^{-1} A_{ff}^{-1}$$

Pre-compute – valid if λ is small

Method

To track an aeroelastic eigenvalue

1. Choose normal mode frequency as a shift

$$S(\lambda) = (A_{ss} - \lambda_0 I) - A_{sf}(A_{ff} - \lambda I - \lambda_0 I)^{-1} A_{fs}$$

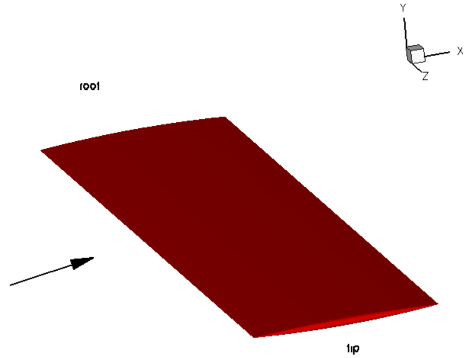
2. Precompute

$$A_{sf}(A_{ff} - \lambda_0 I)^{-1} A_{fs} \qquad A_{sf}(A_{ff} - \lambda_0 I)^{-2} A_{fs}$$

3. Solve nonlinear eigenvalue problem by Newton's method
   - Use **series** approximation to generate function (very cheap)
   - Use **full** evaluation to generate function (more expensive)
   - In both cases use series approximation for LHS
4. Change Altitude and repeat step 3

Comments

- Series expansion to drive convergence
  - Full evaluation for function
    - Can account for aerostatic effects if needed
- Linear system
  - complex variable, block Jacobi – BILU preconditioner
- Mode tracking
  - Cheap solution of nonlinear eigenvalue problems
  - small steps can be taken
- Highly parallel
  - Each mode can be tracked independently

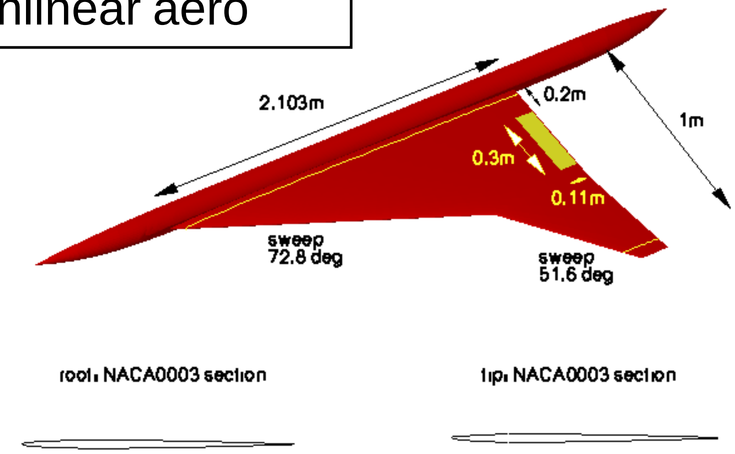


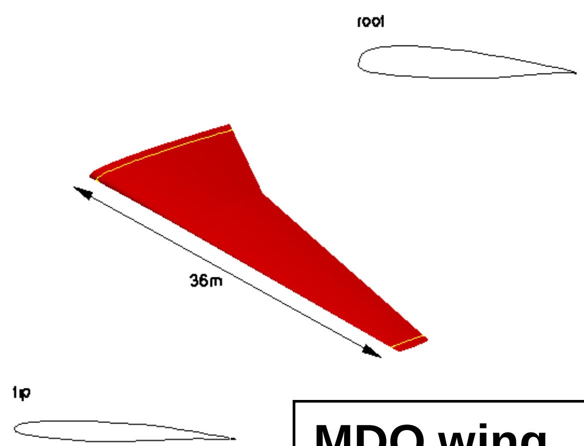
Goland wing

- very flexible – test for series

SST control surface buzz

- driven by nonlinear aero



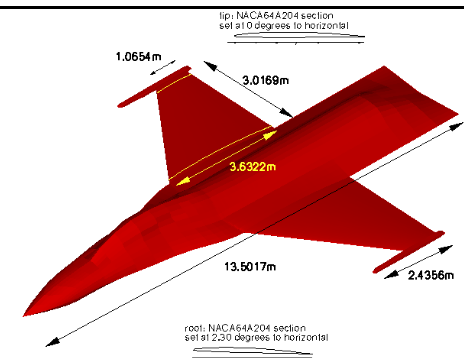


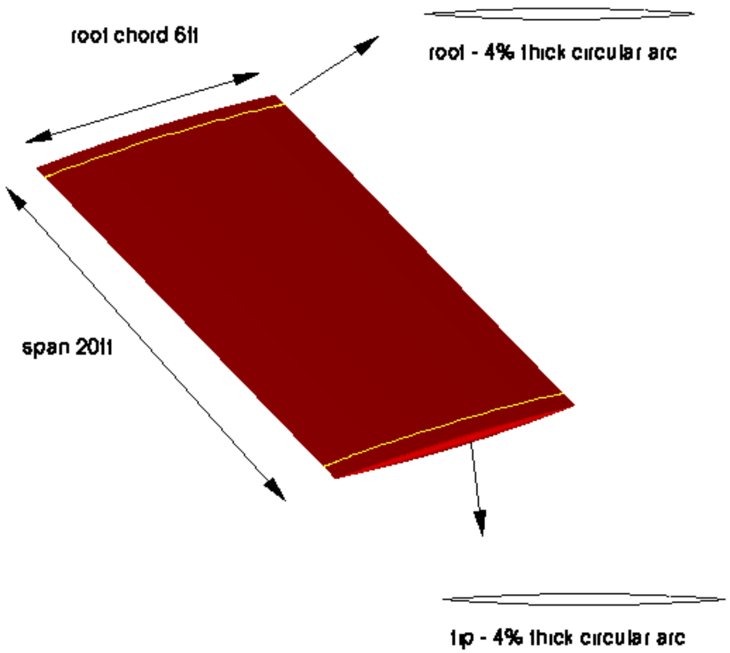
MDO wing

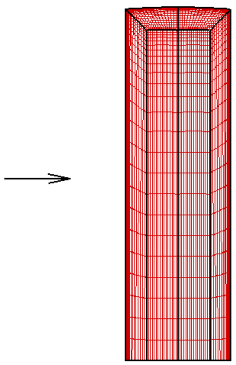
- realistic modes

Generic Fighter

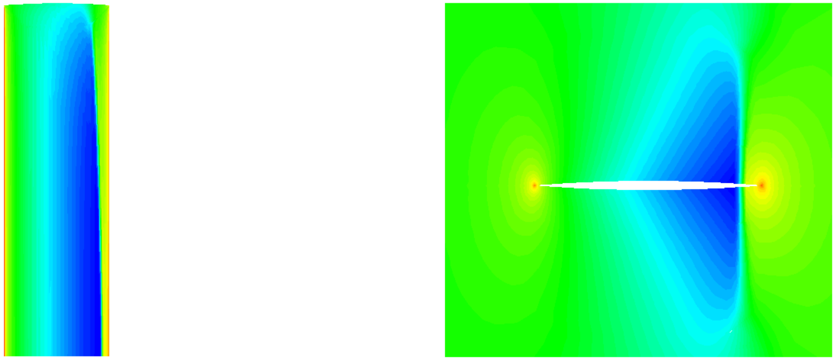
- realistic sized problem



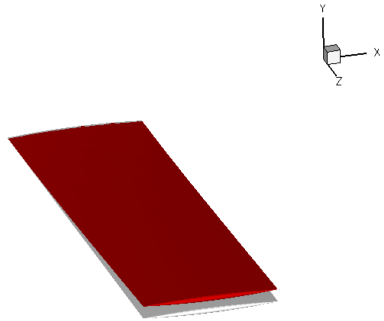




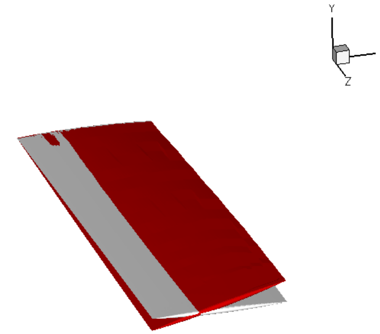
236k points



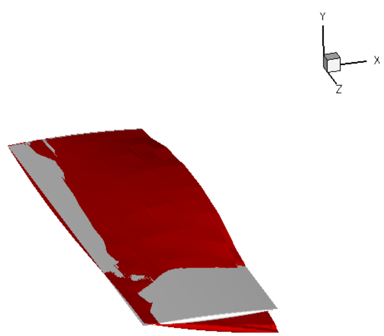
Mach 0.92



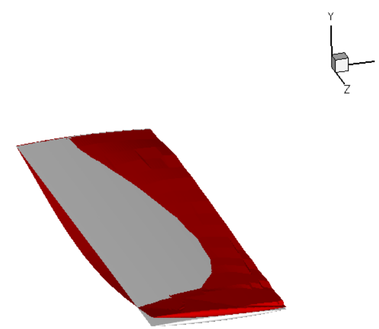
1.72 Hz



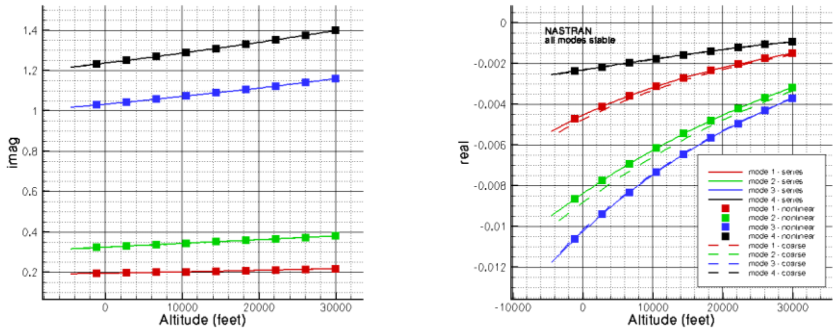
3.05 Hz



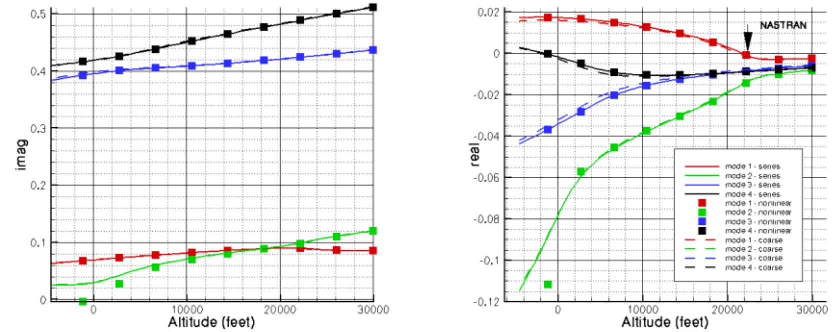
9.18 Hz



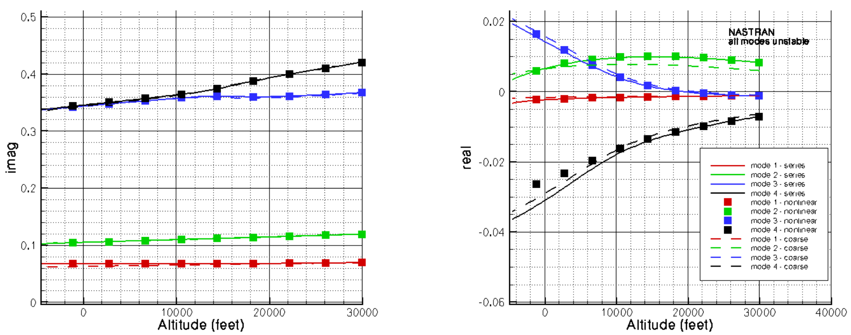
11.10 Hz



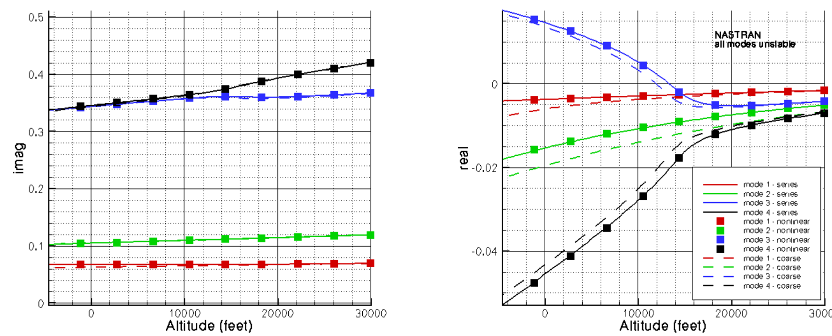
Mach 0.30



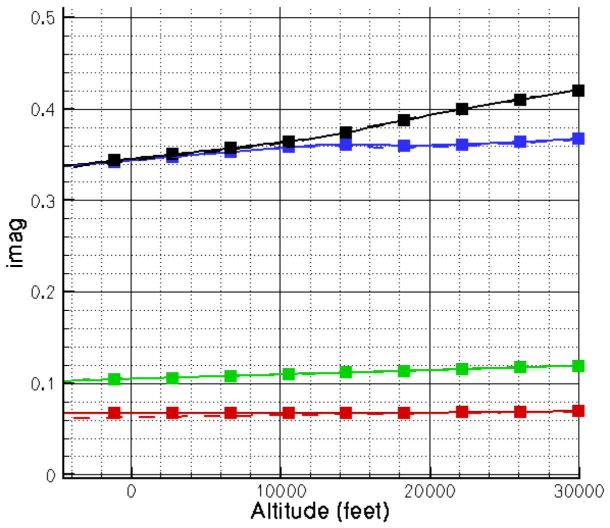
Mach 0.80

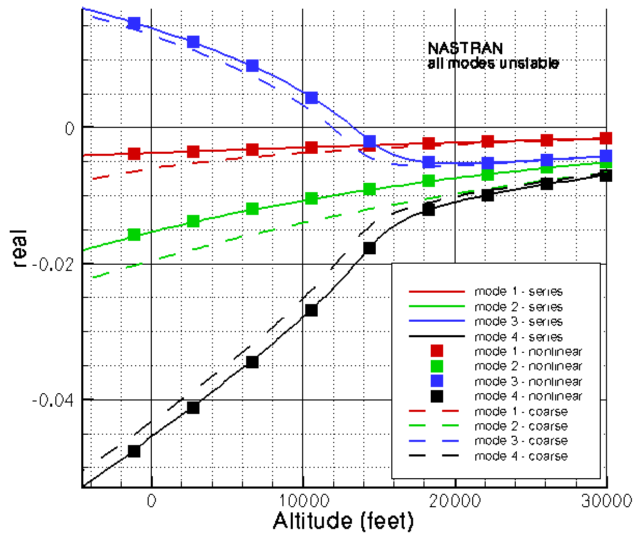


Mach 0.92



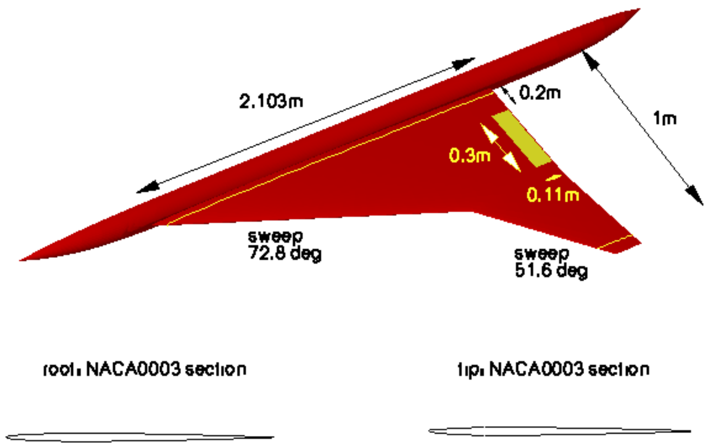
Mach 0.97



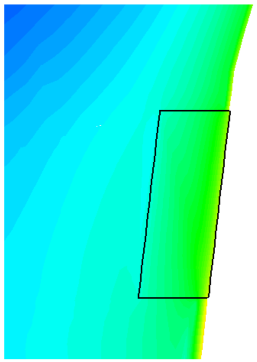


8 processors

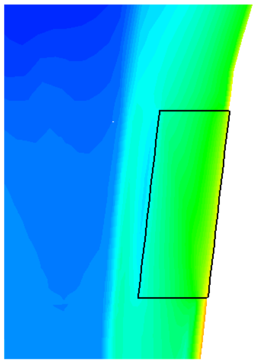
- Steady state in 11 minutes
- 4 modes – generate S - 134 minutes
- Full solution (8) - 202 minutes
- series and full – identical results
- Total cost series method -**13.5 steady state solves**



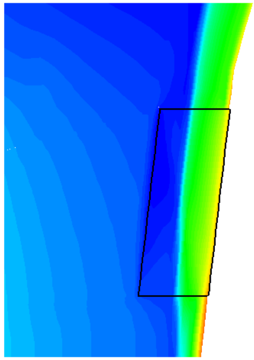
234k points



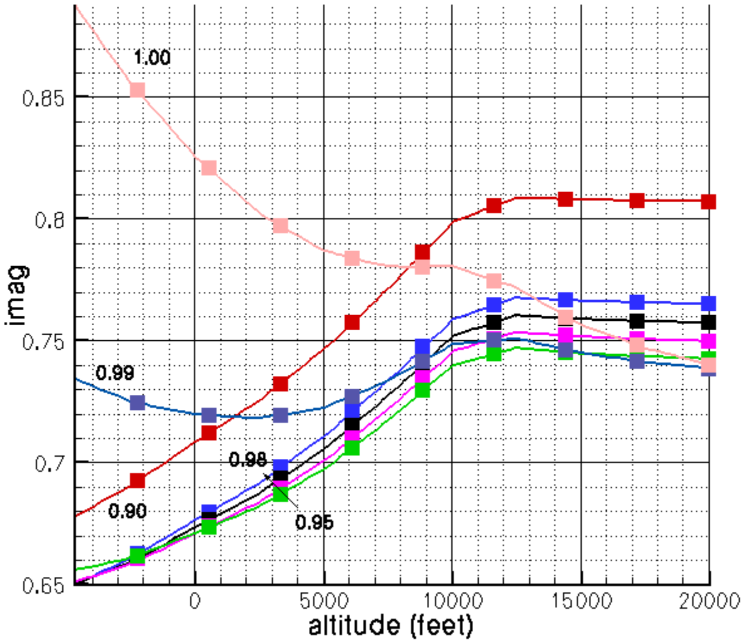
Mach 0.90

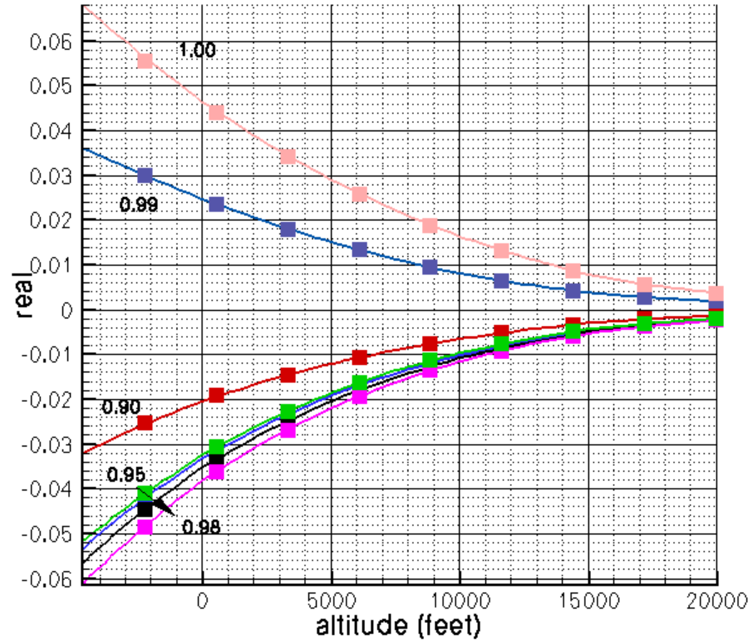


Mach 0.96



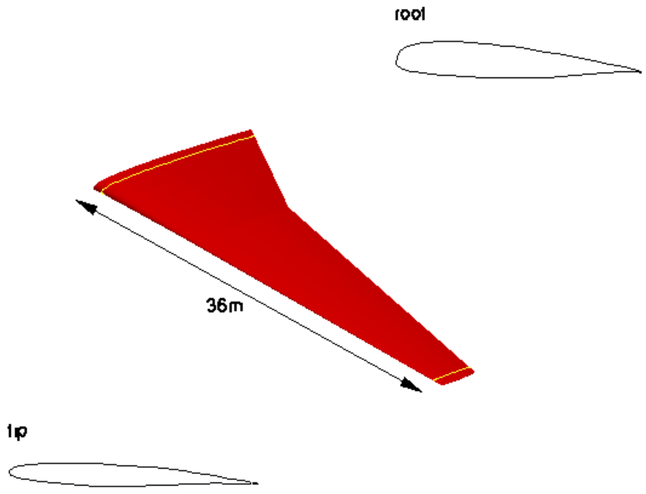
Mach 0.99

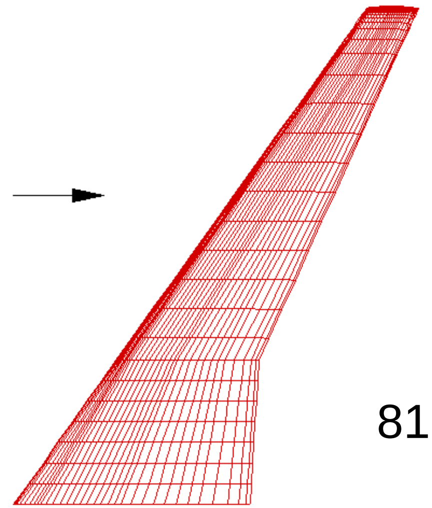




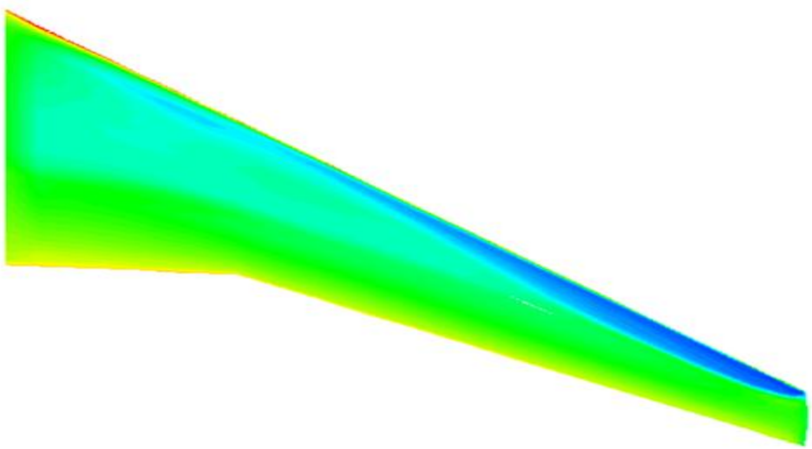
8 processors

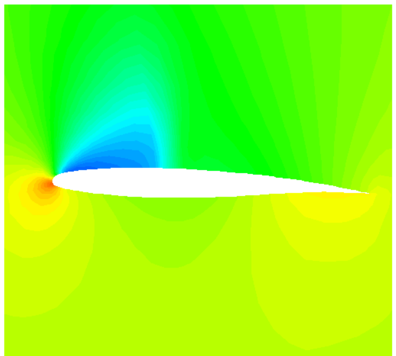
- Per Mach number
    - Steady state in 14 minutes
    - Generate S matrix – 9 minutes
    - Full function evaluation (8) – 49 minutes (23 nonlinear steps)
- series and full solutions identical
- Total cost – series method for 6 Mach nos – **11.5 times steady state**



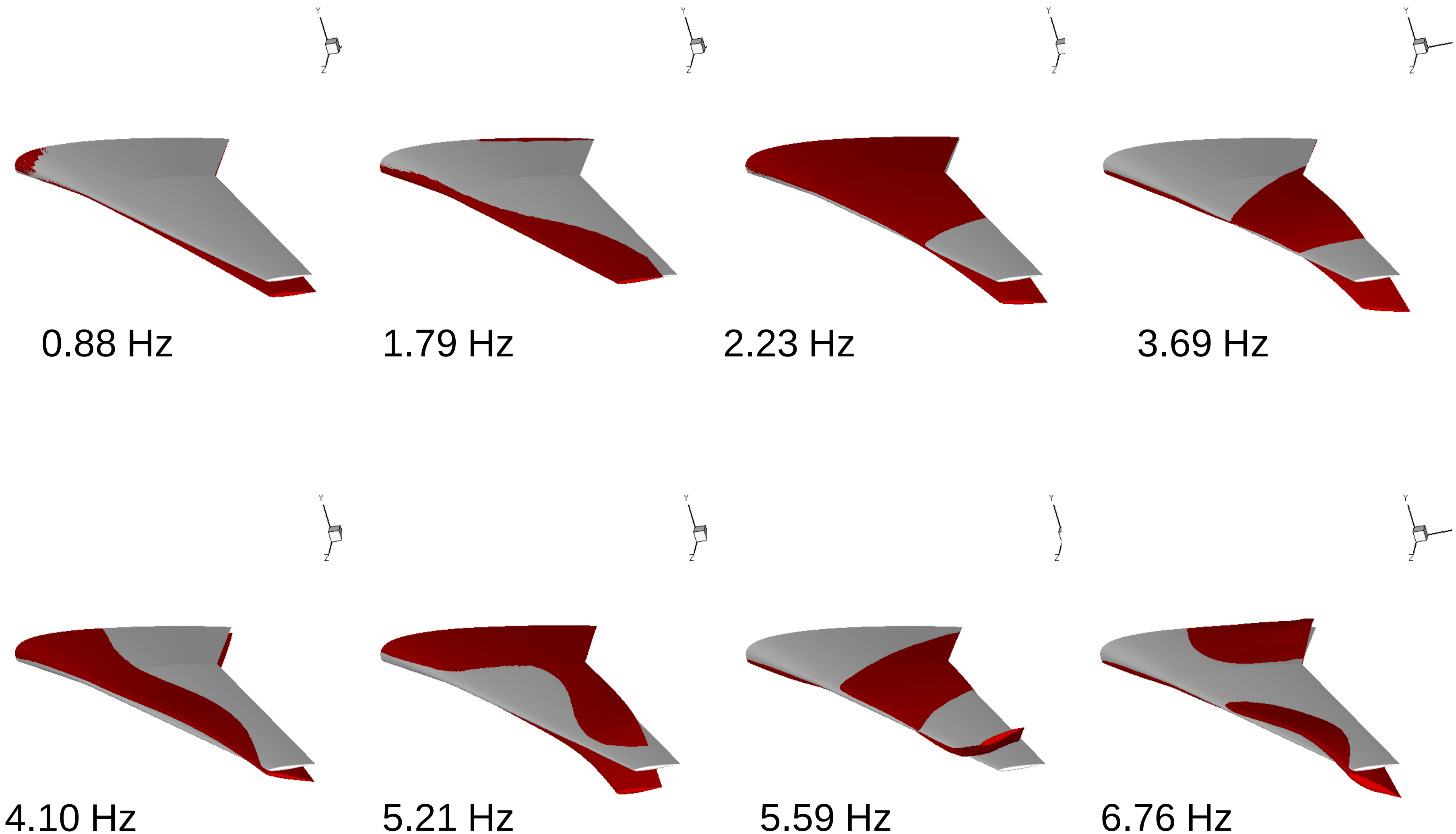


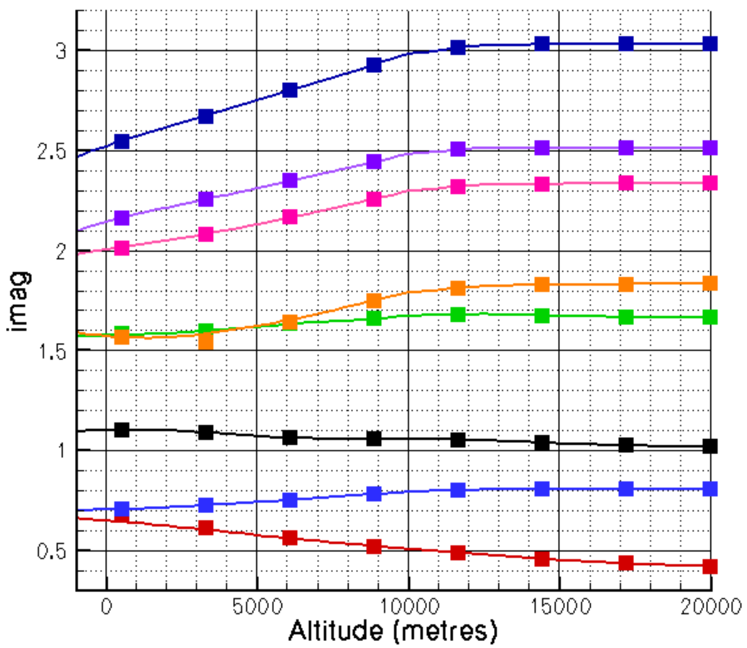
81k points

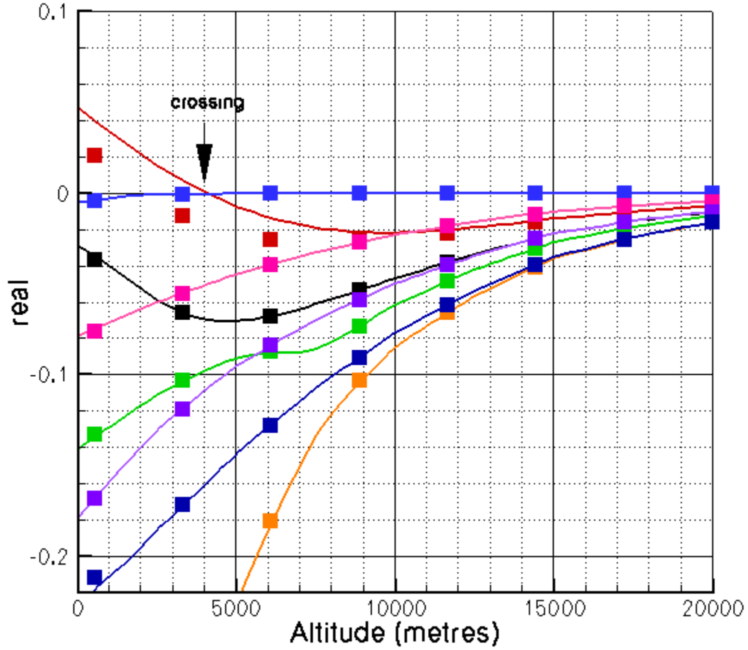




Mach 0.85


0.88 Hz
1.79 Hz
2.23 Hz
3.69 Hz
4.10 Hz
5.21 Hz
5.59 Hz
6.76 Hz





8 processors

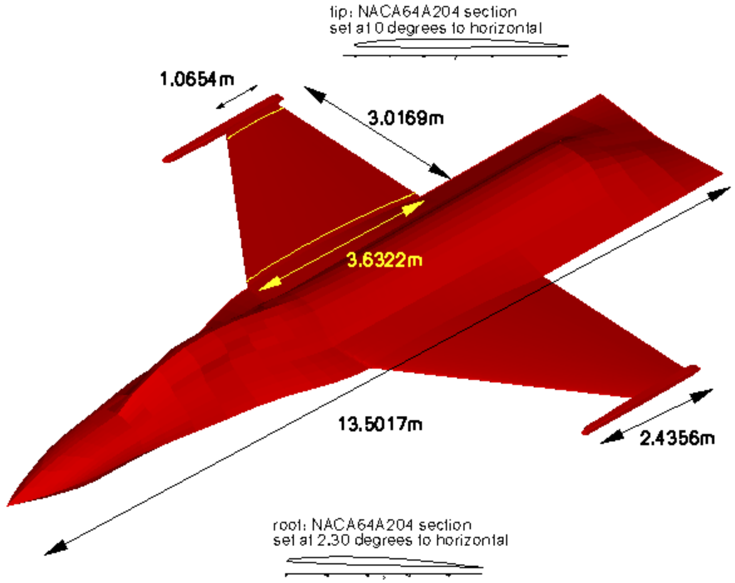
- Steady state in 22 minutes

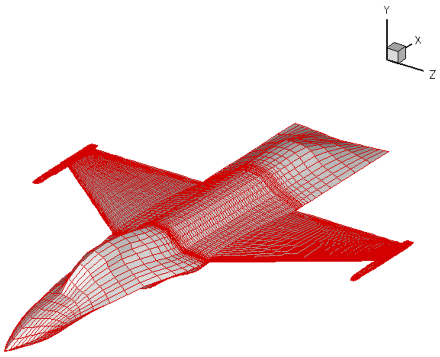
- for 8 modes – generate S – total time 95 minutes

- Full function evaluation - each mode took 72 minutes

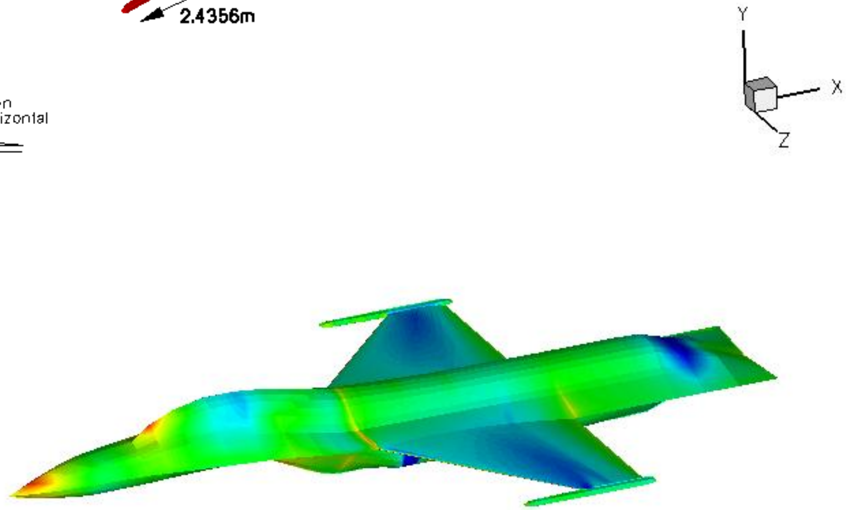
- series and full solutions identical

- Total cost – series method for 8 modes - **5 times steady state**

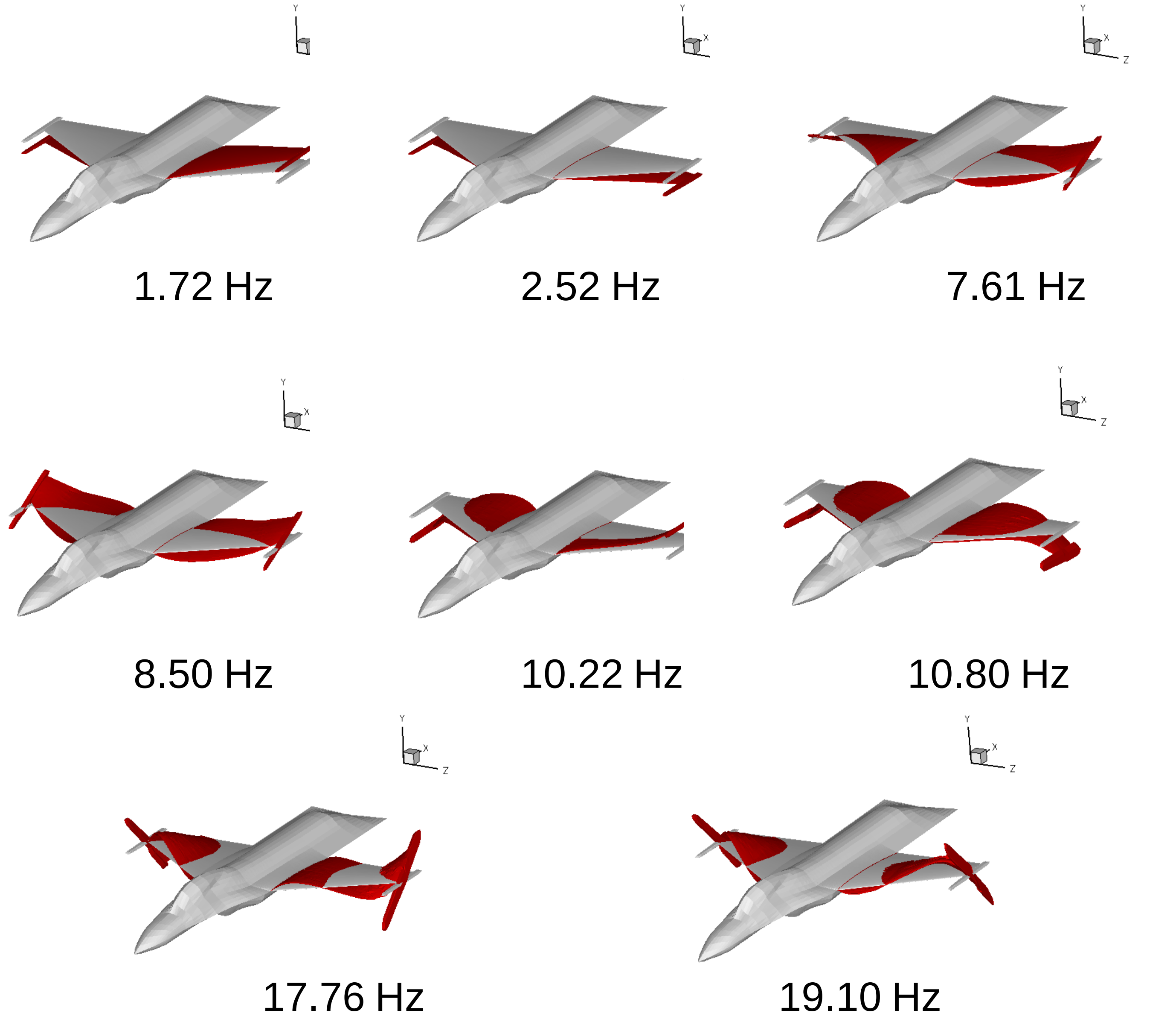


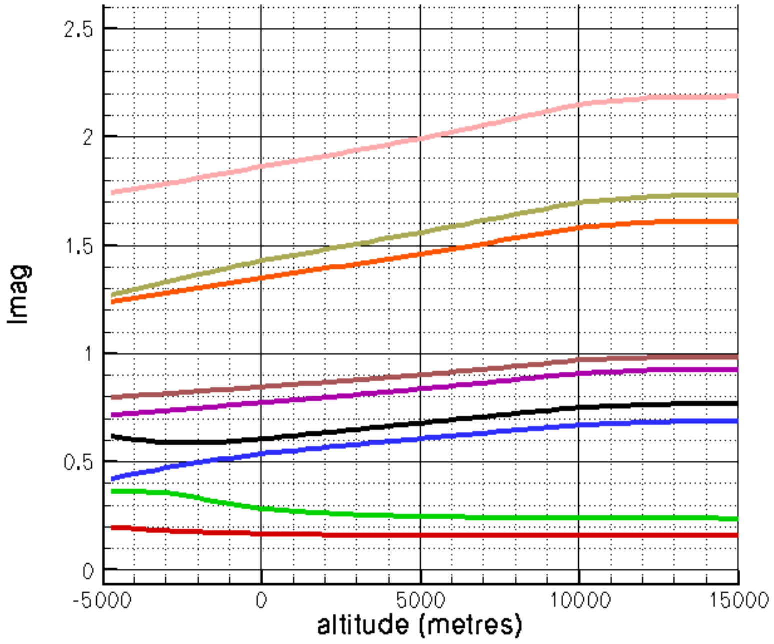


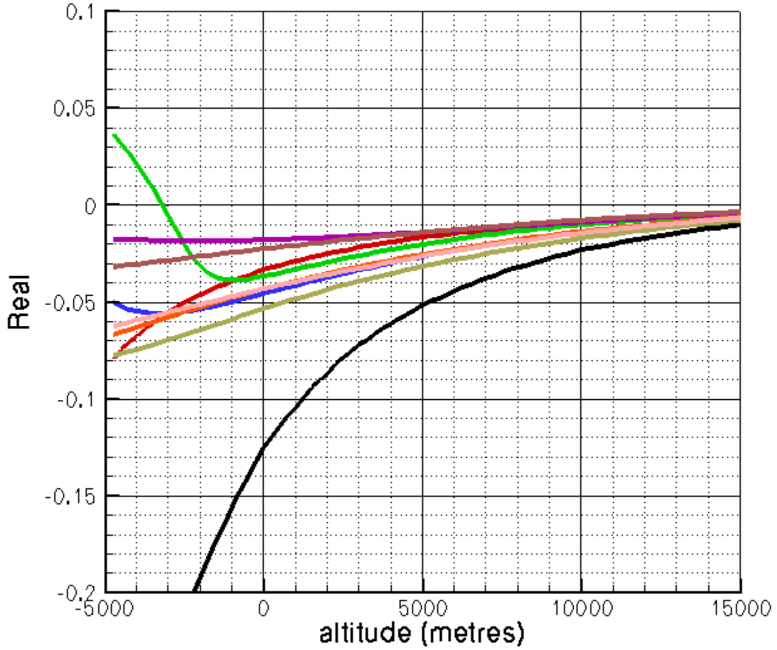
890k points



Mach 0.85


1.72 Hz
2.52 Hz
7.61 Hz
8.50 Hz
10.22 Hz
10.80 Hz
17.76 Hz
19.10 Hz





Mach 0.85

32 processors

- Steady state in 15 minutes

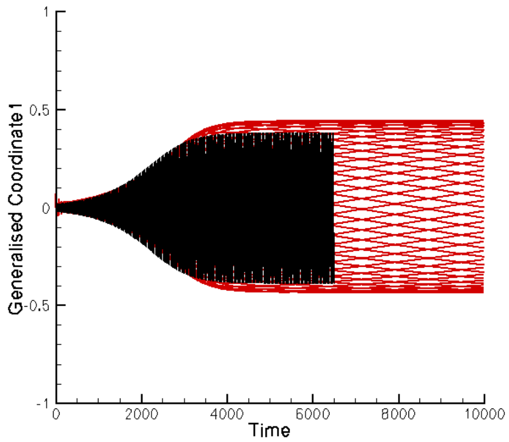
- For each mode – generate S - 45 minutes

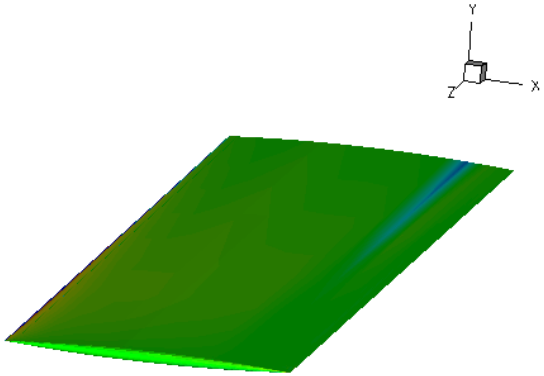
-Total cost - series method for 10 modes – **30 times steady state**

Conclusions

- New method proposed
  - mode tracking (robust for all tests)
  - parallel linear solution
    - **100-220 iterations for all cases shown**
- Application to 4 contrasting test cases
  - Series solution works well in all cases
  - Costs per Mach number
    - **5-30 times steady state cost**
  - Information about mechanisms provided
    - Detailed and familiar

Future Work





- ROM to predict LCO
  - Based on critical eigenvector
- Demonstrated for Goland wing
- Can now develop a parallel version of this approach
  - Application to aircraft LCO